Reflection of Light at an Air/Dielectric Interface

Introduction

The reflection of light from a dielectric surface depends strongly on the angle of incidence of the light and the direction of polarization. One practical application of this is Polaroid sunglasses which exploit the fact that sunlight reflected from the highway is strongly polarized. The Polaroid film in the glasses is oriented (polarization axis vertical) so as to eliminate this glare from a driver's eyes. On a more quantitative level, Maxwell's equations lead to expressions known as the Fresnel equations, which express the intensity of light reflected from a dielectric surface as a function of angle of incidence, the dielectric constants of the media (air and BK-7 glass in our case) on either side of the surface, and direction of polarization. Fresnel derived the equations named after him long before Maxwell by considering the behavior of waves at an interface in elastic material.

If a thin dielectric layer with an index of refraction different from the underlying glass is deposited on the surface of the glass, the reflected light intensity can be altered dramatically. The most common example of this is the anti-reflection coatings found on camera lenses and eyeglasses. The coating layer is designed so that most of the light is transmitted and very little is reflected. If the dielectric layer is made thicker (how much?) the amount of reflected light can be increased dramatically. This property is exploited in producing highly reflective mirror surfaces. A second prism of BK-7 glass with a coating of MgF_2 is available to study the effects of an anti-reflection coating. The analysis for the layered surface is much more complex than for a single surface.

You will need to study the material in your text in order to understand how the Fresnel equations are derived and how they relate to the underlying wave equations. Most of this lab write-up concerns the apparatus and data collection only.

References

1. Hecht, *Optics* (5th ed.), Section 4.6, mainly 4.6.2, The Fresnel Equations and 4.6.3, Interpretation of the Fresnel Equations. Also see section 8.6, Polarization by Reflection. Optional: Section 9.7 Applications of Single and Multilayer Films.

1 Overview of Apparatus and Procedure

Turn on the benchtop ammeter. On the power supply below it, make sure the current knob is turned *all the way down, counterclockwise*, and the voltage knob is turned all the way up, clockwise. (This puts the supply into a "current source" configuration.) Then turn on the power supply for the light source and slowly turn the current knob up to set the current at about 4.3 amps, as read on the digital meter. The lamp current may decrease as the lamp warms up, but you can start using it right away. By the time you are ready to take data the lamp intensity should be quite stable (typically about 4.2–4.4 amps). **Do NOT exceed 4.6 amps of current, or you will blow the fuse for the lamp.**

A divided-circle spectrometer, normally used with a prism or grating as a spectrum-dispersing element, has been adapted for the purposes of this experiment. The principal parts of the spectrometer are a fixed collimating telescope which collimates the incident light beam, a rotating prism support, a movable viewing telescope, and a divided circle. The latter is a precision angle-measuring device, which, on the instrument in the lab, can be read to 10 seconds of arc (with good eyes and some imagination), allowing for very accurate measurement of the viewing telescope position.

There are two prisms which can be studied in this experiment. You should mainly study the *uncoated* prism, as that is the prism which the single-interface Fresnel equations model. **Be sure to measure the uncoated prism first.** Measurements from the coated prism are optional and much more difficult to analyze quantitatively. You need only take measurements from this prism in order to see the qualitative effects of the coating. The hypotenuse of the coated prism has a layer of magnesium fluoride (MgF_2) of thickness 996 ± 20 Å, which corresponds to an optical thickness of $\lambda/4$ for a (vacuum) wavelength of 550 nm. A backing of black paint has been applied to the non-hypotenuse sides of both prisms to reduce reflections from these surfaces.

