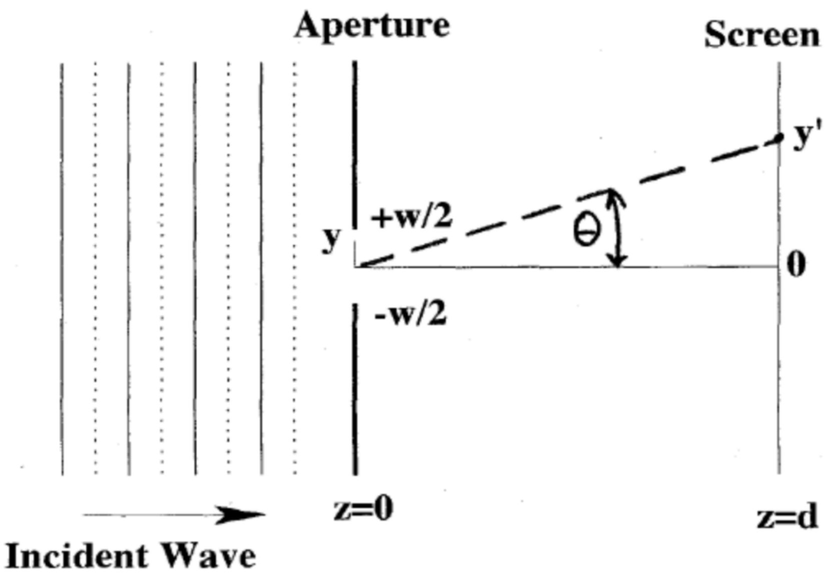
Diffraction Gratings

Labs from Week 3 (your third lab) are due this week

Happy Halloween

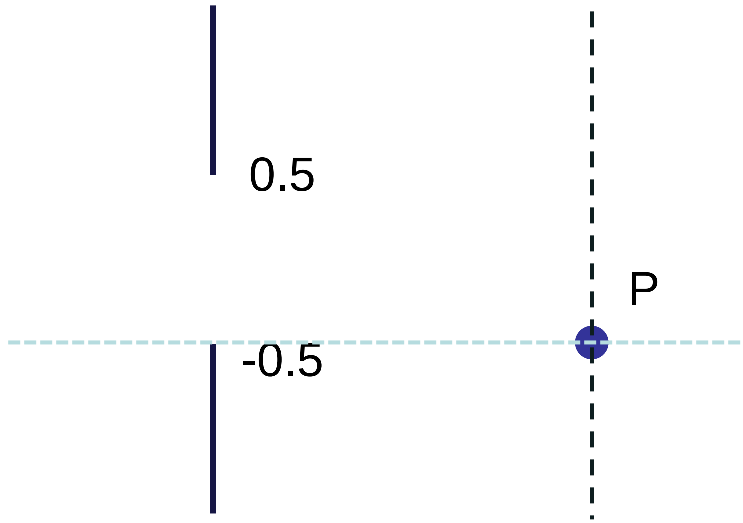
Candy Time..



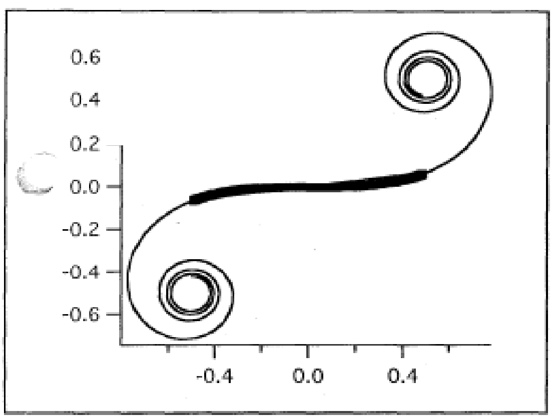
Plane wave diffraction.
Wavelength, 1 μm.
Slit width, 10 μm.
Source to detector plane separation: 1 m.

Question: Is it Fraunhofer or Fresnel Diffraction?

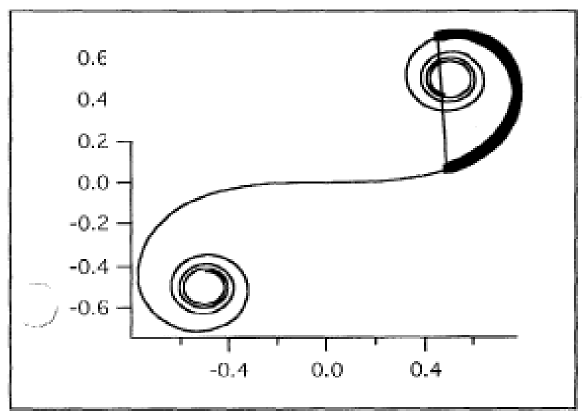
$$d >> \frac{w^2}{\lambda}$$



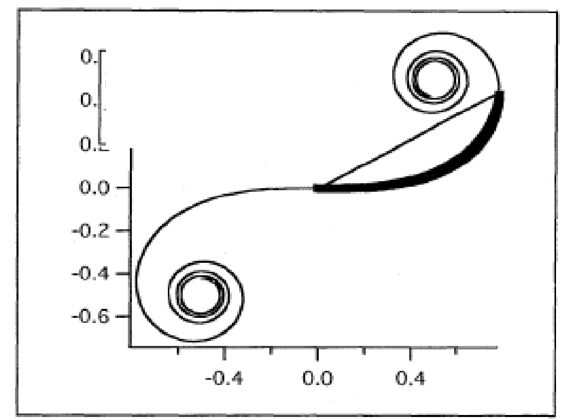
Fresnel diffraction. Normalized slit width is 1. The detector is at point p. Which of the following represents the detection intensity at the detector?



