

The Faraday Effect and Dispersion in Liquids

Introduction

The *Faraday effect*, which is the rotation of the polarization of light due to an applied magnetic field, and *dispersion*, which is the variation in refractive index as a function of wavelength of light, are related to each other through their basis in the phenomenon of absorption.

Absorption is best understood as a kind of resonance response: when a light beam of a particular frequency (i.e., color) impinges on a material that has a resonance near that frequency, some energy may be removed from the beam (absorbed), and the light which propagates will have its speed altered, usually decreased (refracted). Both of these features of absorption may be expressed by a “complex” (in the sense of functions of a complex variable) index of refraction n . The index of refraction is a measure of a material’s polarizability, that is, its response to an applied electric field. This response is frequency dependent because the electrons in atoms and molecules absorb and emit photons—electromagnetic field quanta—at frequencies corresponding to energy differences in their stationary states.

The degree to which refraction and absorption occurs tends to increase as the frequency of the beam gets closer to the resonant frequency. Resonance response has a *width*: a (possibly broad) feature covering a range of frequencies. Many materials that are optically “clear” in the visible region (such as water and glass) have resonances in the ultraviolet. Thus, one sees that the index of refraction n for these materials goes up as the wavelength λ goes down. Since this trend holds for many “clear” materials, it has been given the name *normal dispersion*. But for light of a color very close to a particular resonance, the opposite trend is observed: n decreases as λ decreases. This is known as *anomalous dispersion*, and it is the phenomena underpinning much interesting recent work on superluminal and strongly subluminal light propagation (see Hecht, pp. 296–302).

The absorption frequencies of a material depend upon the quantum states of the molecules which make it up: a photon of energy $h\nu$ may be absorbed when it satisfies Bohr’s famous relation $h\nu = E_f - E_i$, where E_f and E_i are the energies of the final and initial states. When this same material is placed in a magnetic field, the initial and final state energies may be changed, depending on the angular momentum (and consequent magnetic moment) associated with each state. The resulting shift in resonant frequencies is known as the *Zeeman effect*. The Zeeman effect is usually discussed in the context of emission spectra of gases, but the general idea of level shifts due to a magnetic field is the same.

An important concept associated with the theory of the Zeeman effect is the lifting of degeneracy by the applied field. Briefly, “degeneracy” is the term applied whenever states that may be labeled by a distinct set of quantum numbers (for example, particular $n, \ell, m, \dots$ for hydrogen) have the same energy. Under the influence of a magnetic field $\vec{B}$, the energies of the degenerate states become distinct; the different magnetic moments $\vec{\mu}_s$ of each state, which depend upon the different angular-momentum numbers, interact with the field, and give each state an additional energy

$$\Delta E_s = -\vec{\mu}_s \cdot \vec{B} . \quad (1)$$

The simplest example is the spin-1/2 or two-state system. In this case, the energy of a given state E_s is split into two energies $E_s + \mu_s B$ and $E_s - \mu_s B$. It follows that the resonant frequencies involving transitions with this state will also shift by amounts proportional to $\pm \mu_s B / h$.

But this is not the whole story; if it were, one would see no Faraday effect. Along with the

requirement for absorption that the photon energy match the difference in state energies, there is an additional requirement of conservation of angular momentum in the interaction between the photon and the molecule. This means that photons with different polarization states will interact with different resonances, which in turn means that light of different polarizations (but same wavelength) will move through the medium at different speeds: the lifting of the degeneracy by the applied magnetic field produces *birefringence*.

More specifically, it produces *circular birefringence*, where a single index of refraction n must be replaced with two indices: $n_{\mathcal{L}}$ for left-circularly polarized light and $n_{\mathcal{R}}$ for right-circularly polarized light. The effect of circular birefringence is to rotate the plane of polarization of linear polarized light. This may be understood in the following way. The electric field vector of linearly polarized light lies along a fixed axis. At any point in space, this vector grows, shrinks, reverses sign, then grows in the opposite direction with a frequency equal to the frequency of the light. One can imagine (or “decompose”) this vector as the sum of two component vectors of fixed magnitude each rotating in opposite directions about the axis of propagation. Light whose electric field vectors rotate about the axis of propagation is defined as circularly polarized. Each circular component of the linearly polarized light which enters the birefringent medium is subject to a different index of refraction, so as the light propagates the phase relationship between them shifts, and when the light exits the medium, the electric field vector will lie along a different axis.

