Summary of lab reports

Your reports are awesome!

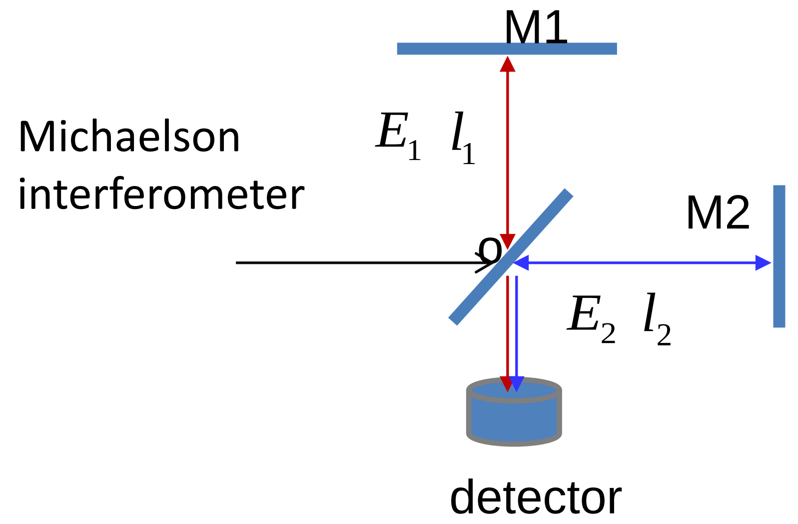
New reports are due next week.

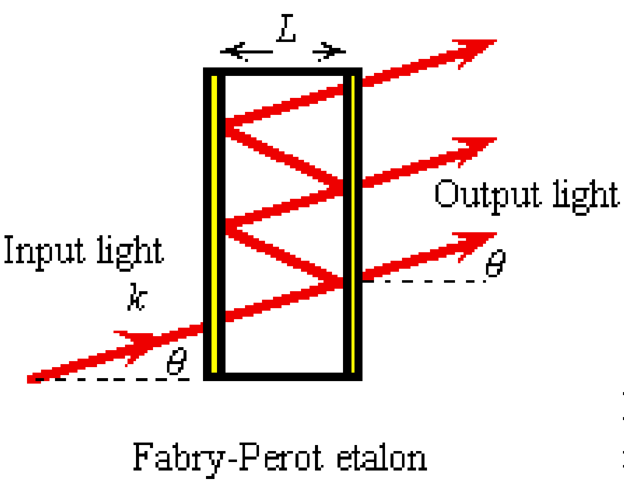
- Units (hand written data sheets)
- Always include your own raw data in the report
- Partner’s name
- Organized data sheet/neat hand writing
- Label the parameters in the figure
- Error analysis (understand what you reported)

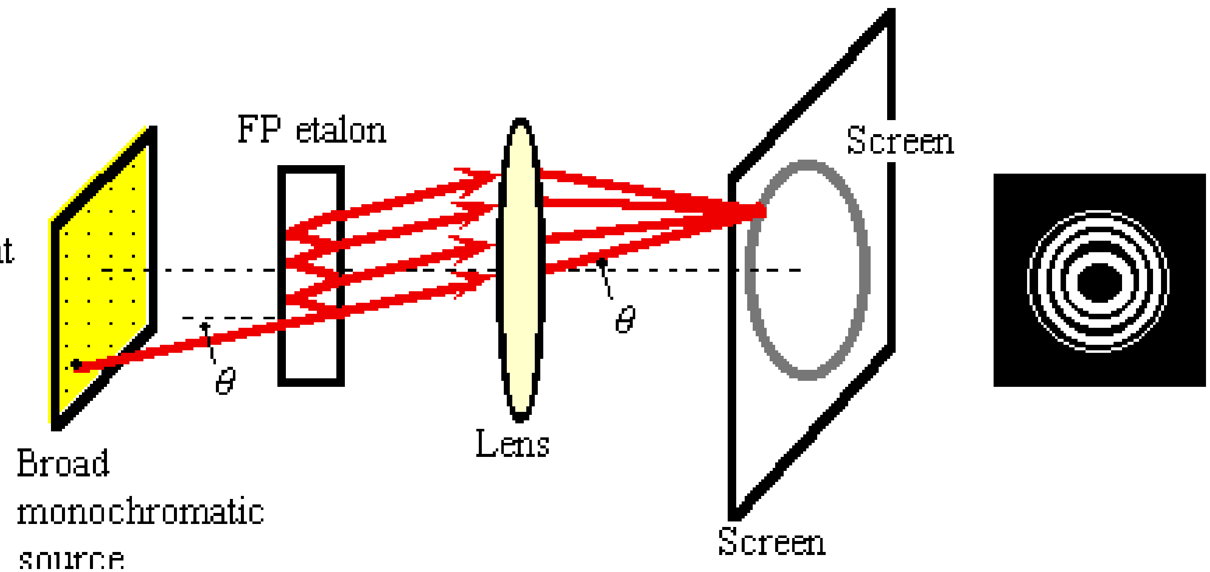
Please use the new report guidelines!