A

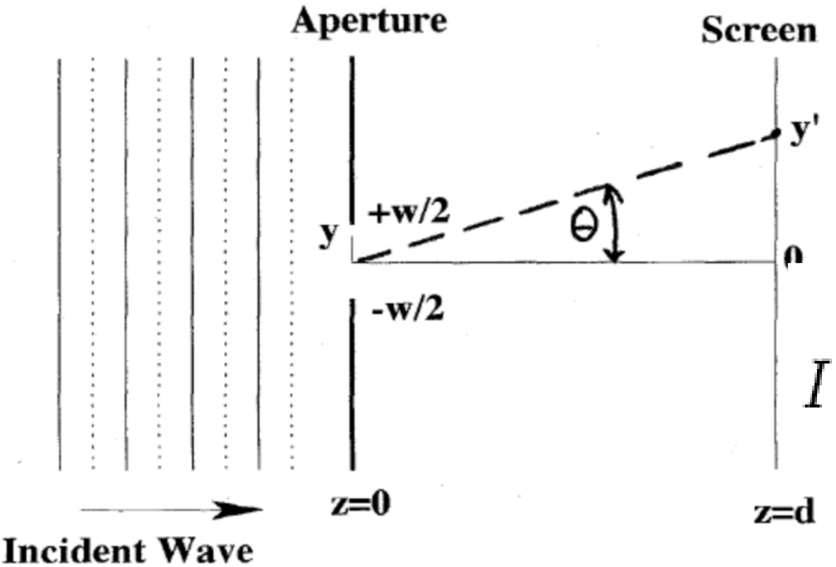


B



C

1-D Fraunhofer Diffraction – Single Slit



$$E \propto \int_{-w/2}^{+w/2} e^{\frac{iky'y}{d}}\,dy \qquad \left(k = \frac{2\pi}{\lambda}\right)$$

Huygens' Recip
Fraunhofer limi

$$I \propto |E|^2 \propto \left|\frac{2d}{iky'} \sin\frac{kwy'}{2d}\right|^2 = w^2 \left[\frac{\sin\frac{kwy'}{2d}}{\frac{kwy'}{2d}}\right]^2$$

SINC sq. function

Criterion for Fraunhofer Diffraction

$$I(y') = I_0 \left(\frac{\sin\beta}{\beta}\right)^2, \qquad \beta \equiv \frac{kwy'}{2d}$$

$$d >> \frac{w^2}{\lambda}$$

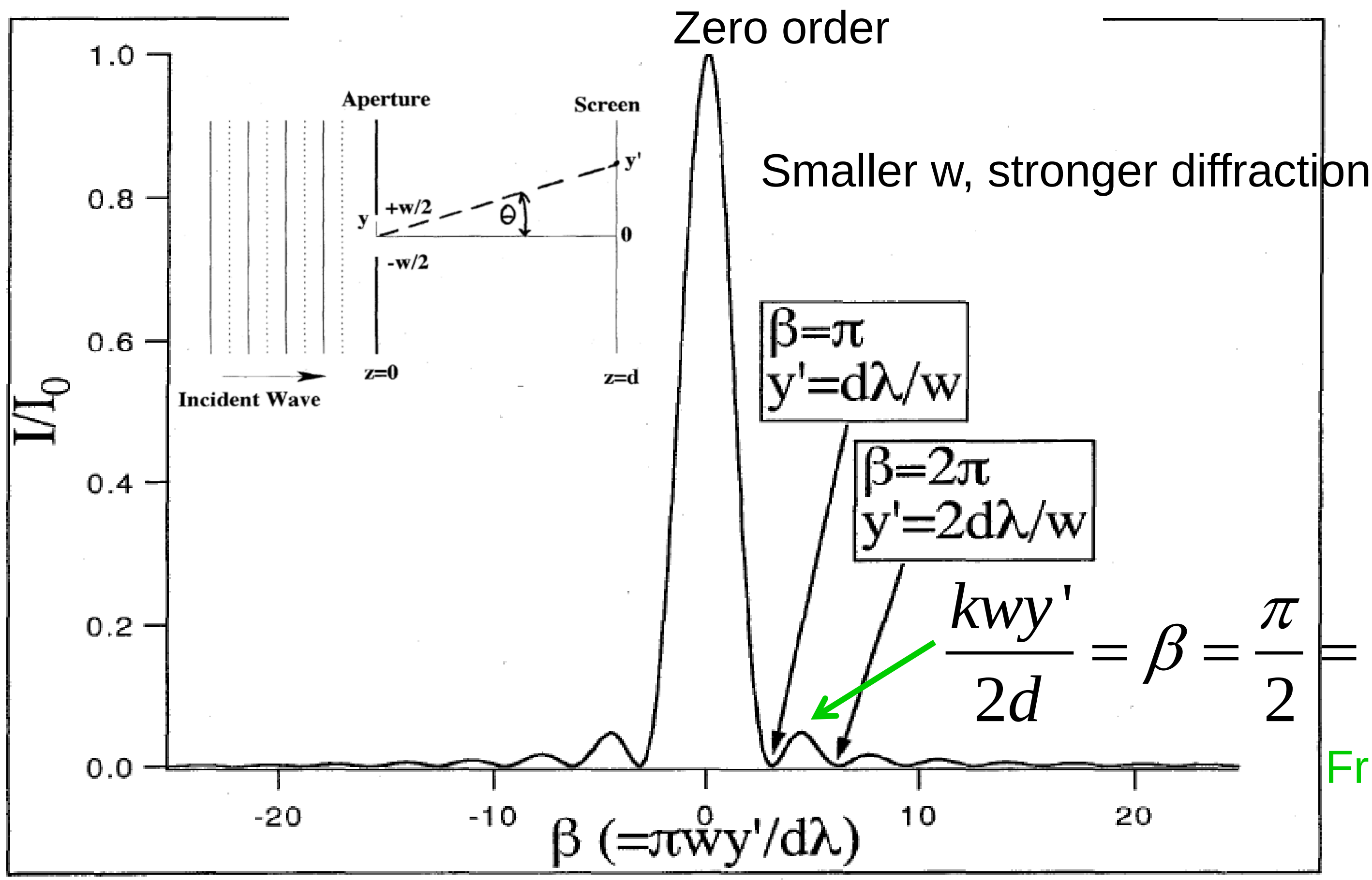
w is largest dim of aperture

$$E \propto \left[\frac{d}{iky'} e^{\frac{iky'y}{d}}\right]_{y=-w/2}^{y=+w/2} = \frac{d}{iky'}\left[e^{\frac{ikwy'}{2d}} - e^{\frac{-ikwy'}{2d}}\right] = \frac{2d}{iky'} \sin\frac{kwy'}{2d}$$

