Holography and Faraday Rotation Fourier Optics

Please turn in your late reports to TAs

Wednesday: Veteran's day. The make up lab will be the Wednesday of the Thanks Giving week.

Fourier Analysis

Fourier series for periodic functions:

$$f(x) = \frac{A_0}{2} + \sum_0^\infty A_m \cos(mkx) + \sum_0^\infty B_m \sin(mkx)$$

$$A_m = \frac{2}{\lambda}\int_0^\lambda f(x)\cos(mkx)dx$$

k=2π/ λ

$$B_m = \frac{2}{\lambda}\int_0^\lambda f(x)\sin(mkx)dx$$

Determine A and B through Fourier analysis.

Fourier's Theorem: a function f(x), having a spatial period λ, can be synthesized by a sum of harmonic functions whose wavelengths are integral of submultiples of λ.

Hecht, Pg 304

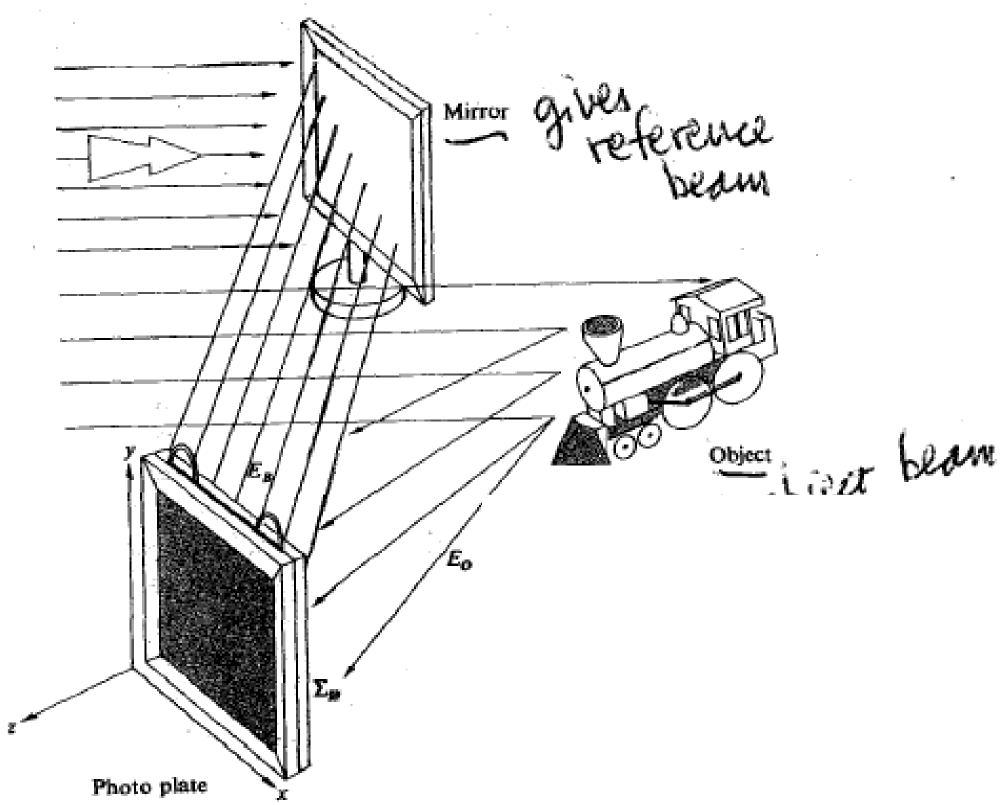
Holograms: Fourier Optics

Dennis Gabor

1960, Leith & Upatnieks @ U. Michigan

Coherent reference wave

Recording amplitude and phase of light field. Gives "3D" picture.