Fabry-Perot Interferometer

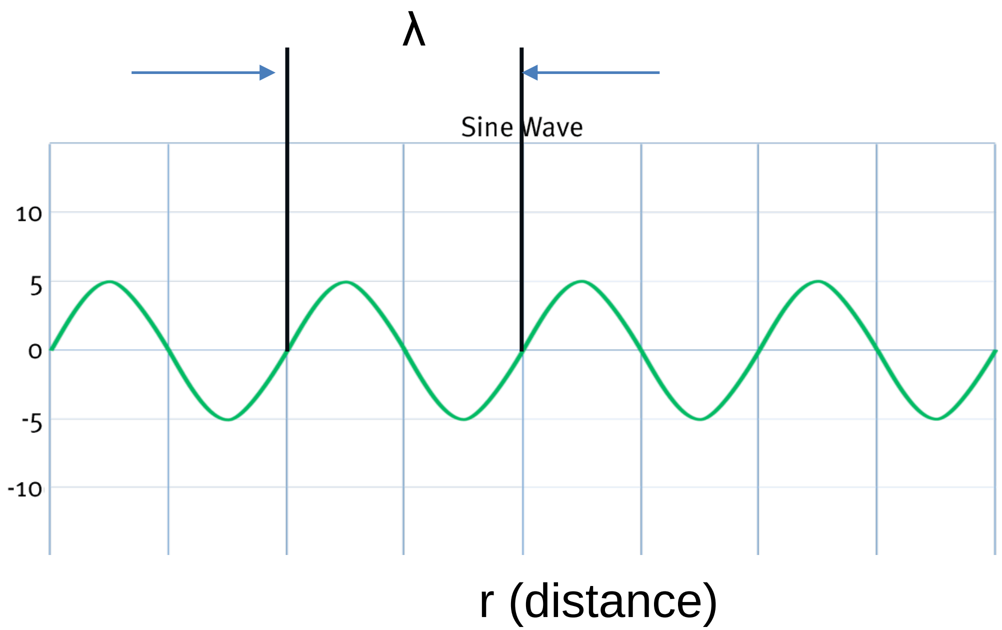
Broad Applications: laser wave meter; Raman spectrometer; Ultra-narrow bandwidth laser; optical filters etc….







Wave Propagation of Light



$$E = Ae^{ik.r}$$

$$k = \frac{2\pi}{\lambda}$$

r= r_o + m λ

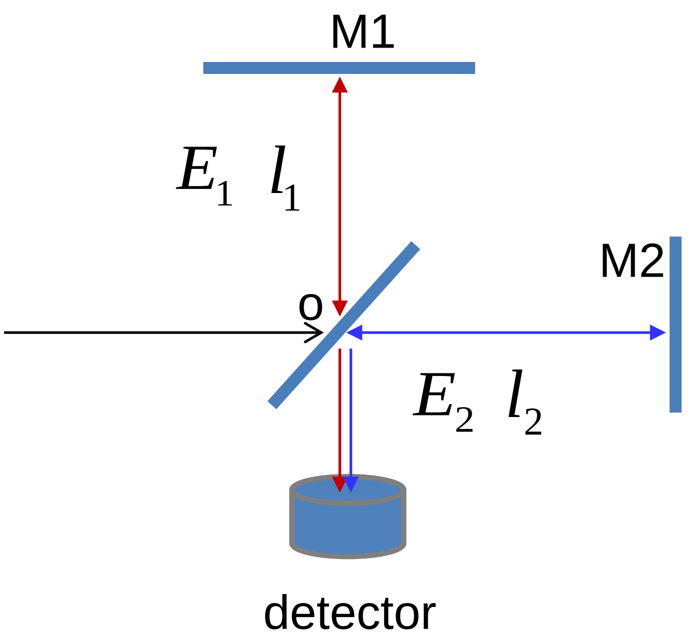
$$E(r) = Ae^{i(k.r_o + km\lambda)}$$
$$= Ae^{ik.r_o}e^{i2m\pi} = E(r_o)$$