Here’s an illustrative analogy: imagine two rowers sitting side by side in a boat. If each rows at the same rate (frequency) and pulls the same length of water (wavelength), the boat will travel straight. But if one rower begins to pull a longer length of water, the boat will turn. Similarly, in the birefringent material, the different wavelengths of the left- and right-circular components (since $\lambda = \lambda_0/n$) cause the electric field vector to twist about the axis of propagation. The net angle of rotation ϕ depends on the difference in phase of the two components as the light passes through the medium:

$$\phi = \frac{\pi L}{\lambda_0} (n_{\mathcal{L}} - n_{\mathcal{R}}) , \quad (2)$$

where L is the distance through the medium and λ_0 is the wavelength of light in a vacuum. (See Hecht, pp. 360–362 for a derivation and useful illustrations.)

To see how the index of refraction n changes as a function of an applied magnetic field, we need to examine the physics of absorption more closely. Hecht derives a model of absorption based upon a classical model of the atom: electrons and nuclei as an oscillating system forced by an applied field. (See pp. 67–73.) The resulting formula for the dielectric constant K may also be found from the methods of quantum mechanics. For purposes of discussion, we’ll consider a simple form of this equation, based on a single resonance, which exhibits the essential features described above:

$$K(\nu) = n^2(\nu) = 1 + \left(\frac{Nq_e^2}{4\pi^2\epsilon_0 m_e} \right) \frac{1}{\nu_0^2 - \nu^2 + i\Gamma\nu} , \quad (3)$$

where N is the number of molecules per unit volume, q_e and m_e are the charge and mass of an electron, and ϵ_0 is the permittivity of the vacuum. The imaginary term in the denominator denotes a damping of the resonance, with a damping coefficient Γ . Note that the refractive index n (here expressed in terms of its square, since the theory gives the dielectric constant K , and $n = \sqrt{K}$) is complex. This reflects the fact that when used with the complex form of a propagating wave, the imaginary part gives the wave’s attenuation due to absorption, and the real part gives the wave’s velocity as changed by the index of refraction.

As an example, Eq. (3) is used to generate a curve of n vs. λ plotted in Fig. 1 for a resonance in the near ultraviolet (300 nm). The plot shows normal dispersion— n decreasing with increasing λ

for wavelengths in the visible region, 400–700nm. The center of the resonance is where the steep center line crosses zero (note the anomalous dispersion here, $dn/d\lambda > 0$), and this is where the incoming light has frequency ν_0 .

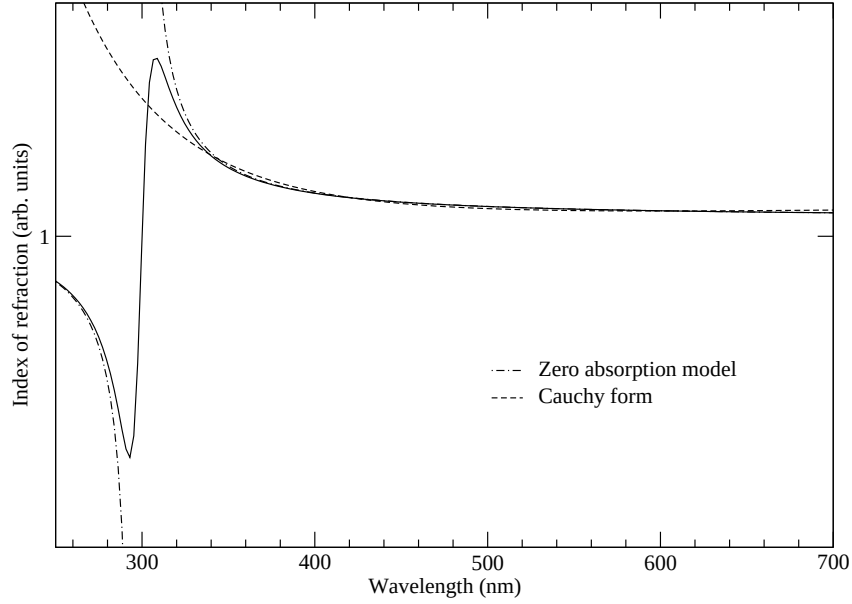


Figure 1: Real part of the refractive index as calculated from Eq. (3) for a model material having one resonance. Two approximations to the model are also shown: the “Cauchy form” is Eq. (11) and the “Zero absorption model” is Eq. (12).

Consider how the figure would change if our model system were placed in a magnetic field. Assume, for the purpose of argument, that the resonance is between a $1S_0$ and $2P_1$ state. The $1S_0$ state is non-degenerate, so it is unchanged by the field, but the $2P_1$ state has three sublevels, corresponding to $m_\ell = -1, 0, +1$, whose degeneracy is lifted by the field. The additional energy causes the single resonance at ν_0 to be split into two resonances with their centers at $\nu_0 + \mu_B B/h$ and $\nu_0 - \mu_B B/h$, where μ_B is the Bohr magneton (i.e., the magnetic moment of a semiclassical electron), $\mu_B = q_e h / (4\pi m_e)$. The reason there are two resonances is that only transitions involving $\Delta m_\ell = \pm 1$ are allowed. So the single resonance curve breaks into two curves, one for each of the two circular polarizations. To the first approximation, the effect is merely to displace the original curve along the x axis. This is shown in Fig. 2, where we have exaggerated the amount of splitting to make the two curves easy to see.