Hecht page 452-469

Single Slit Diffraction Pattern

$$I(y') = I_0 \left(\frac{\sin\beta}{\beta}\right)^2, \qquad \beta = \frac{kwy'}{2d}$$

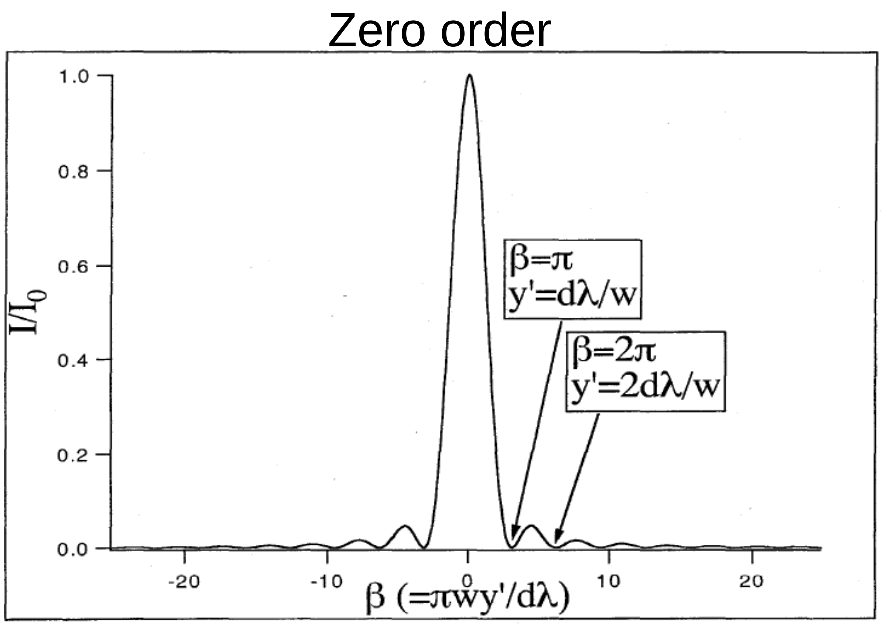


$$\frac{kwy'}{2d} = \beta = \frac{\pi}{2} = \frac{2\pi wy'}{2\lambda d}$$

Can we distinguish different color (wavelength) by the zero order diffraction?

Can we distinguish different color (wavelength) by the first order diffraction?

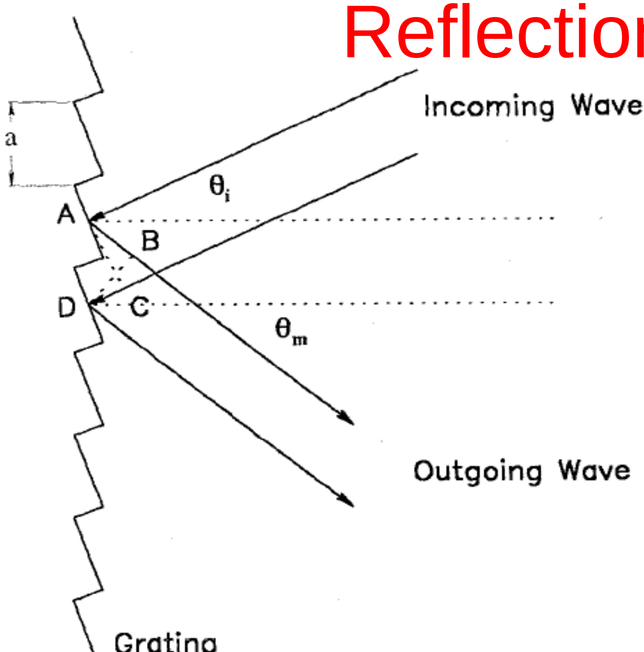
Single Slit Diffraction Pattern

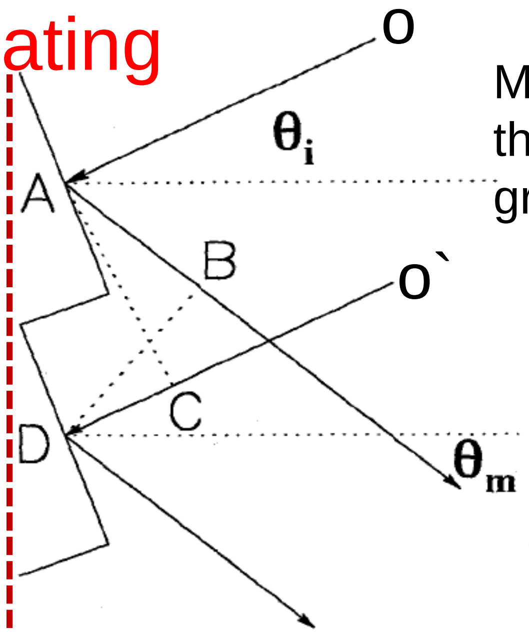


Important concept: most energy is stored in zero order, which is useless for distinguishing different wavelengths.

First order can be used to distinguish different wavelengths. How can we increase the energy in the first order diffraction?