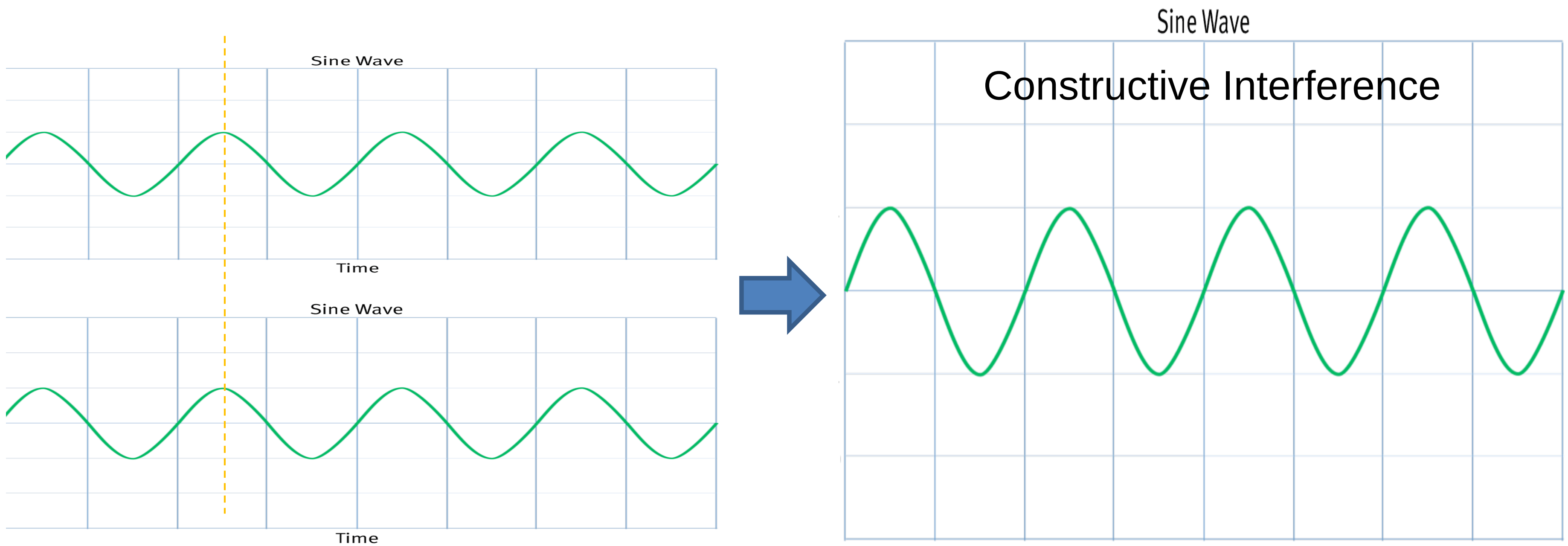
$$E = Ae^{ik.l}$$

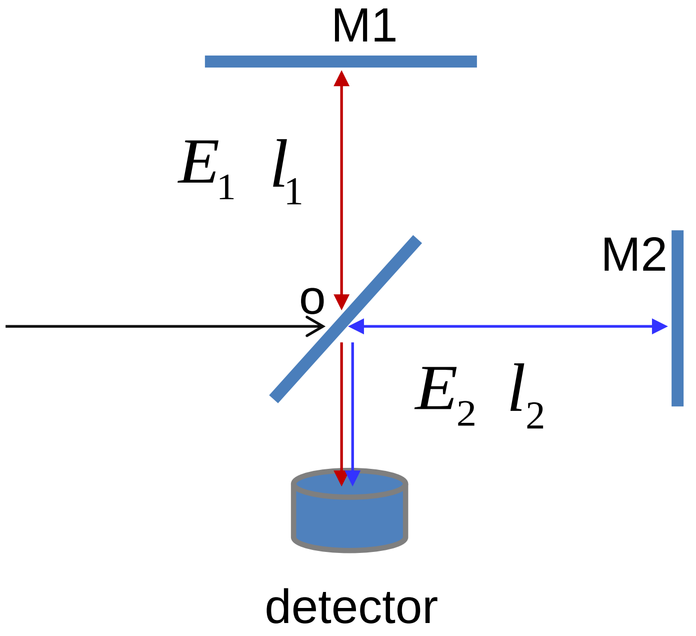
$l_2 = l_1 + 2d$

If $2d = m\lambda$

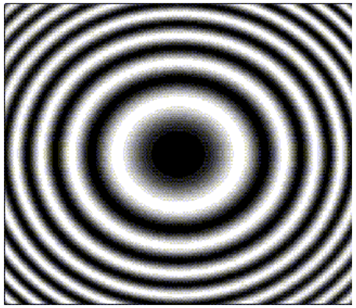
Phase difference $\delta = k.2d = 2m\pi$







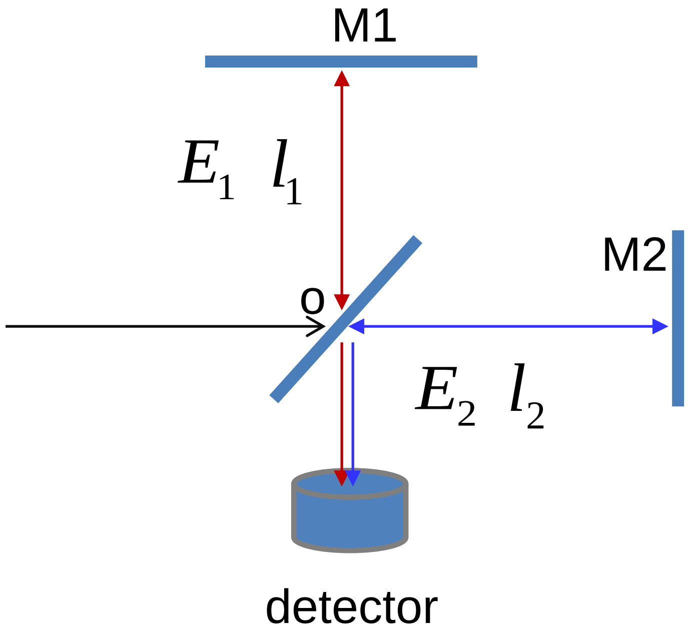
Quiz!



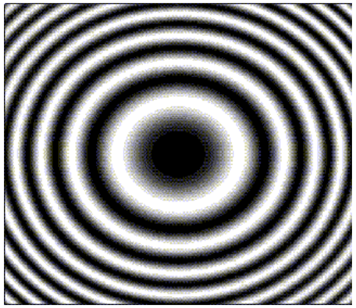
Initially, M2 at position d_o, and it is a dark circle in the middle.

(1)M2 moves by ¼ λ. Is the center bright or dark?

(2) M2 moves by ½ λ. Is the center bright or dark?



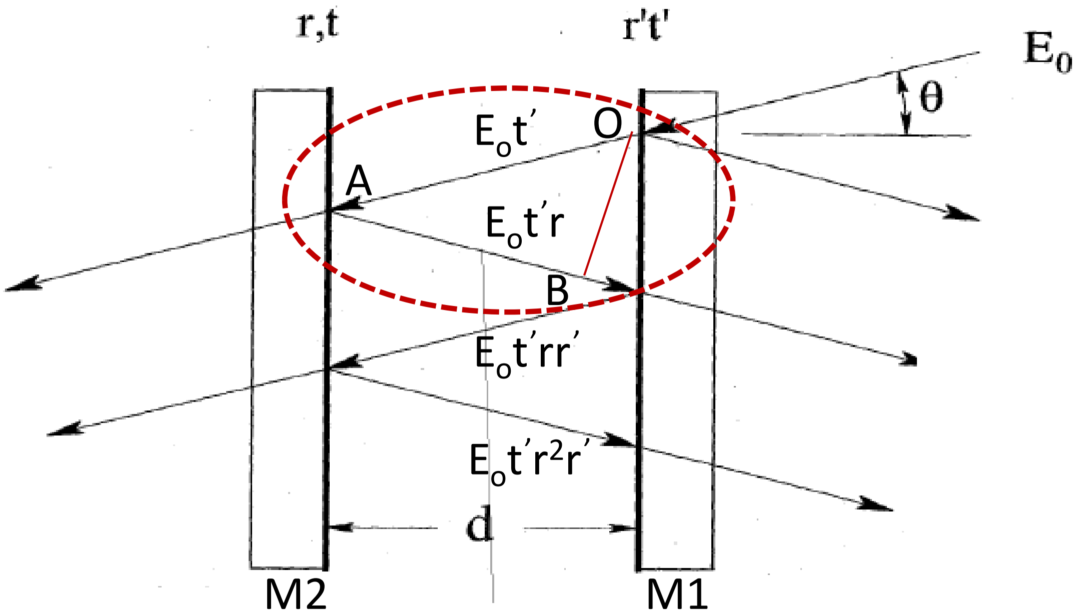
Quiz!



Initially, M2 at position d_o, and it is a dark circle in the middle.

(1) M2 moves 7 μm. The center changes from dark to dark 20 times. What is the exciton laser wavelength?