Note that for wavelengths in the visible region, say 500 nm, there is clearly a difference between $n_{\mathcal{L}}$ and $n_{\mathcal{R}}$. Note also that as the wavelength gets shorter, this difference increases. Thus, *the model predicts that the Faraday effect should be larger for blue light than for green light.*

To give a quantitative estimate of the effect for wavelengths in the visible region, note that if you translate the curve along the horizontal by a small amount $\Delta\lambda$, n changes by an amount approximately equal to $-dn/d\lambda \times \Delta\lambda$. Since the shift is opposite for the two polarizations, we can see that

$$n_{\mathcal{L}} - n_{\mathcal{R}} = -2\Delta\lambda \frac{dn}{d\lambda}. \quad (4)$$

Since $\Delta\lambda$ is due to the energy shift given by Eq. (1), we find by differentiation of the relations

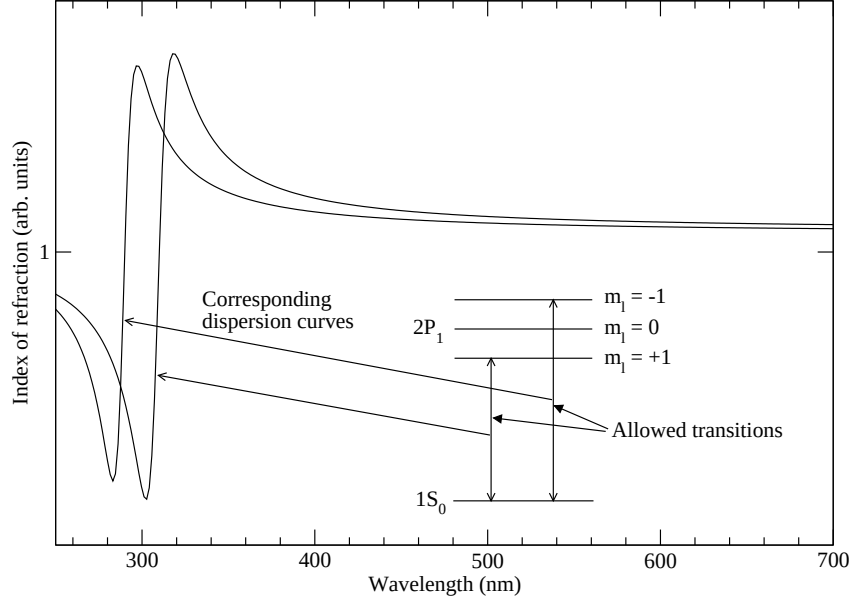


Figure 2: Refractive indices (real part) for the model system under the influence of a very large magnetic field. Note that the shifts in resonant wavelengths due to the magnetic field cause the indices to differ even well away from the resonant center near 300 nm

$\nu = c/\lambda$ and $E = h\nu$ that

$$\Delta\lambda = -\frac{c}{\lambda^2}\Delta\nu = -\frac{c}{h\lambda^2}\Delta E. \quad (5)$$

In our model system with the $1S_0 - 2P_1$ transition, $\Delta E = \mu_B B/h = [q_e/(4\pi m_e)]B$, we find, from Eqs. (2), (4) and (5),

$$\phi = \left(\frac{q_e}{2m_e c}\right) \lambda \frac{dn}{d\lambda} B L. \quad (6)$$

This result was derived by Becquerel back in 1897 from purely classical arguments to explain the Faraday and Zeeman effects.

Our more modern treatment arrives at the same result mainly because we have assumed a particularly simple model of material which has a single resonance between a singlet and triplet state. The main change that a more realistic model would produce is to modify the form of ΔE , as the absorption spectrum should have many more terms, and one may have transitions that involve level splitting in both the lower and higher energy states. (The interested student should study Ref. 4 to see how a more complete treatment would look.) In spite of its simplicity, the Becquerel result comes very close to the experimental result, as you should be able to discover by doing this experiment.

The Faraday effect was, indeed, discovered by Faraday in an experiment in 1845. He found a simple relationship between the rotation of the plane of polarization of light propagating in a medium and an applied magnetic field:

$$\phi = VBL, \quad (7)$$

where ϕ , B , and L are as defined previously. The constant V is known as the “Verdet constant”, and it typically depends strongly on λ , as well as temperature, density, and other material properties

of the medium. Comparison of Eq. (7) with Eq. (6) gives the classical prediction for the Verdet constant, V_C :

$$V_C = \left(\frac{q_e}{2m_e c} \right) \lambda \frac{dn}{d\lambda} = 293.34 \frac{\text{rad}}{\text{T m}} \times \lambda \frac{dn}{d\lambda}. \quad (8)$$

In practice, one finds that measured Verdet constants follow this form, but with a numerical constant somewhat less than the Becquerel result. This is handled by invoking a correction factor called the magneto-optic constant, γ , and writing $V = \gamma V_C$. One of your tasks in this experiment will be to estimate the value of γ for the material used.