Reflection Grating



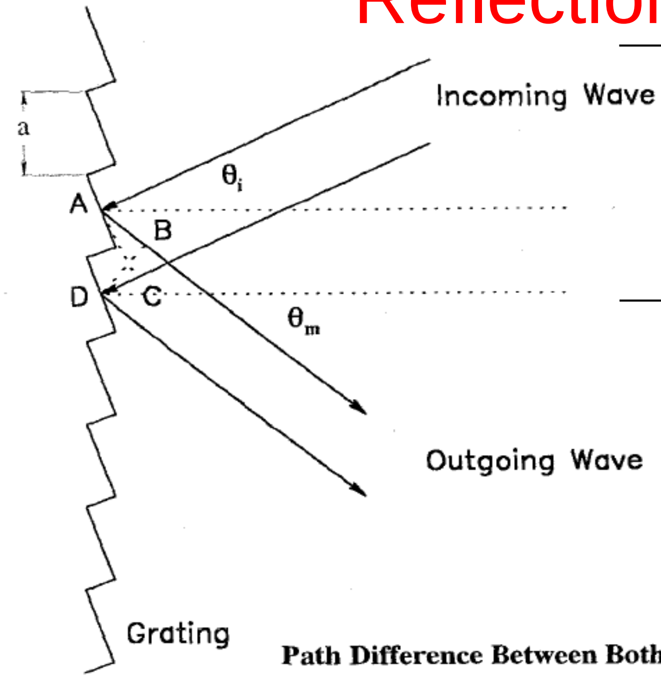


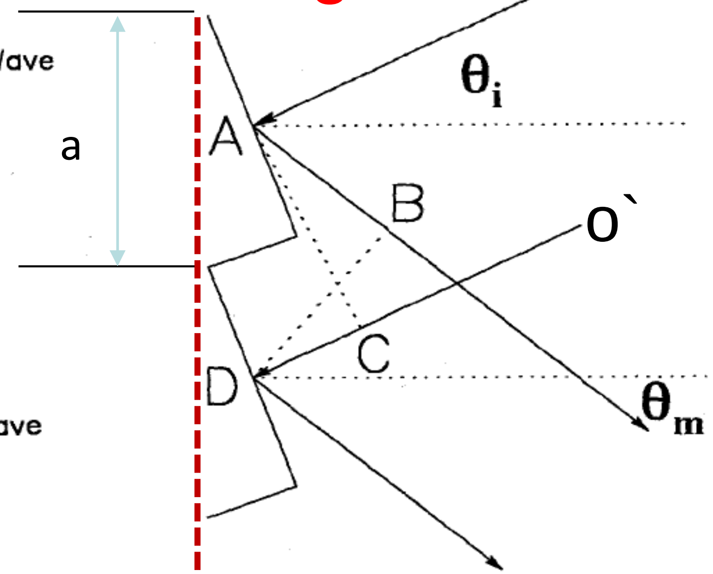
Measured from the normal to the grating plane

Path Difference Between Both Rays =AB-CD=a($\sin\theta_m$- $\sin\theta_i$)

Grating spacing $= a$, **incoming angle** $= \theta_i$, **outgoing angle** $= \theta_m$

Reflection Grating





Path Difference Between Both Rays =AB-CD=a($\sin\theta_m$- $\sin\theta_i$)

Grating spacing $= a$**, incoming angle** $= \theta_i$**, outgoing angle** $= \theta_m$

Constructive Interference from two facets occurs when:

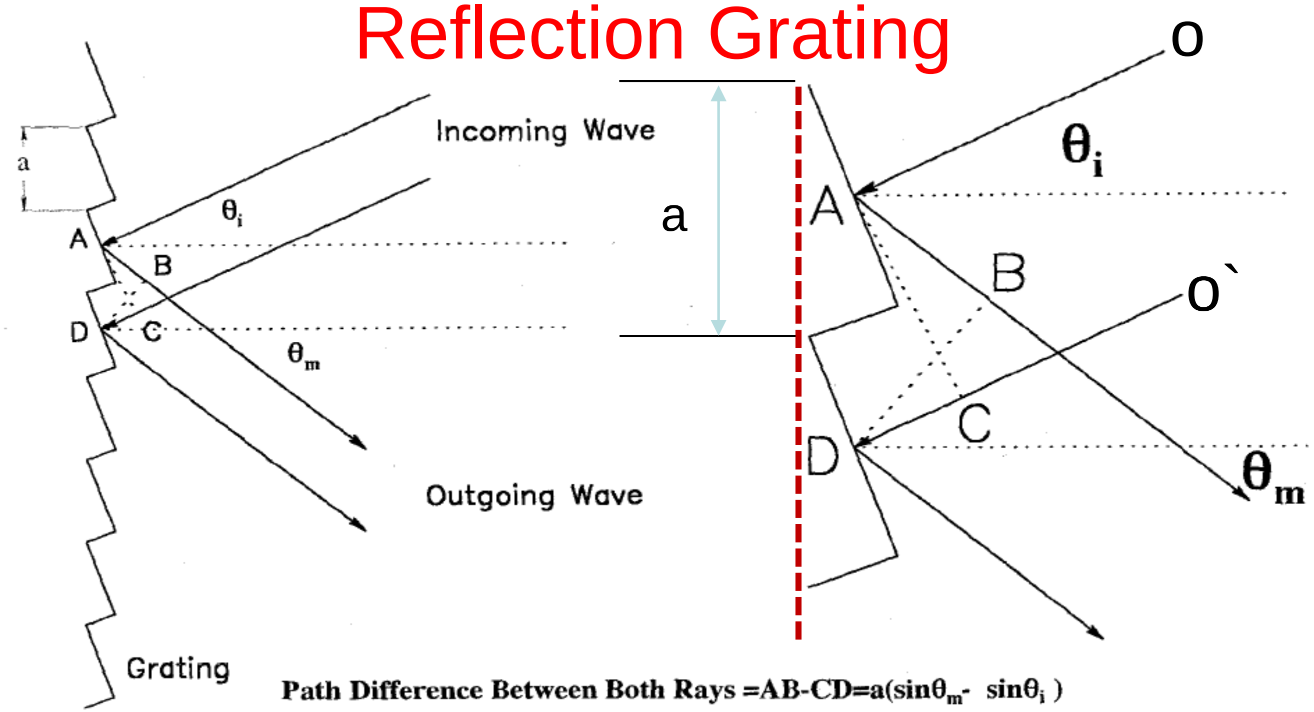
Grating equation Path difference $= AB - CD = a(\sin\theta_m - \sin\theta_i) = m\lambda$ (integral m)