Fabry-Perot Interferometer



r: reflection coefficient
t: transmission coefficient

OA+AB=OA (1+cos2θ)
$=2c\cos^2(\theta)OA$
$=\frac{2c\cos^2(\theta)d}{\cos(\theta)}$ = 2dcos(θ)

Constructive Interference when 2dcosθ = nλ

E-Field Calculation: (Phase-change for **one round-trip** is: $e^{-i\delta}$, where $\delta = 2kd\cos\theta.$ = $\left(\frac{4\pi d\cos\theta}{\lambda}\right)$

Assume: $r = r', t = t'$.

Reflected Field, $E_r = E_0 r + E_0 r t^2 e^{-i\delta} + E_0 r^3 t^2 e^{-2i\delta} + \ldots$

$$E_r = E_0 r + E_0 r t^2 e^{-i\delta}(1 + r^2 e^{-i\delta} + r^4 e^{-2i\delta} + \ldots)$$

Using formula for **geometric series** for terms in parenthesis:

$$E_r = E_0 r \left(1 + \frac{t^2 e^{-i\delta}}{1 - r^2 e^{-i\delta}}\right)$$

Fabry-Perot Interferometer

Transmitted Field,

$$E_t = E_0 t^2 \bar{e}^{i\delta/2} + E_0 r^2 t^2 e^{-3i\delta/2} + E_0 r^4 t^2 e^{-5i\delta/2} + \ldots$$

(**Neglecting** overall phase-factor of $e^{-i\delta/2}$),

$$E_t = E_0 t^2 (1 + r^2 e^{-i\delta} + r^4 e^{-2i\delta} + \ldots)$$

$$E_t = E_0 \left(\frac{t^2}{1 - r^2 e^{-i\delta}} \right) \qquad E_r = E_0 r \left(1 + \frac{t^2 e^{-i\delta}}{1 - r^2 e^{-i\delta}} \right)$$

Intensity:

(*Intensity* reflection coefficient $\equiv R = r^2$ and transmission coefficient $\equiv T = t^2$ **and** $R + T + A = 1$, where A= the **absorptance**).

Transmission:

$$I_t = |E_t|^2 = \frac{E_0^2 T^2}{1 + R^2 - R(e^{i\delta} + e^{-i\delta})} = \frac{I_0 T^2}{1 + R^2 - 2R\cos\delta}$$

$$\frac{I_t}{I_0} = \left(\frac{T}{1 - R} \right)^2 \boxed{\frac{1}{1 + [4R/(1-R)^2]\sin^2(\delta/2)}}$$

Hecht, Pg 422, Eq. 9.65

If there is no absorption, $\dfrac{I_t}{I_o} = A(\theta)$

Fabry-Perot Interferometer

The **second factor** is called the *Airy function*, $\mathcal{A}(\theta)$, where:

$$\mathcal{A}(\theta) \equiv \frac{1}{1 + F\sin^2\delta/2}$$

$$F = \frac{4R}{(1-R)^2}$$

R~0.99->F~40,000

$$\delta = 4\pi d\frac{\cos\theta}{\lambda}$$

$$\frac{I_t}{I_0} = \left[1 - \frac{A}{1-R}\right]^2 \mathcal{A}(\theta)$$

Coefficient of finesse

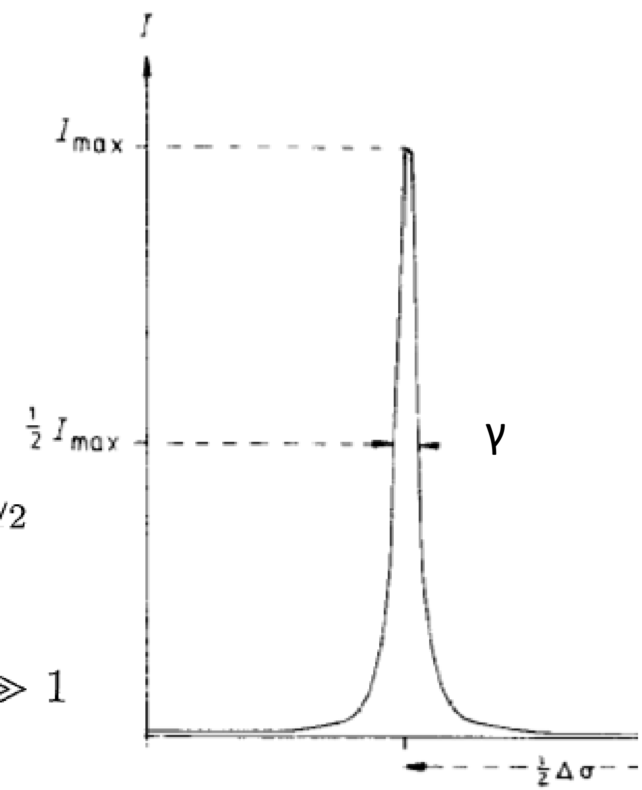
In the presence of *absorption*, $A > 0$ and therefore *peak* $I_t < I_0$ (since the peak of $\mathcal{A}(\theta)$ is 1).

Finesse:

$$\frac{I_t}{I_0} = \left[1 - \frac{A}{1-R}\right]^2 \mathcal{A}(\theta)$$

$$\mathcal{A}(\theta) = \frac{1}{1 + F\sin^2(\delta/2)} = \frac{1}{2} \text{ when } \delta = \delta_{max} \pm \delta_{1/2}$$

$$\delta_{1/2} = 2\sin^{-1}(1/\sqrt{F}) \approx 2/\sqrt{F}, \quad \text{since, usually } F \gg 1$$



Fabry-Perot Interferometer

$$\mathbf{FWHM} \equiv \gamma = 2\delta_{1/2} = 4/\sqrt{F} = \frac{2(1-R)}{\sqrt{R}}$$

The **periodicity** of $\mathcal{A}(\theta)$ (in δ) is 2π. Thus the *Finesse* defined as:

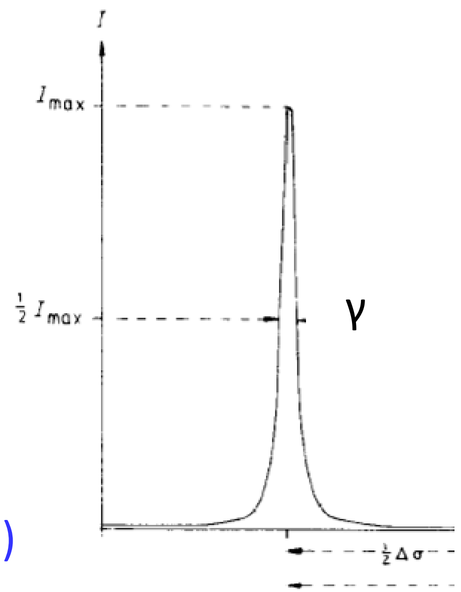
instrumental or experimental (effective) finesse

reflective finesse for $R \approx 1$

$$\mathbf{Finesse} \equiv \frac{\text{peak separation}}{\text{FWHM}} \equiv \mathcal{F} = \frac{\pi\sqrt{R}}{1-R} \approx \frac{\pi}{T} \quad (R \approx 1)$$

Resolving power of instrument ~ frequency width of peaks

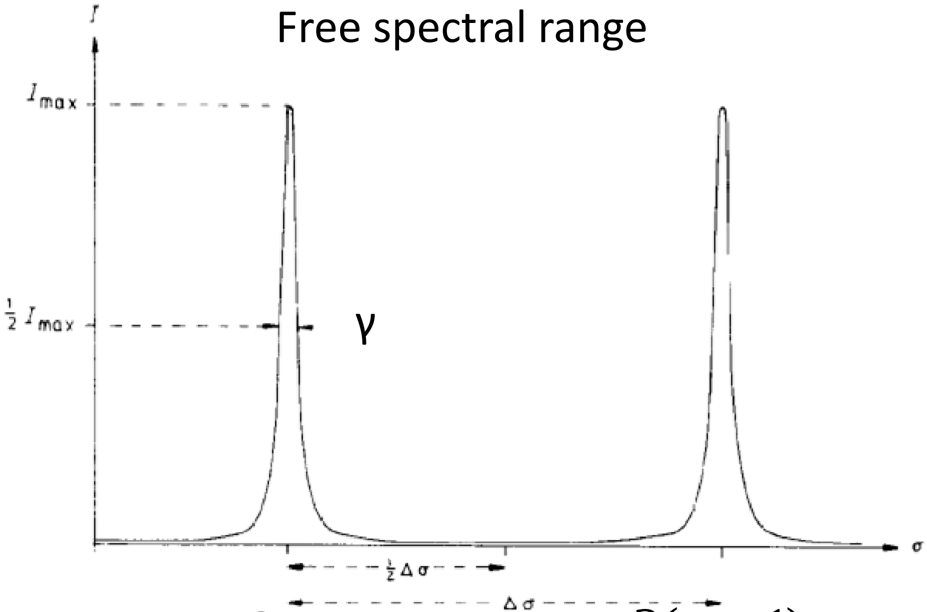
$$\delta = 4\pi d \frac{\cos\theta}{\lambda}$$



Hecht, Pg 423, Fig. 9.45
Fabry-Perot fringes for (monochromatic light)

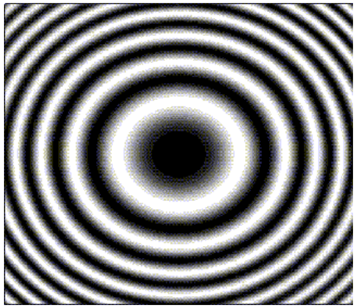
Fabry-Perot Interferometer

Free spectral range



$$\delta = \frac{4\pi d}{\lambda}$$

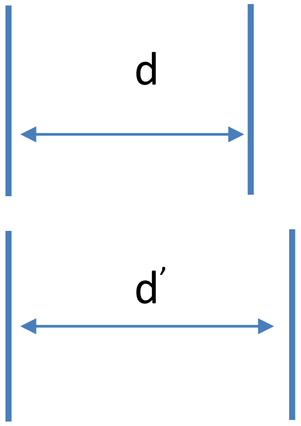
$$\mathcal{A}(\theta) = \frac{1}{1 + F\sin^2(\delta/2)}$$