A final note before we turn to the experiment: The alert reader will have noticed that the Faraday effect should be strongly enhanced for light frequencies very near resonances, since the difference between $n_{\mathcal{L}}$ and $n_{\mathcal{R}}$ becomes quite pronounced there. Indeed, this has been observed. See Ref. 5 for a fascinating and accessible study of the effect in rubidium vapor near the resonance at 780 nm. The paper also gives a good deal of background on the effect generally and fills in steps skipped over in this introduction.

References:

1. Anne P. Thorne, *Spectrophysics* (2nd ed.), pp. 134-143 prism spectrometers.
2. Hecht, *Optics* (5th ed.), Section 8.11.2 The Faraday effect; Section 8.10 Optical activity; Section 3.5.1, Dispersion; Section 5.5.1, Dispersing Prisms.
3. F.L. Pedrotti and P. Bandetti, *Faraday rotation in the undergraduate advanced laboratory*, Am. J. Phys. **58**, 542-5 (June 1990); posted on the class website.
4. R. Serber, *The theory of the Faraday effect in molecules*, Phys. Rev. **41**, 489–506 (1932); posted on the class website
5. D. A. Van Baak, *Resonant Faraday rotation as a probe of atomic dispersion*, Am. J. Phys. **64** (6), 724–735 (1996); posted on the class website.

1 Faraday Effect Measurements

The material used in this experiment is cinnamic acid, ethyl ester-*(trans)*; $\text{C}_6\text{H}_5\text{CH}=\text{CHCO}_2\text{C}_2\text{H}_5$. This liquid, also known as ethyl cinnamate, has a relatively high index of refraction in the visible, and also exhibits substantial dispersion. We will first measure the Faraday rotation in cinnamic acid to obtain a value for the Verdet constant, and then take a series of measurements with a prism spectrometer from which the dispersion can be derived.

Light from a mercury/cadmium (Hg/Cd) lamp is transmitted through a column of cinnamic acid contained in a glass tube. The tube is coaxial with a solenoid coil as shown in Fig. 3. At the entrance end the light is plane polarized by a Glan-Thomson polarizer (the fixed polarizer). At the exit end, the plane of polarization is determined by rotating a second Glan-Thomson polarizer (the analyzer) to obtain extinction of the transmitted light.

You are to measure Faraday rotation in cinnamic acid at two different wavelengths, the cadmium blue line at $\lambda = 480$ nm and the mercury green line at $\lambda = 546$ nm. Separate filters at each of these wavelengths can be moved in and out of the beam path. For the green line, the polarizer angle at which extinction occurs can be easily determined. For the fainter blue line this angle cannot