Electric Field from N **facets:** (Define: $\alpha \equiv$ one half phase difference between successive waves$=\frac{ka}{2}(\sin\theta_m - \sin\theta_i)$

k*(AB-CD)=k*a(sin(θ_m)-sin(θ_i))

Electric Field from N **facets:** (Define: $\alpha \equiv$ one half phase difference between successive waves$=\frac{ka}{2}(\sin\theta_m - \sin\theta_i)$

Reflection Grating



Electric Field from N facets: (Define: $\alpha \equiv$ one half phase difference between successive waves$=\frac{ka}{2}(\sin\theta_m - \sin\theta_i)$

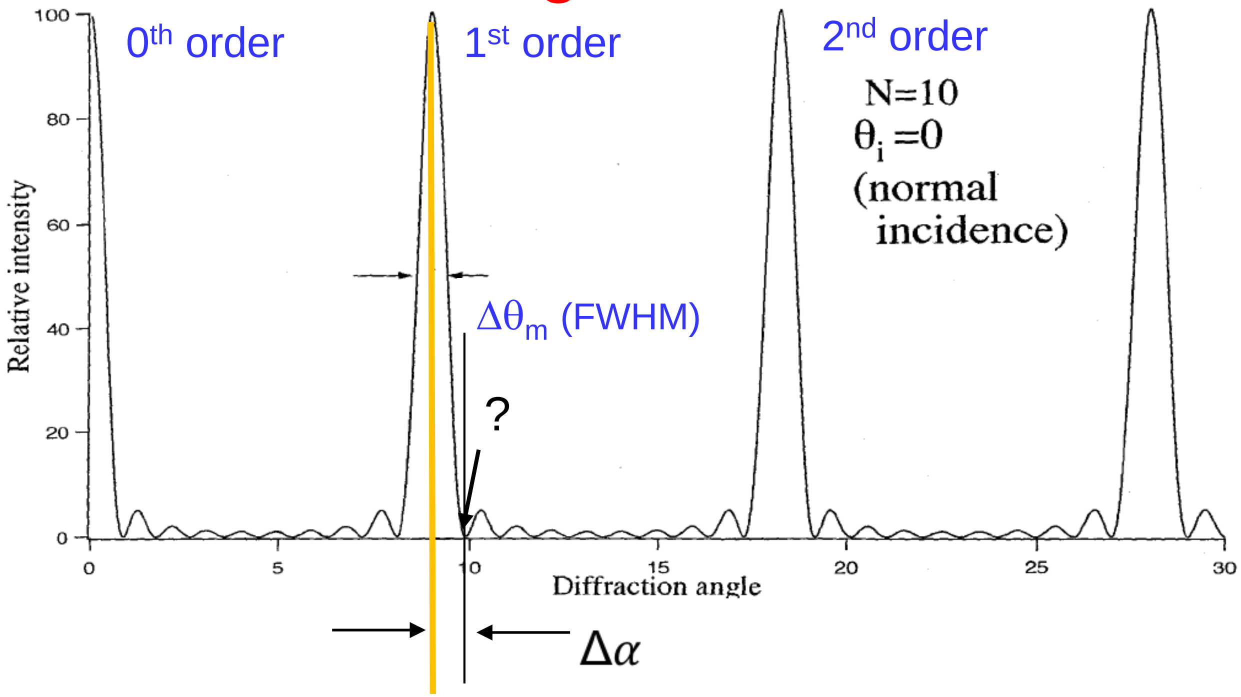
$$E_{total} = E_0 e^{2i\alpha} + E_0 e^{4i\alpha} + E_0 e^{6i\alpha} \ldots E_0 e^{2iN\alpha} \propto \frac{\sin N\alpha}{\sin\alpha}$$

Summing the geometric series

$$\textbf{Intensity:} \quad I \propto \left(\frac{\sin N\alpha}{sin\alpha}\right)^2$$

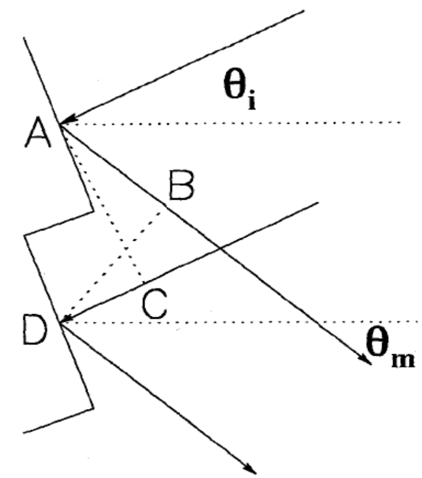
Same expression as **interference factor** in multiple slit diffraction.

