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8.20 am, Tuesday June 5, 2007

Electrodynamics, Physics 323 Spring 2007

Final exam Instructor: David Cobden

Do not turn this page until the buzzer goes at 8.20. You must hand your exam to me before I leave the room at 10.25.

Attempt all the questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. No books, notes or calculators allowed.

$$\Lambda^{\mu}_{\nu} = \begin{pmatrix} \gamma & -\gamma \nu/c & 0 & 0 \\ -\gamma \nu/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$
$$x'^{\mu} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu} \qquad F'^{\mu\nu} = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} F^{\alpha\beta} \qquad \text{Larmor: } P = \frac{\mu_0 q^2 a^2}{6\pi c}$$
$$E'_{\mu} = E_{\mu} \quad B'_{\mu} = B_{\mu} \qquad E'_{\perp} = \gamma (\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}_{\perp}) \qquad \mathbf{B}'_{\perp} = \gamma (\mathbf{B}_{\perp} - \mathbf{v} \times \mathbf{E}_{\perp}/c^2)$$



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1. [20] For electromagnetic fields in a material, one of Maxwell's equations is

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \qquad (\mathbf{i}).$$

Give the other three Maxwell's equations also in the form appropriate inside materials. Show by substituting into Eq. (i) the constitutive relations (hint: one of them gives **D** in terms of **E** and **P**) that the curl of **B** is the sum of four current terms, and say what gives rise to each term.

2. [8] Now instead rewrite Eq. (i) for the case of a linear medium, as a relation between **B** and **E**, and hence deduce the condition on the conductivity σ and dielectric constant ε_r under which the material can be taken as a good conductor at frequency ω .

3. [8] By thinking of a monochromatic plane wave as a beam of photons of frequency *a*, deduce the relationship between the energy density, the intensity, and the pressure exerted at normal incidence on a perfectly absorbing flat surface.

4. [8] In what commonly occurring situation is the speed of light in a dielectric greater than *c*, and why does this not violate special relativity?

5. [8] Define the 4-potential A^{μ} and the 4-current J^{μ} in terms of V, A, ρ , and J in some frame S.

6. [8] Construct the scalar $A^{\mu}J_{\mu}$ and interpret the result.

7. [10] Show that the covariant equation $\partial_{\nu}\partial^{\nu}A^{\mu} = -\mu_0 J^{\mu}$ gives the correct relations between *V*, **A**, ρ , and **J** in the Lorentz gauge (to satisfy Maxwell's equations).

8. [10] Write down (i) the continuity equation and (ii) the Lorentz gauge condition, first in terms of V, **A**, ρ , and **J**, and then in covariant form using 4-vectors.

9. [16] A particle of mass m and charge q is released from rest some distance from a flat metal surface. Find the power radiated as a function of distance z from the surface.

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10. [8] An electromagnetic wave of frequency ω travels in the *x* direction through the vacuum. It is polarized in the *y* direction, and the amplitude of the electric field is E_0 . Write down the electric and magnetic fields, $\mathbf{E}(x, y, z, t)$ and $\mathbf{B}(x, y, z, t)$.

11. [6] Construct the electromagnetic field tensor $F^{\mu\nu}$ for the wave.

12. [10] This same wave is observed from an inertial system S' moving in the +x direction with speed v relative to the original system S. Write down the equations giving the coordinates in S in terms of the coordinates x', y', z', and t' in S'.

13. [16] Find the electric and magnetic fields in S', either by transforming $F^{\mu\nu}$, or using the direct equations given on the cover page, or otherwise. Express them in terms of the S' coordinates.

14. [8] What is the frequency ω' of the wave in S'? Interpret this result.

15. [8] What is the wavelength λ' of the wave in S'? From ω' and λ' , determine the wave speed in S'.

16. [10] An electron and a positron (rest mass *m*) are travelling at equal speed v = 0.8c and opposite directions towards a collision at the origin. Sketch the electric field lines before they collide.

17. [10] When they collide the particles annihilate. Sketch the electric fields a short time t later, including both the regions inside, outside, and on the surface of a sphere of radius ct centered on the origin. Hence deduce in which directions that radiated intensity is maximum.

18. [8] What is the total energy emitted as photons? (Neglect the energy of the EM fields before the collision).

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19. [10] Two identical particles of charge q and mass m are bound to each other by an attractive central force. The system is set in motion and then left alone in vacuum in zero external field. What kinds of radiation can be emitted (electric or magnetic, dipole or quadrupole), and what modes of their relative motion will generate each type?

20. [5] The Lienard-Wiechert scalar potential of a particle of charge q moving on a trajectory $\mathbf{w}(t)$ is $V(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0 R(1-\hat{\mathbf{R}}.\dot{\mathbf{w}}(t_r)/c)}$, where $\mathbf{R} = \mathbf{r} - \mathbf{w}(t_r)$ and $\dot{\mathbf{w}} = d\mathbf{w}/dt$. Give the equation determining the retarded time t_r .

21. [15] A particle of charge q moves at constant speed u along the x-axis, so $\mathbf{w}(t) = ut \,\hat{\mathbf{x}}$. Determine the scalar potential along the x-axis only.