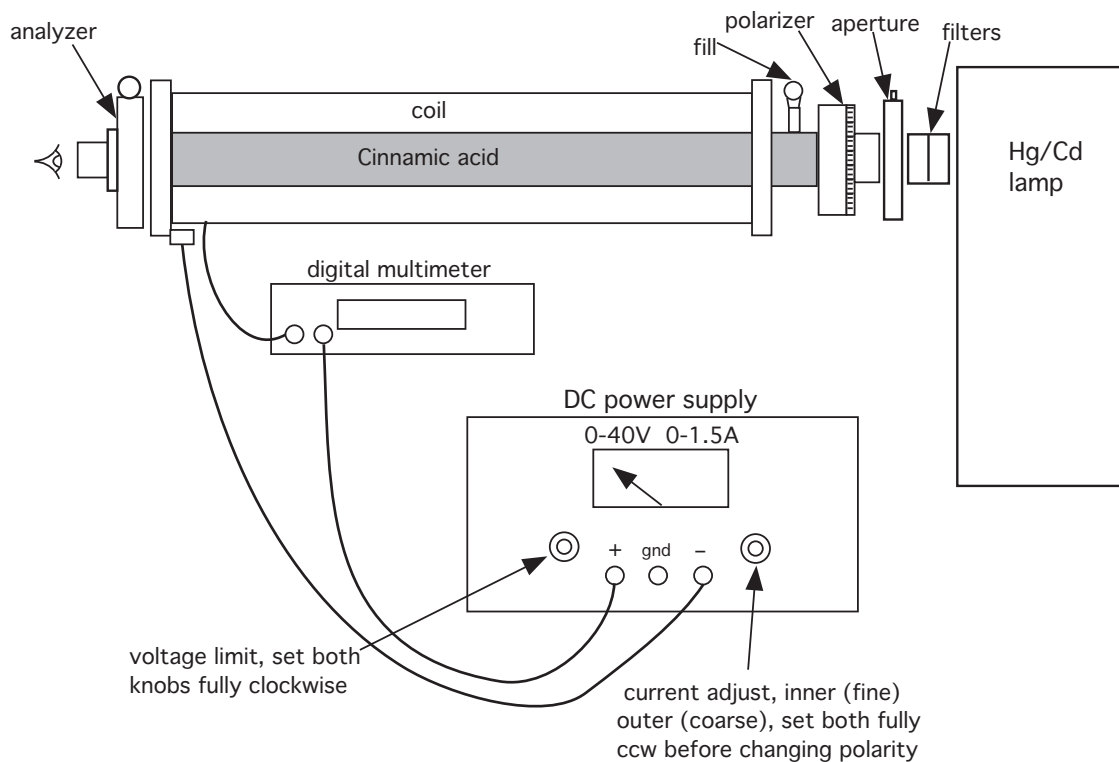


Figure 3: Setup for Faraday effect measurements.

be determined as easily, and there will be a greater uncertainty associated with the extinction measurements at this wavelength. Reducing the ambient light and allowing the eyes to dark-adapt will help with the blue measurements. Hint: take your first set of measurements with the mercury green line. This will help you get used to the apparatus. Then when you take measurements for the blue line, you can use your more-sensitive peripheral vision to help see the extinction of the light.

Place the Hg/Cd lamp at the far end of the Faraday rotation apparatus and adjust the height of the lamp as necessary so that the bright part of the bulb is at the same level as the axis of the coil. Turn on the lamp and allow several minutes for it to warm up and achieve full brightness. Flip the blue filter up out of the beam path and the green filter into the beam path if it is not already there.

Before turning on the solenoid power supply, set the micrometer handle on the polarizer rotator to 8.50 mm, midway between the travel limits of 4.00 mm and 13.00 mm. (Note: micrometer readings are **not** used to record data; angles are read from the $360^\circ + \text{vernier scales}$.) Loosen the clamping knob on the rotator. Look through the polarizer at the central dot and rotate the barrel (marked with blue tape) of the polarizer holder by hand to coarse adjust for extinction of the central dot. Gently tighten the clamping knob. Use the micrometer to fine adjust the polarizer angle for maximum extinction and record the angle of the polarizer. Passing back and forth through extinction will give you a better feeling for achieving this condition. When reading the angle of the polarizer, use the vernier scale to the left of the 0. Each division on this scale represents 5 minutes of arc.

Measure the angle of rotation for a series of coil currents, both positive and negative. The maximum rotation possible given the micrometer travel limits of 4.00 mm and 13.00 mm is approximately $+7^\circ$ and -7° from the 8.50 mm position.

CAUTION: The coil power supply is capable of delivering currents in excess of the steady-state capacity of the coil. High currents will heat the coil and can damage it if left on for long periods of time. For coil currents above 1 ampere, minimize the time the current is on. **Do not exceed a coil current of 1.5 amperes** (which is more than is needed for a rotation of 7°).

2 Dispersion Measurements

The value of the dispersion for cinnamic acid can be derived from measurements of the index of refraction at a series of different wavelengths. The index of refraction at a particular wavelength is derived from a measurement of the angle of minimum deviation at each wavelength as light of that wavelength passes through a prism spectrometer (see reference 2).

The prism in the spectrometer is a hollow glass container with a cross section very close to that of an equilateral triangle. Filled with cinnamic acid, it becomes the dispersing element for the spectrometer. The vertex angle of the prism is an important parameter and has been carefully measured at $60^\circ \pm 5$ minutes.

The Hg/Cd lamp is a convenient light source as it has a large number of bright spectral lines in the visible. The seven lines given below are the brightest and are all readily observed. Other, fainter lines are visible as well.

λ (nm)	Element	Color
643.85	Cd	deep red
579.07, 576.96	Hg	bright yellow doublet
546.08	Hg	bright green
508.58	Cd	turquoise
479.99	Cd	light blue
467.81	Cd	dark blue
435.83	Hg	bright violet-blue
404.66	Hg	dark violet

Table 1: Dominant colors of the Hg/Cd lamp and their wavelengths.

2.1 Using the Spectrometer

Note: the knobs on the spectrometer marked white do not in general need to be adjusted and should be left in their present position.

Please handle the prism assembly carefully. The prism is made of glass, is very fragile and costs \$400!!

Place the Hg/Cd lamp in front of the slit on the collimating telescope. If the prism assembly is in place on the spectrometer, remove it: carefully lift it off by holding the heavy aluminum disk and pulling straight up. Set it on the table for now.