Resolving Power of a Diffraction Grating



$$\theta_1 \rightarrow \alpha = \pi$$

$$I \propto \left(\frac{\sin(N\alpha)}{\sin \alpha} \right)^2$$

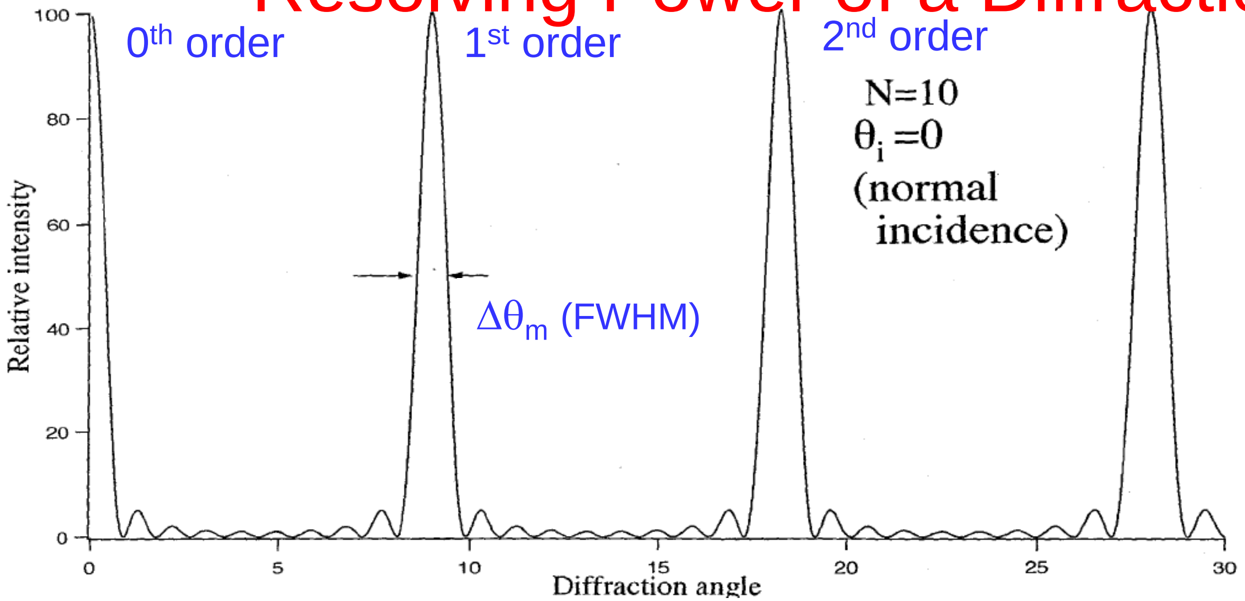


First minimum: $N\Delta\alpha = \pi$ $\quad \Delta\alpha = \frac{\pi}{N}$

$$\alpha = \frac{ka}{2}(\sin\theta_m - \sin\theta_i) \Rightarrow \Delta\alpha = \frac{\pi a}{\lambda}\cos\theta_m \Delta\theta_m$$

$$\Delta\theta_m = \frac{\lambda}{Na\cos\theta_m}$$

Resolving Power of a Diffraction Grating



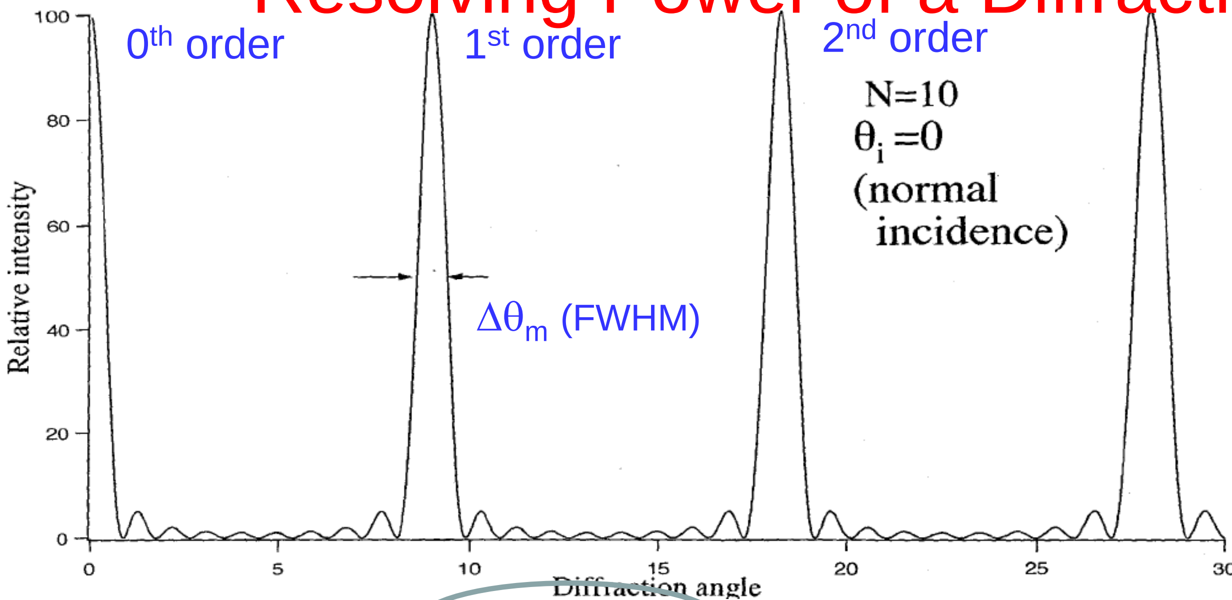
$$I \propto \left(\frac{\sin(N\alpha)}{\sin\alpha}\right)^2$$

$$\alpha = \frac{ka}{2}(\sin\theta_m - \sin\theta_i)$$

Angular Dispersion $\equiv \mathcal{D} \equiv d\theta_m/d\lambda = m/a\cos\theta_m$ (from $a(\sin\theta_m - \sin\theta_i) = m\lambda$)

Resolving Power $\equiv \mathcal{R} \equiv \lambda/(\Delta\lambda)_{min}$

Resolving Power of a Diffraction Grating



$$I \propto \left(\frac{\sin(N\alpha)}{\sin\alpha}\right)^2$$

$$\alpha = \frac{ka}{2}(\sin\theta_m - \sin\theta_i)$$

Angular Dispersion $\equiv \mathcal{D} \equiv d\theta_m/d\lambda = m/a\cos\theta_m$ (from $a(\sin\theta_m - \sin\theta_i) = m\lambda$)

Resolving Power $\equiv \mathcal{R} \equiv \lambda/(\Delta\lambda)_{min}$

$$\Delta\alpha = \frac{ka}{2}\cos\theta_m \Delta\theta_m = \pi/N \Longrightarrow \Delta\theta_m = \frac{\lambda}{Na\cos\theta_m}$$

$$\Delta\theta_m = \mathcal{D}\Delta\lambda = \Delta\lambda\left(\frac{m}{a\cos\theta_m}\right) = \frac{\lambda}{Na\cos\theta_m} \Longrightarrow mN\Delta\lambda = \lambda$$

$$\Longrightarrow \boxed{\mathcal{R} = mN} = \frac{Na(\sin\theta_m - \sin\theta_i)}{\lambda}$$

$$\frac{a(\sin\theta_m - \sin\theta_i)}{\lambda} = m$$

Resolving Power = (mode number)×(number of lines illuminated)