Central spot scanning

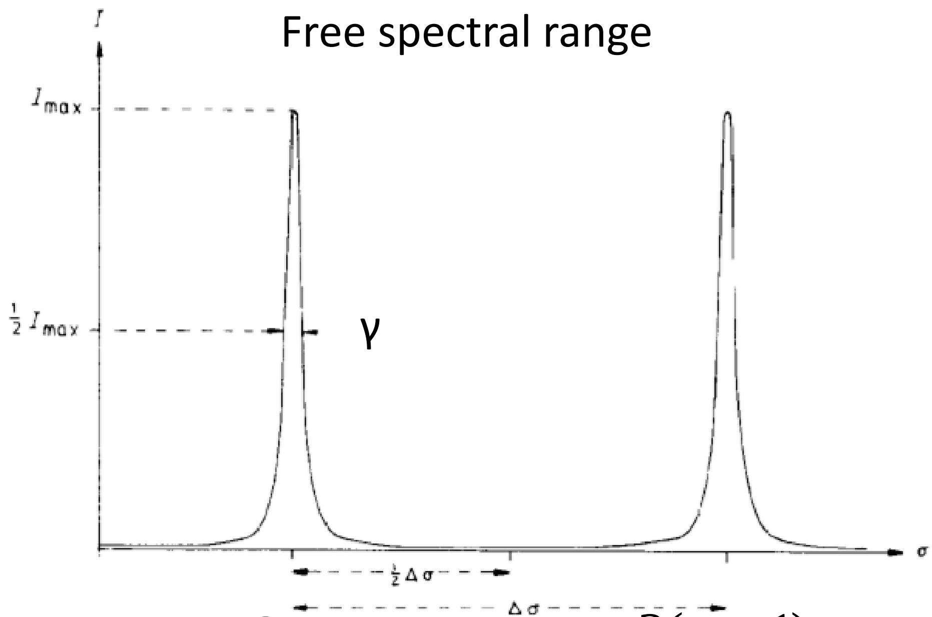
$$d' = d + \lambda/2$$

	$2m\pi$	$2(m+1)\pi$
same λ	$\frac{4\pi d}{\lambda}$	$\frac{4\pi d'}{\lambda}$



Fabry-Perot Interferometer

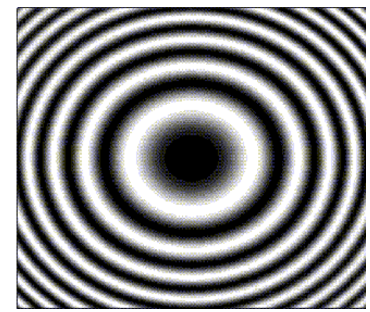
Free spectral range



$2m\pi$ $\qquad$ $2(m+1)\pi$

$$\delta = \frac{4\pi d}{\lambda}$$

$$\mathcal{A}(\theta) = \frac{1}{1 + F\sin^2(\delta/2)}$$



Central spot scanning

$$d' = d + \lambda / 2$$

same λ $\qquad \frac{4\pi d}{\lambda} \qquad \frac{4\pi d'}{\lambda}$

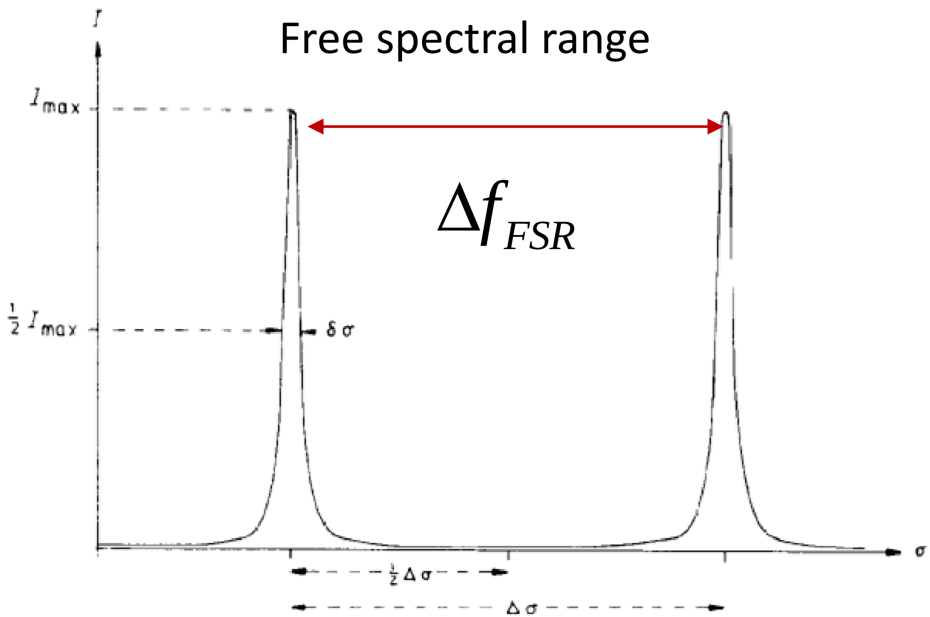
Different λ $\qquad \frac{4\pi d}{\lambda} \qquad \frac{4\pi d}{\lambda'}$

In this case, we cannot distinguish these two frequencies apart

$$\lambda - \lambda' = \Delta\lambda$$

Free spectral range

Fabry-Perot Interferometer



$$\frac{4\pi d}{\lambda'} = 2(m+1)\pi = \frac{4\pi d}{\lambda} + 2\pi$$

$$\frac{\lambda - \lambda'}{\lambda'\lambda} = \frac{1}{2d} \qquad \lambda f = c \qquad \lambda = \lambda'$$

$$\frac{\dfrac{c(f - f')}{ff'}}{c^2 / ff'} = \frac{1}{2d}$$

$$\Delta f_{FSR} = \frac{c}{2d}$$

The round-trip time for light travels in the cavity

Fabry-Perot Interferometer

resolving power ~ free spectral range

$$\mathbf{FWHM} \equiv \gamma = 2\delta_{1/2} = 4/\sqrt{F} = \frac{2(1-R)}{\sqrt{R}}$$