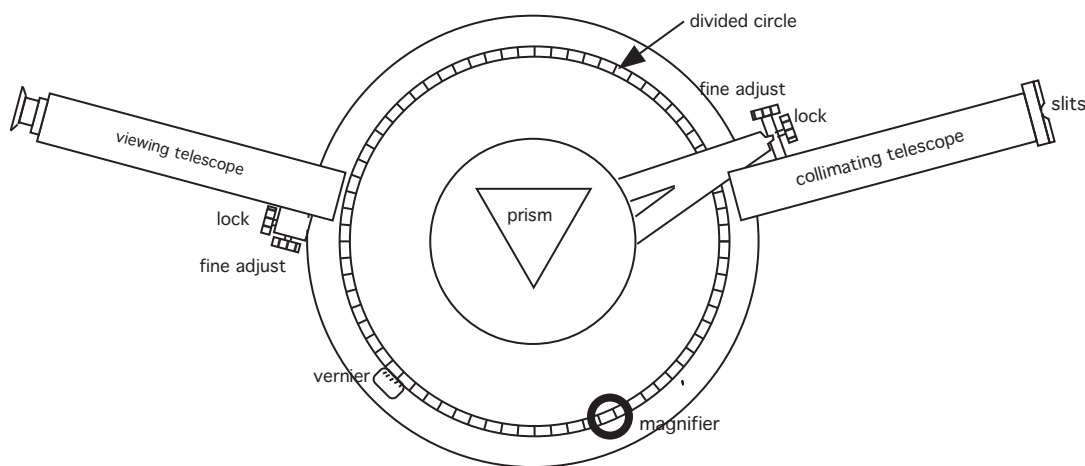


Figure 4: Spectrometer used for measuring dispersion of cinnamic acid.

With the red knob loose, move the viewing telescope so as to align the cross hairs with the undeviated image of the slit. Adjust the focus of the slit image with the blue knob. (If the cross hairs are out of focus, move the eyepiece in/out until they are in focus.) Adjust the slit (the knurled ring at the end of the collimating telescope) so that the image of the slit is narrow, but still readily observable. Clamp the coarse adjust for the viewing telescope (red knob) and make any necessary adjustment to center the cross hairs on the slit image with the fine adjust (gray) knob.

With the undeviated beam aligned in the cross hairs, check that the orange knob is loose, and by hand adjust the inner circle for a reading of 0 degrees. The 0 degree reading obtains when the 0 on the inner circle (which goes from 0 to 360 degrees) is exactly opposite the 0 on the vernier scale on the left side of the spectrometer. After moving the inner circle to near 0 degrees, tighten the orange knob and make any necessary adjustment to achieve the 0 degree reading with the fine adjust (yellow) knob. The inner circle will remain in this position for the rest of the experiment.

Carefully place the prism assembly on the stand; the locator pin goes through the hole in the metal base supporting the prism, and the strip of yellow tape on the opposite side of the base should line up with the strip of yellow tape on the stand. Loosen the stand clamp (purple knob) so the stand rotates freely. To find the emission lines, position the purple knob at about 8 o'clock (collimating telescope is at 12 o'clock), and look directly into the prism on the right side of the viewing telescope, at a position between 4 and 5 o'clock. Rotate the stand with the purple knob until the lines come into view.

Now unclamp the viewing telescope (loosen red knob), move it to the right and look through it to find the emission lines, again between 4 and 5 o'clock. The focusing will need to be adjusted (blue knob) to bring the lines into best focus. By moving your eye to the side while looking through the telescope, you may find the lines to appear sharper than when viewed straight on. Rotate the prism stand and then move the viewing telescope as necessary to keep the lines in view. While rotating the stand, the lines will first travel in the same direction until the angle of minimum deviation is reached, whereupon the direction of travel will be reversed for the same sense of prism rotation.

At the angle of minimum deviation for each line, set the cross hairs of the telescope on the line and measure the angle (see below). **Note: the prism must be adjusted for minimum deviation at each wavelength when measuring the deviation angle.** Make your measurements carefully,

as you need as much precision as possible in order to derive an accurate number for the dispersion.

A comment on using the vernier scale to read the angle:

The inner circle is marked from 0 to 360 degrees. Between adjacent degree marks are six divisions, each representing 10 minutes of arc. The vernier scale has 10 major divisions, each representing 1 minute of arc, with three minor divisions, each representing 20 seconds of arc, between the major divisions. Here is an example of how to read an angle: suppose the 0 on the vernier is between the second (20 minute) and third (30 minute) marks going from 315 to 316 degrees, and that the first minor division past the seventh major division on the vernier scale exactly lines up with a mark on the inner scale. 7 minutes and 20 seconds of arc are added to 315 degrees and 20 minutes of arc for an angle reading of 315 degrees, 27 minutes, 20 seconds.