(order number)

$$I \propto \left(\frac{\sin(N\alpha)}{\sin\alpha}\right)^2$$

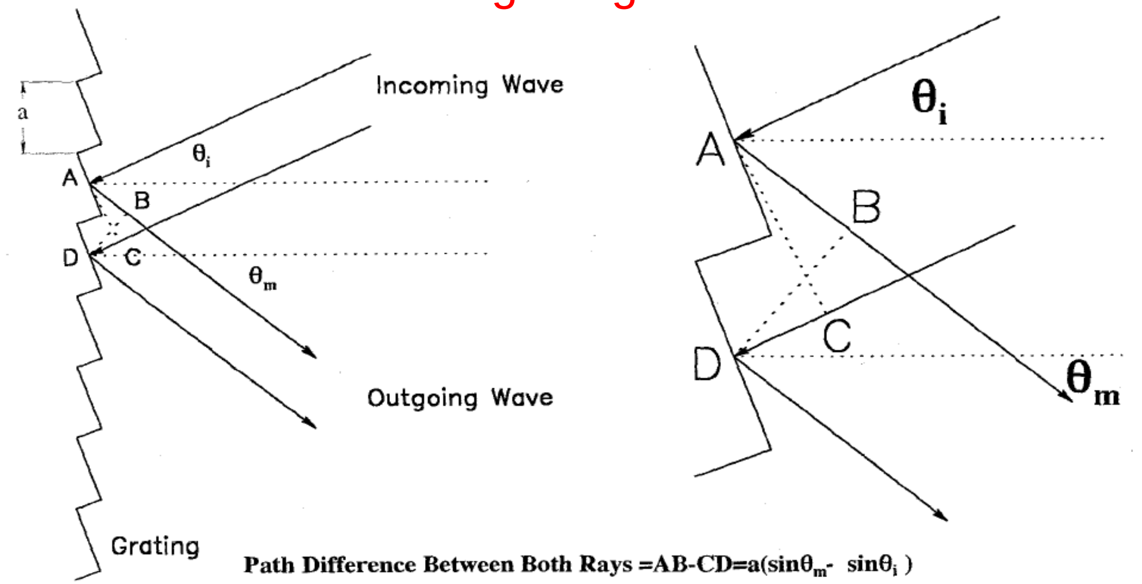
$$\Delta\theta_m = \frac{\lambda}{Na\cos\theta_m}$$

Na: the size of the grating

$$\Longrightarrow \boxed{\mathcal{R} = mN} = \frac{Na(\sin\theta_m - \sin\theta_i)}{\lambda}$$

Resolving Power = (mode number)×(number of lines illuminated)

Blaze grating



$$\text{Path difference} = AB - CD = a(\sin\theta_m - \sin\theta_i) = m\lambda \quad (\text{integral } m)$$

Question: is it a useful grating if most of energy is concentrated in the zeroth order? Zeroth order: $\theta_m = \theta_i$ and m=0.

Blazed Gratings

Blaze is defined as the concentration of a limited region of the spectrum into any order other than the zero order. Blazed gratings are manufactured to produce maximum efficiency at designated wavelengths.

http://www.horiba.com/scientific/products/optics-tutorial/diffraction-gratings/

Blaze Grating: Littrow Condition

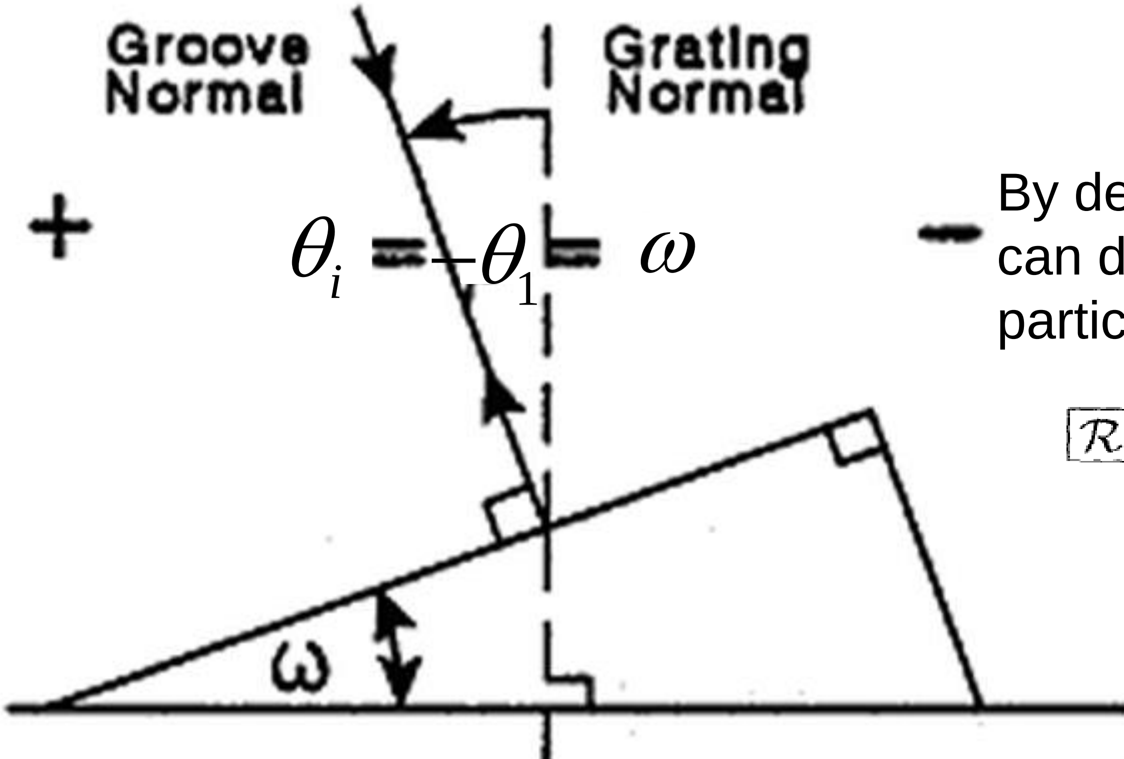
The incoming wave is normal to the groove face