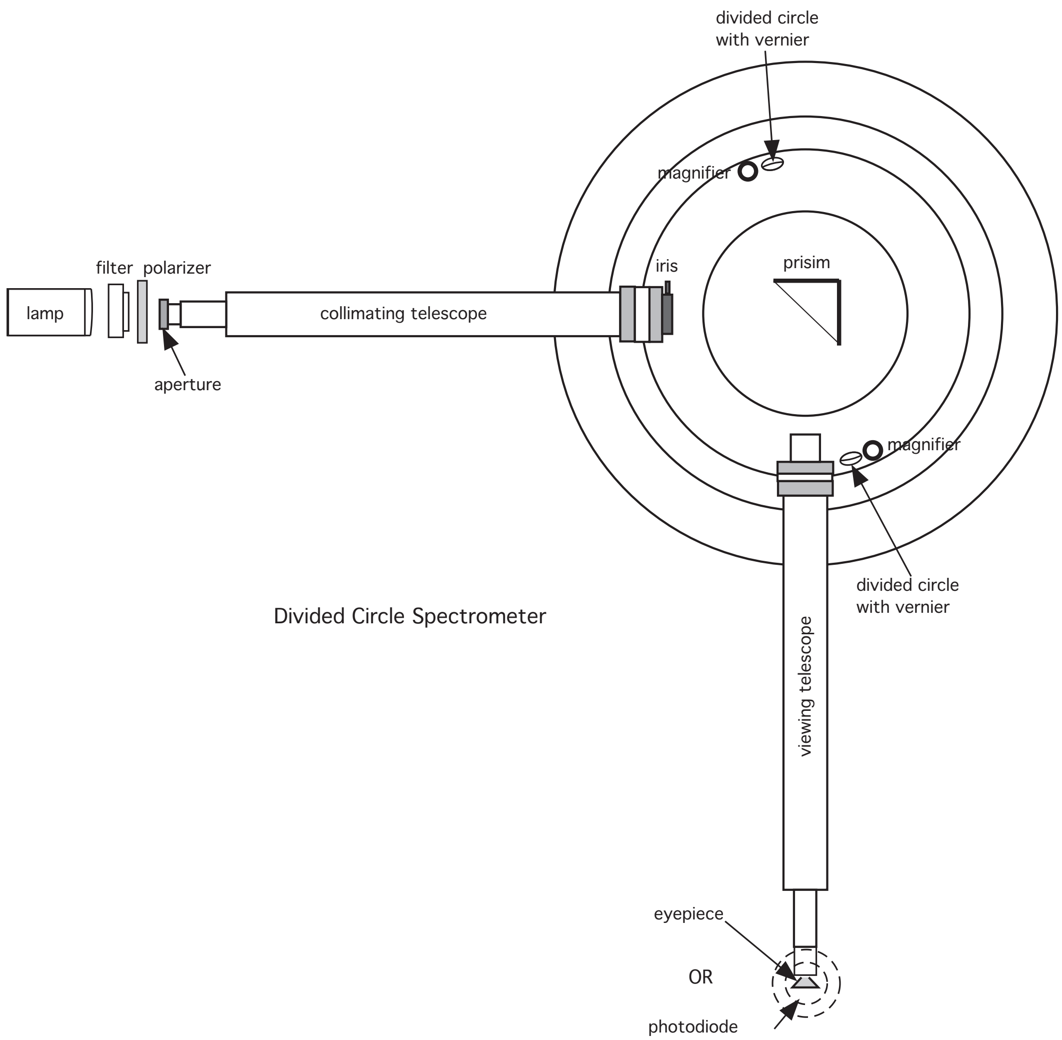

Figure 1: Spectrometer used for reflection experiment

Unpolarized white light is generated by the incandescent lamp which is positioned along the axis of the collimating telescope. Filters for different wavelengths, λ= 541 nm (green), and λ= 648 nm (red), can be flipped in and out of the beam path. Following the filters is a Glan-Thomson polarizer which can be rotated 90° so as to transmit either parallel (||) or perpendicularly ($\perp$) polarized light. Polarizer orientation is defined with respect to the the plane of incidence, which is the plane that contains the vectors of the incident light direction and the direction normal to the reflecting surface.

To make a measurement, the prism support and viewing telescope are brought into alignment at the desired angle of incidence by visually centering the disk of reflected light in the viewing telescope. (The angle of incidence is derived from the position of the viewing telescope; the prism support rotates independently of any scale.) Once aligned, the viewing eyepiece is (*carefully!!*) removed and replaced with the photodiode mounted in the aluminum housing. The output current of the photodiode is proportional to the reflected light intensity for the selected combination of filter and polarizer orientation.

The procedure is repeated for different angles of incidence by rotating the viewing telescope and prism support, replacing the photodiode housing with the eyepiece, centering the disk of reflected light in the viewing telescope, replacing the eyepiece with the photodiode housing, and then recording another set of reflected light intensities.