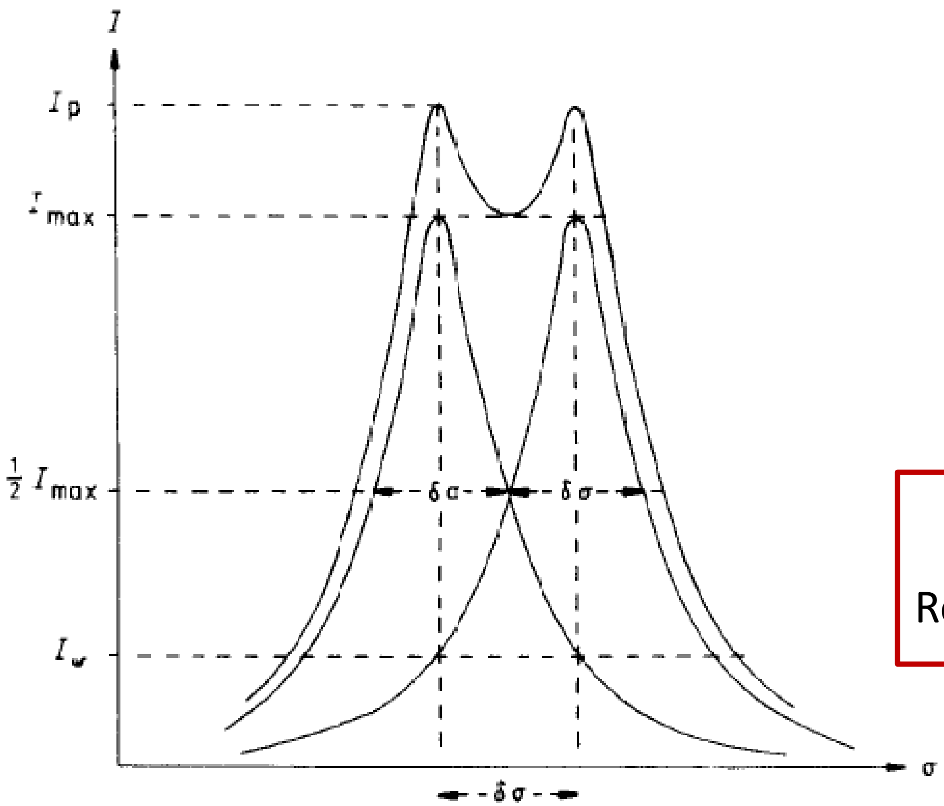
$$Finess = 2\pi / \gamma$$

$$\lambda f = c$$

$$\delta = \frac{4\pi d}{\lambda} = \frac{4\pi d f}{c}$$

$$\Delta\delta_{1/2} = \frac{4\pi d \Delta f}{c} = \gamma$$

$$\frac{2\pi}{\gamma}\Delta f = \frac{c}{2d}$$



Resolving power: $\Delta f = \dfrac{\Delta f_{FSR}}{Finesse}$

Fabry-Perot Interferometer

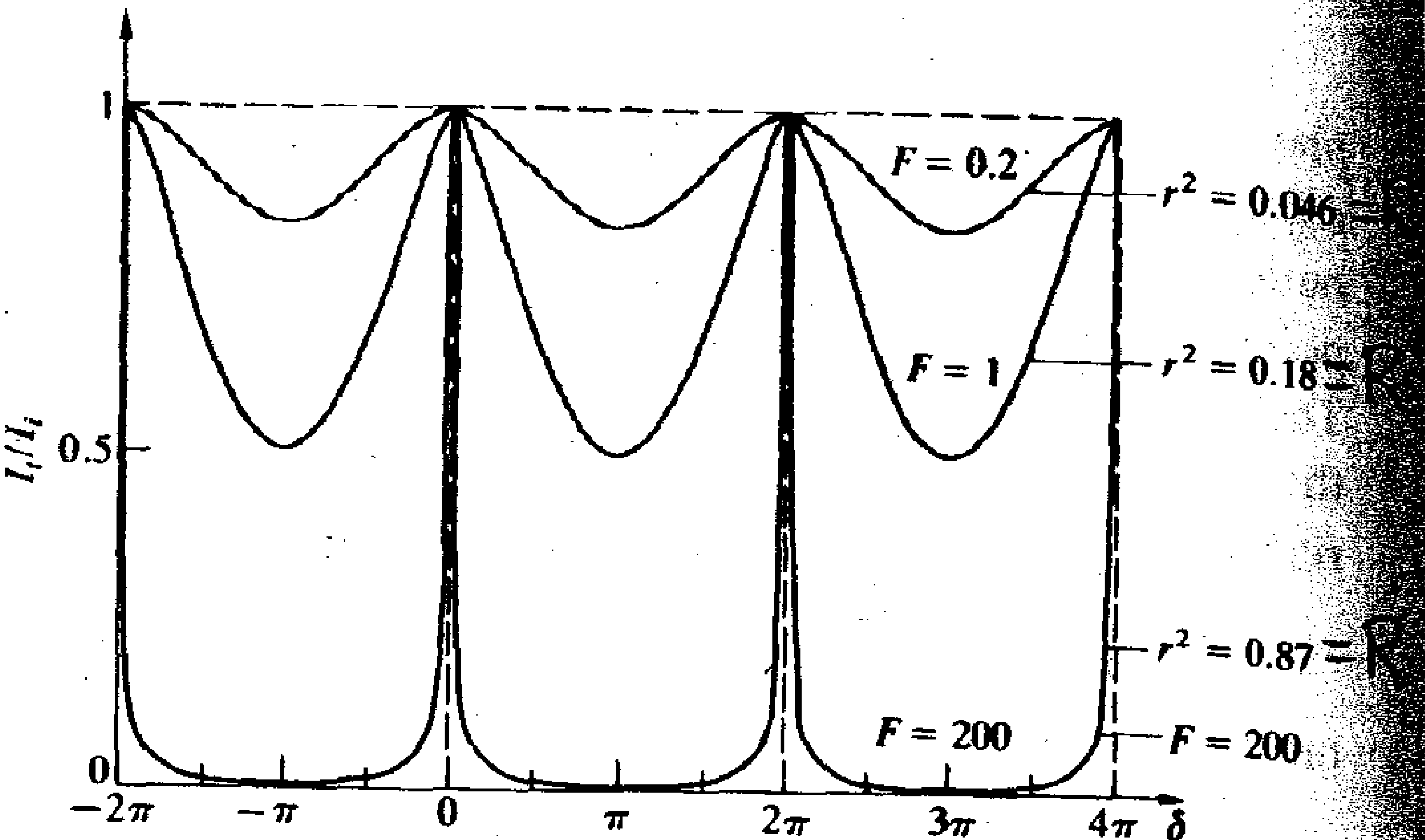
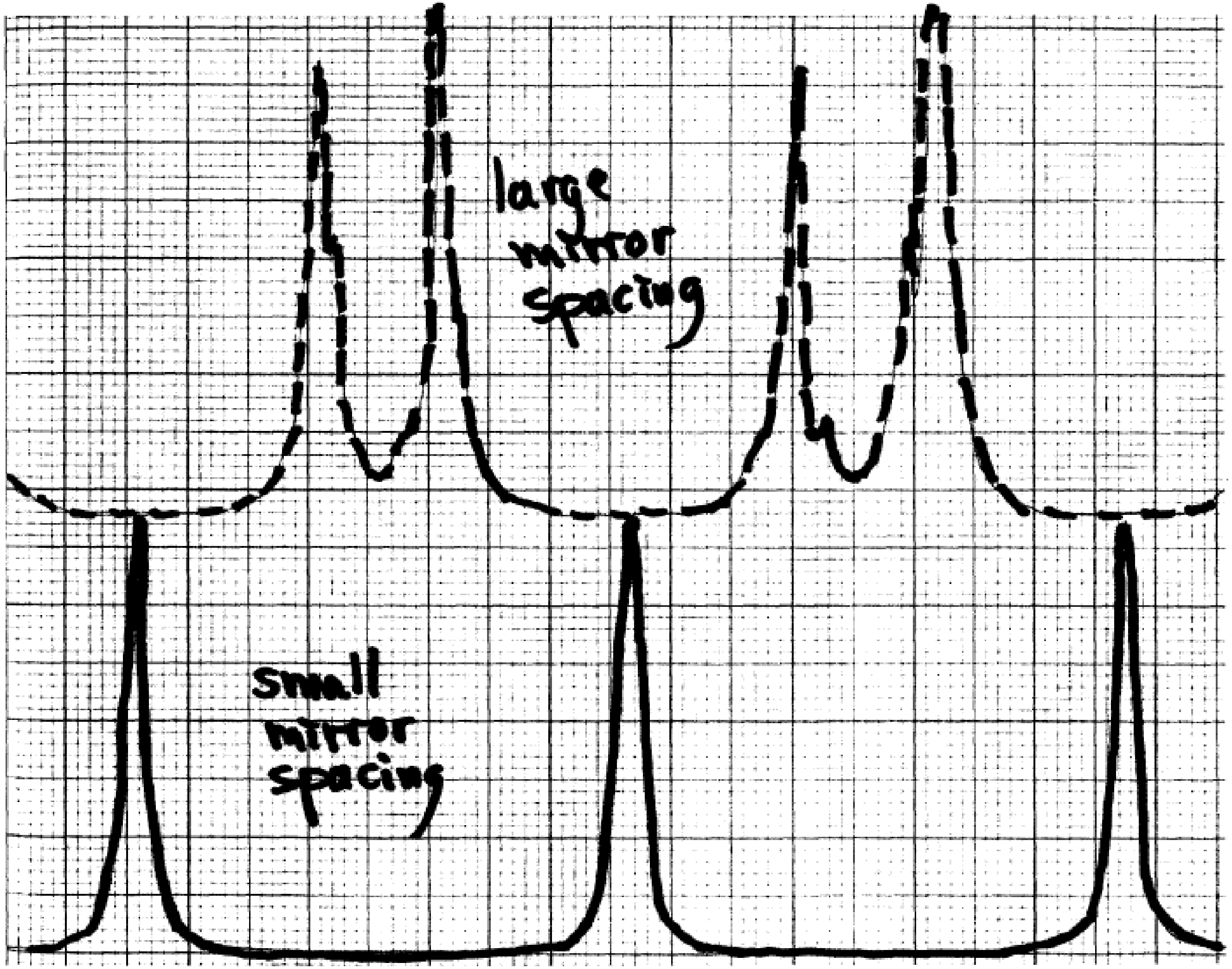