if $\theta_i = -\theta_1$ It is called Littrow Condition

$$a(\sin(\theta_i) - \sin(\theta_1)) = \lambda$$

$$\theta_i = \arcsin(\frac{\lambda}{2a})$$

Blazing angle



$$\theta_i = -\theta_1 = \omega$$

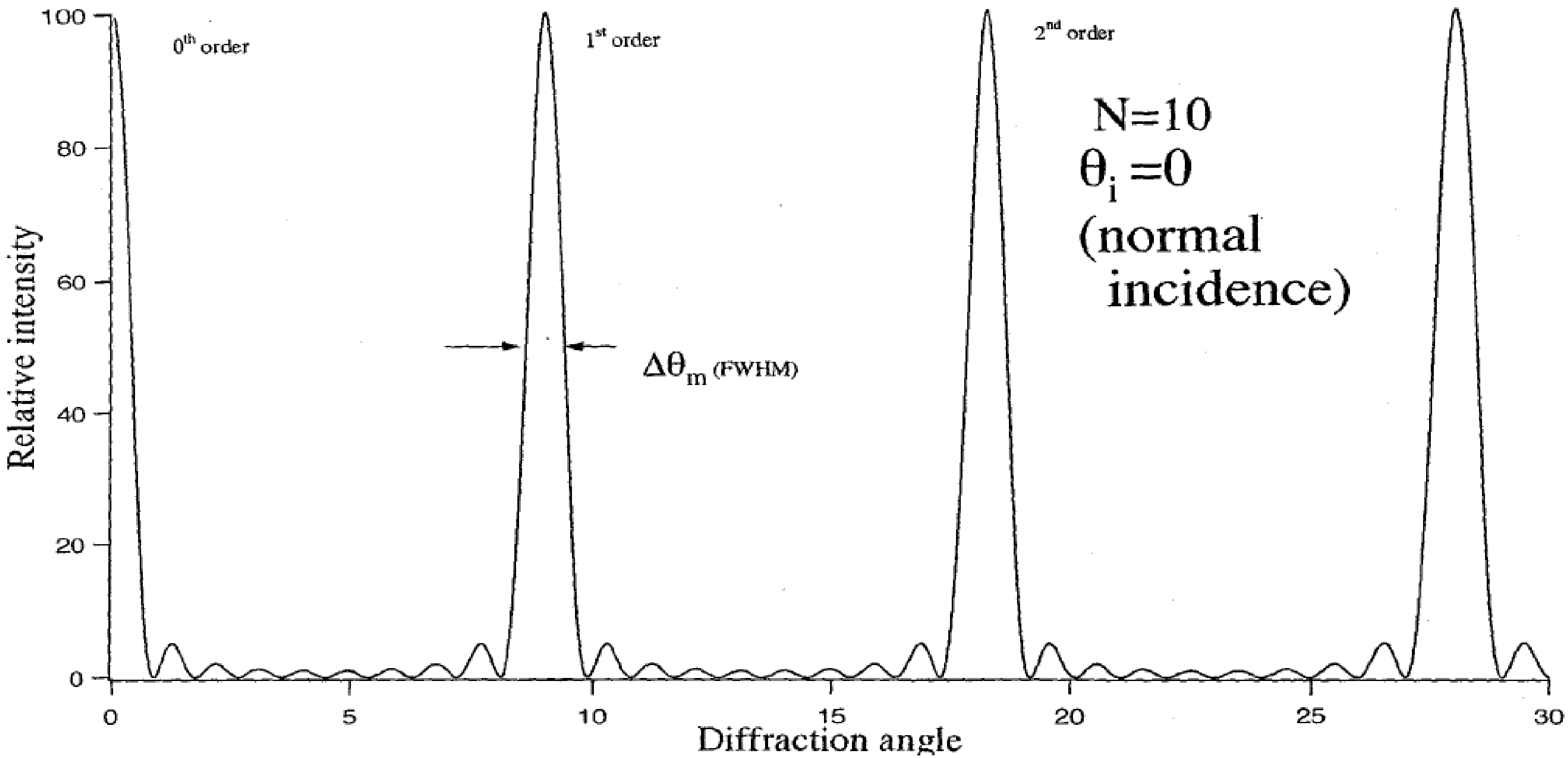
By designing a and wedge angle, we can design a grating optimizing for a particular wavelength.

$$\boxed{\mathcal{R} = mN} = \frac{Na(\sin\theta_m - \sin\theta_i)}{\lambda}$$

Harrison’s 260 mm wide gratings with blaze angle 75°

λ=500 nm

λ=500nm-> R>10^6



Which is the best diffraction order for spectroscopy?

Blaze Grating: Free Spectral Range

$$a(\sin(\theta_i) - \sin(\theta_1)) = \lambda$$ (eg 600nm)

$$a(\sin(\theta_i) - \sin(\theta_1)) = 2 \times \frac{\lambda}{2}$$ (300nm)

$$a(\sin(\theta_i) - \sin(\theta_1)) = 3 \times \frac{\lambda}{3}$$ (200nm)

First of 600nm overlaps with the second order of 300nm, and overlaps with the third order of 200nm…..

If two lines of wavelength λ and λ+Δλ in successive orders (m+1) and m overlaps,

$$a(\sin(\theta_i) - \sin(\theta_m)) = (m+1)\lambda = m(\lambda + \Delta\lambda)$$

Free Spectral Range

$$(\Delta\lambda)_{fsr} = \frac{\lambda}{m}$$

$$\boxed{\mathcal{R} = mN} = \frac{Na(\sin\theta_m - \sin\theta_i)}{\lambda}$$

Example

White light falls normally on a transmission grating that contains 1000 lines /cm. At what angle will red light (650nm) emerge in the first order spectrum?

Hint: $a(\sin\theta_m - \sin\theta_i) = m\lambda \quad (\text{integral } m)$

$$a = \frac{1}{1000} cm$$

$$\theta_i = 0$$

$$m = 1$$

$$\lambda = 650nm$$

$$\theta_m = \arcsin(\lambda / a)$$