A second photodiode is located near the entrance aperture of the collimating telescope. The output of this photodiode is proportional to the intensity of the incident light, and it is known as the reference detector. As each reflected light intensity reading is recorded, the reference light intensity reading should also be recorded. These readings will be used to normalize the reflected light intensity readings in the event that the intensity of the light source changes during the course of your measurements (which it always does).

The red and green filters can each be flipped in and out of the path of the light beam. Each of the filters transmits a narrow spectral range of light. The index of refraction of the prism is different at the center wavelength of each of the filters (see table for BK-7 glass), so the normalized reflected intensity curves will be different for the different filters. Graphs of the bandwidth and transmission characteristics for each of the filters are posted on the class website and available in a binder in the lab.

The Glan-Thomson polarizer transmits highly polarized light. The extinction ratio (fraction of light transmitted by two crossed polarizers) for this type of polarizer is less than 10^{-5}. The polarizer can be rotated 90°. Part of your analysis should be to determine which polarization corresponds with which lever position. When using this polarizer, the lever with the yellow tape should always be all the way up against the top stop, or all the way down against the lower stop.

1.1 Specifics on Apparatus and Procedure

Spectrometer knobs:

The spectrometer has a large number of adjusting and locking knobs. They are numbered and/or color-coded for identification. (Where have you seen this color code before?) The following key should help you identify the appropriate knob and its function.

1. **Brown**: Locks the coarse angular position of viewing telescope arm.

2. **Red**: Fine adjust of angular position of viewing telescope arm.

3. **Orange**: Locks outer divided circle (marked from 0 to 360 degrees) to viewing telescope arm.

4. **Yellow**: Fine adjust of outer divided circle with respect to (w.r.t.) viewing telescope arm. (Note: this adjustment moves the outer divided circle, not the viewing arm.)

5. **Green**: Locks coarse angular position of inner divided circle (vernier scale).

6. **Blue**: Fine adjust of angular of position of inner divided circle. **This adjustment is used only during the initial alignment procedure, and should not be changed while you are taking reflection measurements.**

7. **Violet**: Height lock for brass plate support (3 legged piece). **DO NOT ADJUST. This knob should be tight at all times during experiment**.

8. **Gray**: Locks brass plate support to shaft. **DO NOT ADJUST. This knob should be loose at all times during experiment**.

9. **White**: Level adjust screws (3) for brass plate supporting the prism holder. Do not adjust these knobs unless you see a problem after following the procedure below.

Alignment check:

Flip the green filter into the light path and check that the red filter is out of the light path. To remove a filter from the light path, rotate/flip the filter housing upward, **being careful to avoid touching the glass surface of the filter.**

Check that the iris at the end of the collimating telescope is open to about 6 mm (1/4") diameter.

If the prism is on the spectrometer, remove it by lifting it by its black base off of the brass support plate and set it aside. **Do NOT touch the surface of the prism.**

Check that the eyepiece is in place in the viewing telescope arm.

Caution: you will be handling the eyepiece many times as you take measurements in this experiment. **Please handle the eyepiece with great care, as it is very fragile**. Dropping it will likely break the lens and cross-hairs! The eyepiece should be inserted and removed from the viewing telescope by pushing or pulling on the knurled flange adjacent the ring of orange tape. **When you remove the eyepiece from the viewing telescope, be sure to put it in the box provided.** This will prevent it from rolling off the table (ouch!).

Check that knob #1 (brown) is loose. Move the viewing telescope **by pushing on the base just above where the brown tape is, not the telescope itself** until it is aligned parallel the collimating telescope and the disk of light is visible in the eyepiece. (If you push on the telescope itself, you may twist it in its vertical support, causing the instrument to go out of alignment.) Center the disk of light in the cross-hairs as best you can and then tighten knob #1. (If the cross-hairs are not in focus, move the outermost piece of the eyepiece in or out so as to improve the focus.) The horizontal centering of the disk of light can now be fine tuned, if necessary, with knob #2 (red).

Note: if the disk of light is not vertically centered in the eyepiece, consult with TA, Professor, or Lab Manager to correct this problem. A *slight* vertical misalignment is acceptable, but a significant misalignment may compromise the reflection measurements and should be dealt with.

