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Electrodynamics, Physics 323 **Final exam** **8.20 am, Tuesday 10 June 2008**
Spring 2008 Instructor: David Cobden

Do not turn this page until the buzzer goes at 8.20. You must hand your exam to me before I leave the room at 10.25.

Attempt all the questions.

Please write your name on every page and your SID on the first page.

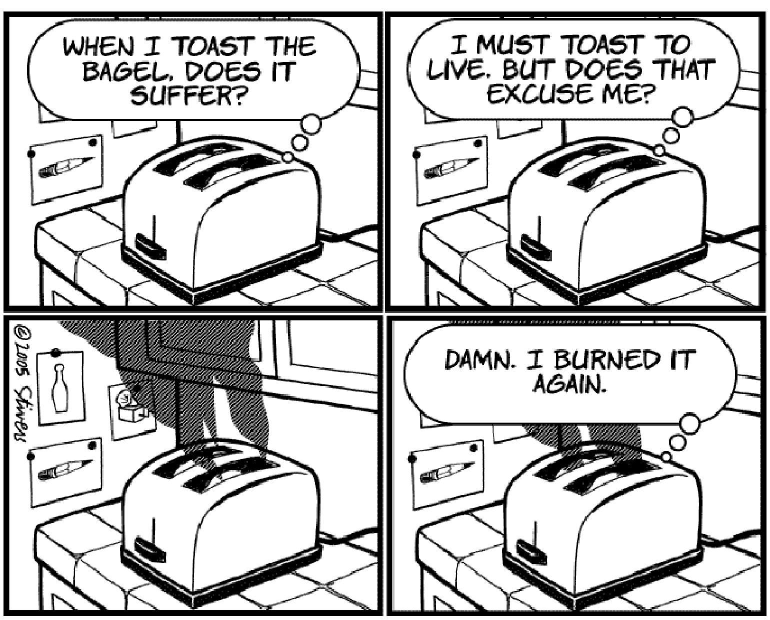
Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. ***No books, notes or calculators are allowed.***

$\nabla \times (\nabla \times \mathbf{X}) = \nabla(\nabla.\mathbf{X}) - \nabla^2\mathbf{X}$

Larmor formula: $P = \frac{\mu_o q^2 a^2}{6\pi c}$



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1. [6] Two of Maxwell's equations in matter are (i) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ and (ii) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_{\mathrm{f}}$. State the others.

2. [15] A nonzero charge density is induced at time $t = 0$ in a material with conductivity σ and permittivity ε. Using continuity of charge and Gauss's law show that the charge density subsequently relaxes exponentially on a timescale $\tau = \varepsilon/\sigma$.

3. [15] Starting from Maxwell's equations (i) and (ii) above and the linear constitutive relations, show that at high enough frequencies the dispersion relation for a monochromatic plane electromagnetic wave of frequency ω in a linear medium can be written $k \approx \frac{\omega}{v}\left(1 + \frac{i}{2\omega\tau}\right)$, where $v = 1/\sqrt{\epsilon\mu}$.

4. [10] Hence find the absorption coefficient (the inverse length scale for decay of power in the wave) in this limit, assuming ε and σ are real (this is actually not valid, but please do it anyway!)

5. [10] A linearly polarized laser beam with frequency ω propagating along the z-axis passes normally through a birefringent slab of thickness a, which has different refractive indices n_x and n_y for electric field polarizations in the x and y directions. For what orientation of the input polarization and what thicknesses will the beam that emerges from the other side be entirely circularly polarized?

6. [15] By applying the boundary conditions for $E_{//}$ and $H_{//}$, derive the Fresnel relation

$$\frac{E_R}{E_I} = \frac{\alpha - \beta}{\alpha + \beta}$$

giving the reflected amplitude E_R for a plane wave of amplitude E_I incident in vacuum on the surface of a dielectric with polarization in the plane of incidence. Here $\alpha = \frac{\cos\theta_T}{\cos\theta_I}$, where θ_I and θ_T are the angles to the normal for the incident and transmitted waves, and $\beta = Z_0/Z_1$, where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{E_I}{H_I}$ is the impedance of free space and Z_1 is the impedance of the dielectric whose refractive index is n.

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7. [10] Express the equation for E_R/E_I in terms of θ_I and n only? (Take $\mu = \mu_0$.)

8. [12] Use the Fresnel relation in Q.6 to deduce the transmission coefficient T (the fraction of incident power transmitted) in terms of α and β.

9. [10] A particle of charge q and mass m moving in a uniform magnetic field of magnitude B has initial velocity u perpendicular to the field axis. Show that it moves in a circular orbit and determine the cyclotron frequency ω_c.

10. [8] Sketch the distribution of radiated power in a plane perpendicular to the orbit. (Note: you need to average over the different orientiations of the acceleration.)

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11. [16] Show that the kinetic energy of the particle decays exponentially due to dipole radiation with a time constant given by

$$\tau = \frac{3\pi m^3 c}{\mu_0 q^4 B^2}.$$

12. [10] What are the components of x^μ, $\frac{\partial}{\partial x^\mu}$, A^μ and J^μ in a particular frame S?

13. [6] State the Lorentz gauge condition and the continuity equation for charge in covariant form.

14. [4] The electromagnetic field tensor is defined as $F^{\mu\nu} = \frac{\partial A^\nu}{\partial x^\mu} - \frac{\partial A^\mu}{\partial x^\nu}$. Show that it is antisymmetric.

15. [10] Show that $F^{01} = \frac{E_x}{c}$ and that $F^{23} = B_x$.

16. [10] Fill out the remaining elements: $F^{\mu\nu} = \begin{pmatrix} . & . & . & . \\ . & . & + & -B_y \\ . & . & . & . \\ . & . & . & . \end{pmatrix}$

17. [8] Using the fact that $F^{\mu\nu}$ is a 4-tensor, or otherwise, show that $E^2 - c^2B^2$ is invariant under Lorentz transformations.

18. [8] In frame S a static infinite line charge of linear density η lies along the *z*-axis. Find the electric field around it, using old tricks (including cylindrical coordinates).

19. [6] Another inertial frame S´ moves at constant velocity *v* along the *z*-axis relative to S. Find the electric field in S´ using the same old tricks (remember Lorentz contraction).

20. [10] Find the magnetic field in S´ using Ampere's law. Show that $E^2 - c^2B^2$ is indeed invariant in this particular case.