Key Idea – record interference pattern between object beam and reference beam

Hecht pg 626, fig. 13.45

Dennis Gabor

Holograms: Fourier Optics

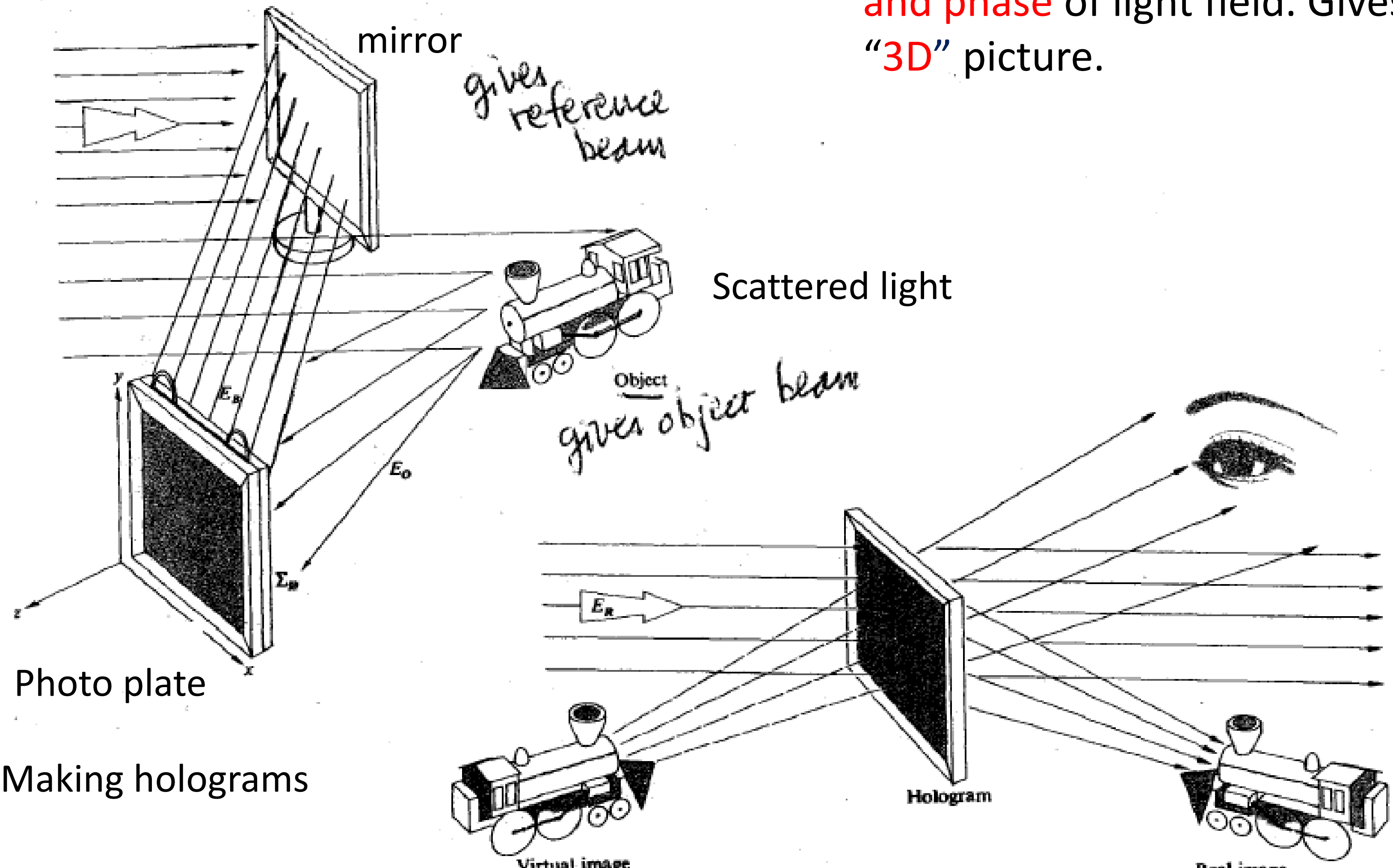
1960, Leith & Upatnieks @ U. Michigan

Recording amplitude and phase of light field. Gives "3D" picture.