3 Data Analysis

3.1 Faraday effect for the cadmium blue line and mercury green line

The magnetic field produced by a wire in any shape is directly proportional to the current in the wire (the Biot-Savart law). Because of this, the Faraday rotation angle ϕ should be linearly dependent on the current. Plot your data and fit straight lines to the data for the angle of rotation ϕ versus coil current i for the two sets of data. In order to find a value for the Verdet constant, it is necessary to convert coil current to magnetic field and determine the average value of $\vec{B}$ over the tube containing the cinnamic acid.

As a first approximation, calculate the average field by assuming that the simple result for an infinite solenoid, $B = \mu_0 Ni$, where N is the number of turns per unit length, holds. The solenoid has a 50.0 cm length and contains 7349 turns. Use this result for B and the slope $d\phi/di$ from your data to obtain V . Hint: differentiate both sides of Eq. (7) with respect to i .

The approximation causes the value of V to be underestimated because it overestimates the magnitude of the average field. In a real solenoid the field falls off near the ends, and our tube of cinnamic acid sticks out of one end of the solenoid. A better value of V can be found by using a direct measurement of the field as a function of position for a given current. In this case, the average value B of the field would be found by integrating the measurement along the length of the cinnamic acid and then dividing by the length:

$$B = \frac{1}{L} \int_L \vec{B} \cdot d\vec{l} \quad (9)$$

The value of the field along the axis of the coil has been carefully mapped for a current of 1.000 amperes and is shown in Fig. 5. The column of liquid starts at $x = 20.0$ mm, and ends at $x = 566.0$ mm. The integral $\int_L \vec{B} \cdot d\vec{l}$ has been calculated and found to be

$$8.887 \times 10^{-3} \text{ Tm/A.}$$

You may confirm this value by studying the Excel spreadsheet available on the class website.

Use the improved value of B from the measurement at 1.000 amp to recalculate V for the blue and green wavelengths. Question to ponder: Why is it valid to use the field measured at only one value of i to obtain V ? The answer is given in this section.

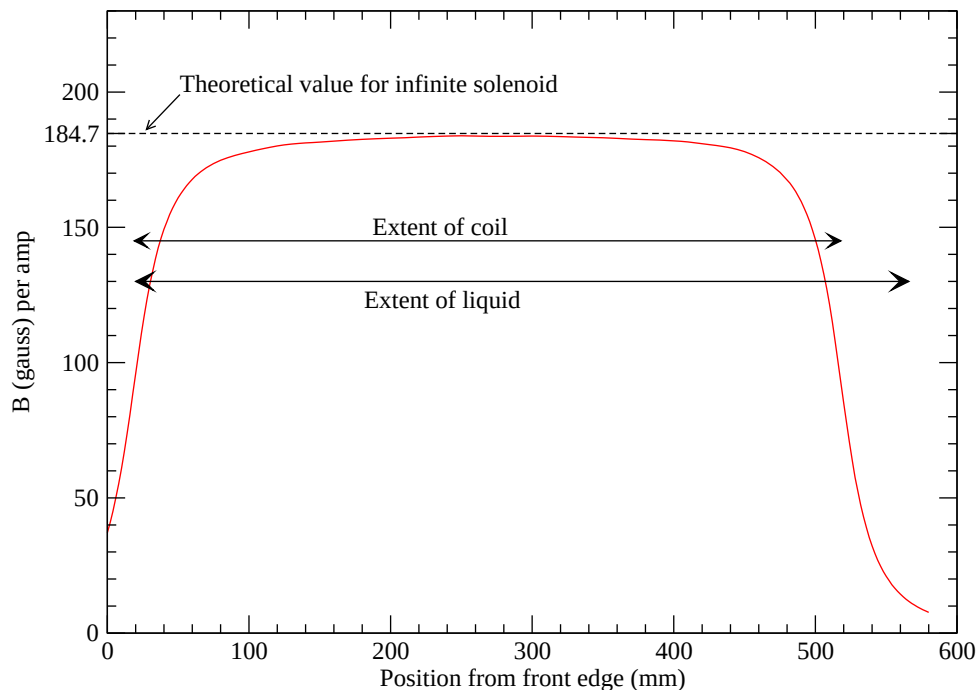


Figure 5: Plot of the axial field of the solenoid used in the Faraday effect measurement, as measured by a gaussmeter from the front edge of the coil assembly. The double-headed arrows show the extent of the coil windings and the cinnamic acid liquid. The theoretical value for the infinite solenoid is calculated from $\mu_0 Ni$ where $i = 1$ A.

3.2 Dispersion of cinnamic acid at Cd blue line and Hg green line wavelengths

From your measurements of the angle of minimum deviation for the different wavelengths λ of Hg/Cd light, calculate the index of refraction $n(\lambda)$ by using the result for dispersing prisms given in Hecht, Section 5.5. The apex angle of our prism is 60° , which will allow you to simplify the equation somewhat before plugging in your numbers. Don't forget to convert your readings to the deviation angle—the angle between the incoming and outgoing beam—and correctly convert your degrees, minutes and seconds to radians!