Next check that knob #3 (orange) is loose and rotate the outer (black) divided circle scale (by gently pushing on the ribs on the underside of the piece on which this scale is marked) until the yellow tape on the edge of this piece is approximately above the larger piece of light blue tape on the base of the spectrometer. Tighten knob #3. With knob #5 (green) loose, rotate the upper plate (one with three red dots on it) attached to the inner (brass) divided circle until the arrow on the yellow tape on the edge of this piece is lined up with the arrow on the yellow tape below. Using a flashlight to illuminate the scale, look through the scale magnifier (scale B) and line up the 180° mark on the outer scale with the 0° mark on the inner scale as best you can by rotating the upper plate by hand. Tighten knob #5 and fine tune the alignment of the 0° and 180° marks, if necessary, by adjusting knob #6 (blue).

Note: after this alignment is complete, the only knobs you should touch are knobs **knobs #1 and #2**, which control the output telescope position, for the remainder of the experiment.

Remove the eyepiece and put it in the plastic box. Place the photodiode housing in the viewing telescope. Before taking any readings, both the reflected light and reference detector readout devices need to be zeroed. To do this, turn off *all* the lights in the experiment area except the light source, draw the curtains, put both filters in the beam path so as to prevent any direct beam light from reaching the detectors, and zero the readings on both readout devices with the zero or offset knobs. (Flashlights should be off when making the zero adjustment.) After doing this, any non-zero reading will be due to light in the beam path.

Flip the red filter out of the beam path. With only the green filter in the path, the reading for the reflected light detector should be between 15 and 20 nanoamperes. If the detector current is not in this range, check that the lamp current is about 4.4 amps, and adjust the iris at the end of the collimating telescope to achieve a detector current in this range. After adjusting the iris it should not be changed for the rest of the experiment, as changing it will affect the normalization of your measurements.

Next, check to see that with the settings you have on the current meters that you will read at least *three digits* of accuracy on each meter for all four possible combinations of filter color and polarizer position: green + polarizer-up, green + polarizer-down, red + polarizer-up, and red + polarizer-down. It is important for data consistency that you do not change the current meter switches or zero knob between the different measurements. If you find that you have one setting that gives too few digits or is off scale, make small adjustments to the lamp current or iris in order to compensate.

With the detector current in the above range, take readings for all four combinations of filter and polarizer (and also record the associated reference detector reading for each filter/polarizer combination). These readings correspond to the readings at an angle of incidence of 90°, where the value for all the reflectances is equal to one. The ratio of the reflected light intensity reading at smaller angles of incidence to the intensity reading at 90° is then the value of the reflectance at that angle.

Put the *uncoated prism* back on the brass plate by carefully lowering the aluminum base back ovet the alignment posts.

If you have mistakenly removed the brass plate from the spectrometer, note that it should be put onto the three supports such that the plate is properly located: Line up letters A′, B′ & C′ on both pieces. The pointed supports (A′ & B′) should match up with the recess (position A′) and groove (position B′) on the underside of the brass plate. When properly seated on the supports, the plate will be level and secure, and exhibit no movement or wiggle relative to the supports.

Rotate the support piece (by pushing on the shaft with the white tape at the end) so that the light from the collimating telescope hits the prism at an angle of incidence of about 45°. Hold a white index card up to the prism surface and check that the disk of green light is incident on the prism at about mid-height. (Room lights will need to be off to clearly see the disk.) If the disk of light is not hitting the prism face at or near mid-height, consult with Lab Manager to get height adjusted properly.

Loosen knob #1 (brown) and rotate the viewing telescope so it is as close as possible to the collimating telescope. (The angle as read on scale B should be about 320°.) Rotate the prism support piece, again by pushing on the shaft with the white tape at the end, until the reflected disk of green light is visible in the eyepiece. If the disk is not vertically centered, adjust the white leveling screw at C′ (directly behind the exposed face of the prism) until the disk of light is vertically centered in the eyepiece. Check that this condition holds for several different angular positions of the prism, including an angle where the angle of incidence is 80° or greater. If you cannot level the prism so that the disk of light remains centered or nearly centered at different angles using leveling screw C′, please consult with Lab Manager.

2 *Measurements*

Reflected light intensity measurements can be made for angles of incidence from about 20° to more than 80°, in intervals of about 5°, a range which easily covers the very different behaviors for the two polarizations.

Measurements for both polarizations and both wavelengths should be made at a particular setting of spectrometer angle by flipping the appropriate filters in and out of the light path and rotating the polarizer. As you flip the filters in and out and rotate the polarizer, take care not to touch the glass surfaces of these devices.