Figure 9.41 Airy function.

Fabry-Perot Interferometer



ACTUAL FABRY PEROT SCAN WITH LARGE (TOP) AND SMALL (BOTTOM) MIRROR SPACING

5. **Fabry Perot** Suppose a Fabry Perot interferometer has surfaces with a reflectivity of 0.850 and the surfaces are perfectly parallel.

(a) (3 pts) What is the Finesse ($\mathcal{F}$) of this instrument?

$$\mathbf{FWHM} \equiv \gamma = 2\delta_{1/2} = 4/\sqrt{F} = \frac{2(1-R)}{\sqrt{R}}$$

(b) (4 pts) Suppose the plates are 5.00 cm apart. Calculate the free-spectral-range (in MHz) and explain in words what that means.

$$\Delta f_{FSR} = \frac{c}{2d}$$

(c) (3 pts) Would this Fabry-Perot be able to resolve the mode structure of a laser whose modes are 800 Mhz apart (give reasons for your answer)?

Resolving power: $\dfrac{\Delta f_{FSR}}{Finesse}$

5. Fabry Perot Suppose a Fabry Perot interferometer has surfaces with a reflectivity of 0.850 and the surfaces are perfectly parallel.

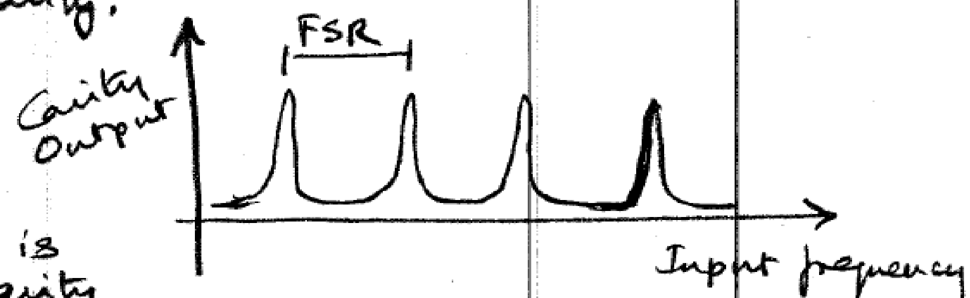
(a) (3 pts) What is the Finesse ($\mathcal{F}$) of this instrument?

$$\mathcal{F} = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi\sqrt{0.85}}{1-0.85} \simeq 19.3.$$

(b) (4 pts) Suppose the plates are 5.00 cm apart. Calculate the free-spectral-range (in MHz) and explain in words what that means.

$$FSR = \frac{c}{2L} = \frac{3\times 10^{8}\,\mathrm{m/s}}{2\times 5\times 10^{-2}\,\mathrm{m}} = 3\times 10^{9}\,\mathrm{Hz} = 3\,\mathrm{GHz}$$

This is the frequency spacing between consecutive longitudinal modes supported by the cavity.



Note that a similar figure is obtained when scanning the cavity length & keeping the laser frequency fixed

(c) (3 pts) Would this Fabry-Perot be able to resolve the mode structure of a laser whose modes are 800 Mhz apart (give reasons for your answer)?

$$\text{Resolving power/Resolution} \simeq \frac{FSR}{\mathcal{F}} = \frac{3\,\mathrm{GHz}}{19.3}$$

$$\simeq 155\,\mathrm{MHz} < 800\,\mathrm{MHz}$$

YES. This would resolve the mode structure of a laser whose modes are 800 MHz apart.