Plot your values of $n(\lambda)$ in two ways: as n vs. λ and as n vs. $1/\lambda^2$. In the first plot, you should notice that the shape of the curve is similar to the curve in Fig. 1 in the visible region, that is, your material exhibits normal dispersion. In the second plot, fit the data to a straight line using the relationship

$$n = a + b \left(\frac{1}{\lambda^2} \right), \quad (10)$$

which, with some effort, can be derived from the dispersion equation (3) by an expansion assuming $\nu \ll \nu_0$. This form of the dispersion equation is called the “Cauchy” form, and is shown in Fig. 1.

From the fit parameters a and b determine $\lambda \frac{dn}{d\lambda}$ at the values of λ corresponding to the cadmium blue line (480 nm) and the mercury green line (546 nm).

3.2.1 Optional analyses

Alternate analysis of dispersion. You may notice that the equation above does not fit your dispersion data exceptionally well. You can get a much better fit by adding another term, but the problem would no longer be a linear fit. If you have access to a more powerful fitting program that can handle non-linear or polynomial fits, try fitting the dispersion data to

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} , \quad (11)$$

and then extracting the values of $\lambda \frac{dn}{d\lambda}$ from these fit constants.

Estimate of resonance wavelength. As discussed in Hecht (p. 70), in the approximation of a single resonance and negligible absorption (i.e., $\Gamma = 0$) in the dispersion equation, Eq. (3) may be rewritten in the form

$$\frac{1}{n^2 - 1} = \frac{C}{\lambda^2} - \frac{C}{\lambda_0^2} , \quad (12)$$

where λ_0 is the resonance wavelength. Transform your data to plot $1/(n^2 - 1)$ vs. $1/\lambda^2$. From the slope and intercept of a line fitted to the result, estimate the value of λ_0 . Equation (12) is shown in Fig. 1.

3.3 Combining the Dispersion and Faraday Effect Measurements

As discussed in the theoretical introduction, the Verdet constant is expected to obey the form

$$V = \gamma \frac{e}{2mc} \lambda \frac{dn}{d\lambda} .$$

Where γ is the magneto-optical constant, typically less than 1. Use your analyses above to calculate the ratio of

$$\frac{V}{\lambda \frac{dn}{d\lambda}}$$

for both the blue and green lines. Compare the values to see how well they match, and discuss possible reasons for their difference, focusing on the effects of *experimental* factors (i.e., experimental uncertainty, analytical approximations used to reduce the data).

Finally, calculate a value for γ , and discuss. Does it seem reasonable? Previous measurements on a variety of materials have found values of γ to lie between 0 and 1.

4 Postscript – A Practical Application

Although the free electron (i.e., Becquerel) model does not provide a good quantitative description of the Faraday effect, it is useful in providing qualitative insight into some of the features of this effect. According to the model, the index of refraction for a given sense of circular polarization increases or decreases according to the direction of rotation of the electric field about an axis parallel to the external magnetic ($\vec{B}$) field for that sense of polarization. For light propagating parallel to

$\vec{B}$, the index of refraction for right circularly polarized light ($\vec{E}$ appears to rotate clockwise as light comes toward observer) increases as the magnetic field increases, and the index decreases for left circularly polarized light propagating parallel to $\vec{B}$. Now consider light propagating in the opposite direction, i.e., anti-parallel to $\vec{B}$ and away from the observer. The electric field $\vec{E}$ now appears to rotate clockwise for left circularly polarized light; the index of refraction increases for this polarization and decreases for right circularly polarized light. The net effect is that the rotation of the plane of polarization does not reverse as the direction of light traversing the material is reversed, but continues to rotate in the same direction.

In many optics setups (e.g. Michelson and Fabry-Perot interferometers) the desired beam alignment often requires that the light reflect back on itself. A beam reflecting back into a laser can adversely affect the laser operation, and in some situations it is necessary to prevent this from occurring. Question: how can you use the Faraday effect to construct a device that passes light in one direction only? Such a one-way light valve is called an isolator.

Finally, it is to be noted that in materials which exhibit natural optical activity (plane of polarization rotates with no magnetic field), the rotation of the plane of polarization reverses when the propagation direction of the light is reversed, i.e., the light exactly retraces its path if reflected back through the material. Natural optical activity results from a helix-like structure in the material, and this structure has the same handedness independent of which direction you look at it. It is like a nut on a screw—the nut rotates one way to move one direction along the screw, and rotates the opposite way to move in the other direction. The same symmetry does not hold in the Faraday effect, with the result that the plane of polarization rotates the same direction independent of which way the light propagates relative to the direction of the magnetic field.

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