Since you will be recording nine (9) numbers at each angle, it is useful to set up a large table to keep the measurements organized. You will need a column for the angle and one for each value of the reference and detected light for all combinations of polarizer and filter settings.

With the disk of light centered in the viewing telescope, the angle of incidence can be derived from the reading taken at either scale A or scale B. Given the geometry of the apparatus, it is most convenient to work with scale B, but this is not feasible for angles of incidence between approximately 40° and 50°, as the viewing telescope arm is in the way of the scale. If θ is the angle of incidence and α is the reading on scale A and β the reading on scale B, then

$$\theta = 90^\circ - \alpha/2 = 180^\circ - \beta/2 \; . \tag{1}$$

As mentioned earlier, angles can be read to 10 arc seconds of resolution on this spectrometer. Given that the light incident on the prism is not completely parallel (how can you demonstrate this?), it suffices to read the angle to the nearest minute of arc. Each degree of arc on the outer (black) scale is divided into six equal segments, each representing 10 minutes of arc. The major divisions on the inner (brass) scale each represent 1 minute of arc, and each of these divisions is further divided into six segments, each representing 10 seconds of arc. To read the angle to the nearest minute of arc, find the major division between 0 and 10 on the inner scale which best lines up with a mark (any mark) on the outer scale. Add this number of minutes to the number of minutes represented by which 10 minute segment the zero on the inner scale falls in on the outer scale. For instance, if the 0 falls in the third 10 minute segment of the outer scale and the 5 on the inner scale best lines up with a mark in the outer scale, the angle is the whole number of degrees (on outer scale) + 25 minutes of arc.

Both the reference detector and the reflected light detector (when installed in the viewing telescope) are sensitive to ambient light. It is best to note the detector readings with all lights off, and then turn on a flashlight to record the readings.

Important: **If you have not already done it, record the reference and detector currents for the 90° incident angle, that is, the *straight through beam without the prism in place*, for all four settings of the filters and polarizer. You will need these numbers in order to properly analyze the data.**

Take your first series of measurements with the **uncoated** prism; measurements from the coated prism are optional. Begin with an angle of incidence of about 20° (reading on scale B will be about 320°). Clamp the viewing telescope in place (tighten knob #1) and rotate the prism support until the disk of light is visible in the eyepiece. The goal now is to center the disk of light in the eyepiece cross-hairs. This can be accomplished by fine-tuning the position of the viewing telescope with knob #2 (red), or by coaxing the prism support into just the right position. The first alternative will change the angle reading, but is the easiest way to center the disk of light. The second alternative will not change the angle reading, but is more difficult to perform, as the rotation of the prism support can be somewhat jerky due to stickiness. If you are trying to center the disk by rotating the prism support, it may help to "walk" the disk into the centered position by gently moving the rod with the white tape at the end up and down as you apply a slight rotational force.

Once the disk is centered, remove the eyepiece (put it in the box) and replace it with the photodiode housing. Make sure to put the photodiode housing into the end of the viewing telescope with the same orientation each time. The photodiode is weakly sensitive to the polarization of light.

Record the reflected and reference light intensities for all four combinations of filter and polarizer orientation. The currents generated by the reflected light detector will range from picoamperes to 10's of nanoamperes. The current generated by the reference detector is typically a fraction of a microampere (with a filter in the beam path). You should avoid changing the ranges on the current meters because different ranges can have different offsets, and this will require resetting the zero.

Repeat the alignment and measurement procedures described in the above two paragraphs at angles of incidence increasing in 5° increments through $\theta = 80°$. ***Remember:*** **to increase the angle of *incidence* by 5°, you must rotate the viewing telescope by 10°!** If the disk of light moves off center vertically as you change the angle of incidence, it can be recentered with small adjustments of the leveling screws at A′ & B′.

Finally, at the end of your measurements, you should double-check your initial "straight-through" measurements ("90 degrees," no prism) to (1) check consistency—how much has the light source changed over the time of your measurements, and (2) give you another set of good readings to normalize by. You may discover that you made a mistake earlier, since you have now gained practice with your measurement procedure.

2.1 Normalizing your measurements

First, calculate the angle of incidence θ from your measured values of the scale readings α or β. See Eq. (1).

Next, as mentioned earlier, the value of the reflectance at a given angle of incidence can be derived from dividing the reflected light detector current at that angle of incidence by the reflected light detector current at 90° (straight through), *as long as the lamp intensity does not change from what it was at the 90° reading.* But the lamp intensity does fluctuate somewhat slowly over time, so we correct for that problem by first normalizing each measurement with the reference beam, since these two readings (for each angle, color, and polarization) are taken at nearly the same time. Then we normalize the whole data set against the (normalized) intensity at 90°.

To repeat, there are *two steps* you need to do in order to normalize your intensity data in order to compare it to the theory:

1. Divide each detector measurement by its associated reference measurement, including the ones you made at "90° incidence" (straight through, no prism) for each color/polarization combination. This normalizes each point to its reference intensity, and corrects for fluctuations in the light source brightness.

2. Then divide the each point in the set of resulting normalized measurements by the associated normalized measurement at 90° incidence. This normalizes the reflectance at each angle to its value at 90°.

3 Data Analysis

From your suitably normalized data, determine the angle of incidence and the reflectance for both wavelengths and both polarizations and enter them in a table. Plot the reflectance against the angle for the four cases. On the same graphs, plot curves (i.e., lines, not points) from the appropriate Fresnel equations, as described below.

Note: By construction the reflectance at 90° is 1.00, so **this point should not be plotted.** If you include it in your plot, the point will dominate the graph since the reflectance falls very quickly between 90° incidence and 80° incidence. It is much easier to see the features of the plot and the agreement between the data and theory without the point at 90° incidence.

Determine a preliminary value for the index of refraction n from Brewster's angle which you should be able to find from your curves. Estimate an uncertainty in n from this method by considering how well you are able to locate the minimum in the appropriate reflection curve. (See Hecht, pp. 348–350 for a definition of Brewster's angle.)

You must also compare the resulting curves to the Fresnel equation by using a computer program. There are two ways you may do this.

The first, and best, way is to fit the data to the theoretical curve by using a nonlinear least-squares fit, as can be performed with a computer. Treat the index of refraction n as one parameter to be determined by the fit. You will also need two other parameters: an overall multiplier to normalize your data (if all goes perfectly well the value of this parameter should be close to 1) and an additive constant to adjust the baseline (which should come out close to 0).

If you cannot carry out the first method, you may try an alternative which relies on your ability to see the agreement by eye. Use a spreadsheet program to plot the data points along with a theoretical curve generated by the appropriate Fresnel formula (shown below). In the theoretical curve that you overlay on the data, estimate the value of the index of refraction n by varying its value and observing the resulting curve.

After a few attempts, you should have a range of n that appears to work. Your final plot should show curves for three values of n: one at the best value n_{best} and two more at n values that are different by an amount Δn, i.e., $n_{hi} = n_{best} + \Delta n$ and $n_{low} = n_{best} - \Delta n$. The value of Δn is your uncertainty estimate.

Note: Since it is primarily the *magnitude* of the reflectance $R_\perp$ for light polarized perpendicular to the plane of incidence that depends on the index, the fit (using either method) will not be very sensitive to the value of the index for this polarization. Consequently, it is best to use the index as an adjustable parameter only for the curves of reflectance $R_{\parallel}$ for light polarized parallel to the plane of incidence in which n determines the Brewster angle. Then use the index found for $R_{\parallel}$ as a fixed parameter for the $R_{\perp}$ fit. This will leave only the normalization and the baseline as adjustable parameters for this fit.

These simplified forms of the Fresnel equations, where we are assuming that the index of refraction of the medium carrying the incident ray $n_0 \approx 1$ (i.e., air), the index of the glass $n > 1$, and the incident angle is θ may be used for your computer work. For polarization perpendicular to the plane of incidence,

$$R_{\perp} = \left(\frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \right)^2 ,$$

and for polarization parallel to the plane of incidence,

$$R_{\parallel} = \left(\frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \right)^2 ,$$

Once you have made your fits, compare the value of the indices you find at the green and red wavelengths to the values given by the manufacturer of the prism glass. A table of these values is given on the class website and in a notebook in the lab.

If you have made measurements of the coated prism, plot these on the same graph as the results for the uncoated prism, and comment on the measurements: How much reduction in reflectance does the coating produce? Discuss as a function of wavelength and polarization. Extrapolate the reflectance to normal incidence and estimate the percent reflectance to the manufacturer's claim: 1.28% at 550 nm and 1.38% at 650 nm.

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