Coherent reference wave

mirror

gives reference beam



Scattered light

gives object beam

Photo plate

Making holograms

Virtual image

Reconstruct wavefront

(conjugate image)

Key Idea – record interference pattern between object beam and reference beam

Hecht pg 626, fig. 13.45

Hololens

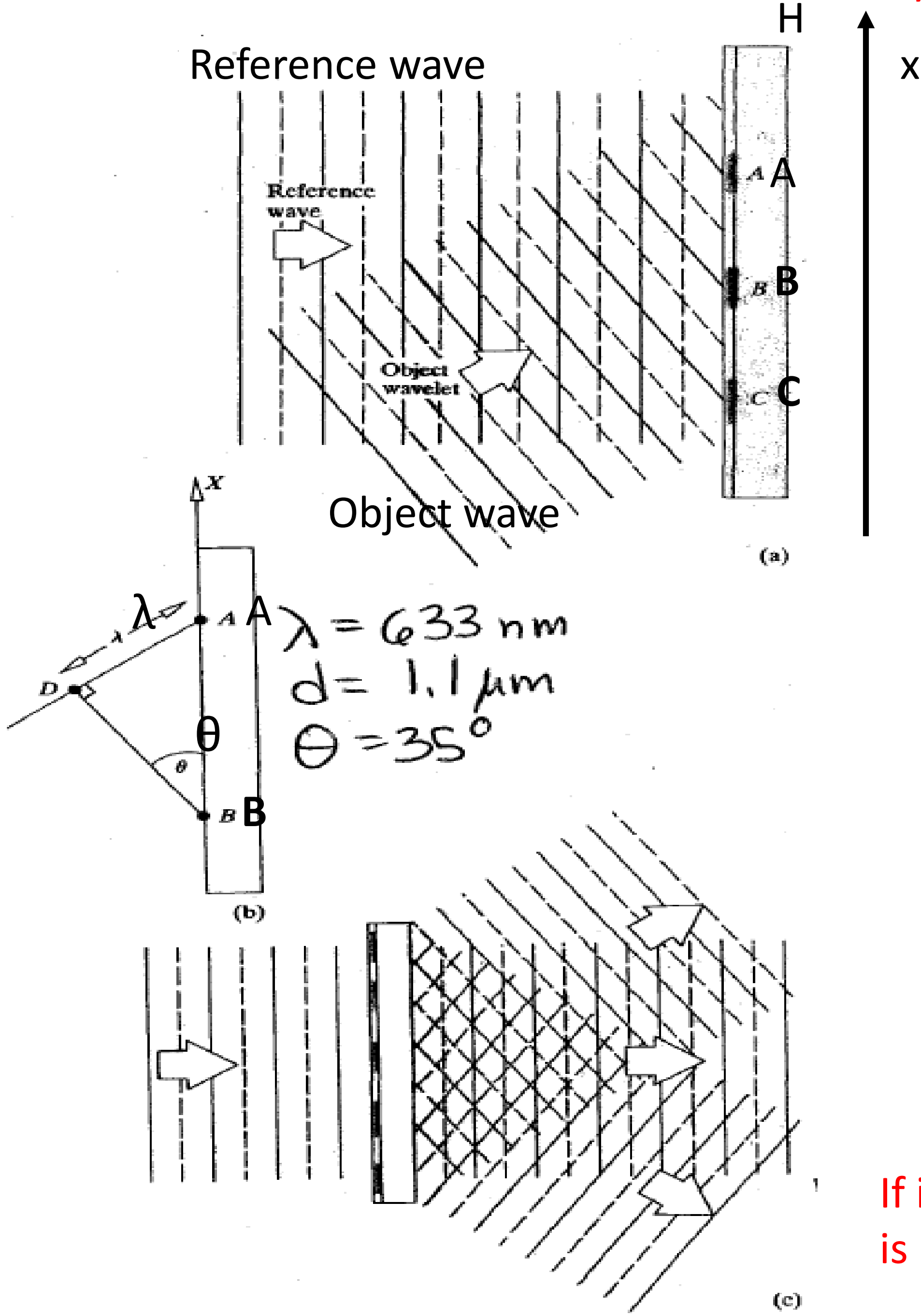
What does standard photograph do?

Holograms: Fourier Optics

- Standard photographs – only record intensity distribution of light.

- Holography, Fourier transform between real image to spatial frequency domain. Spatial frequency of the objective is recorded by the film-> hologram. Fourier transform of the hologram reconstruct the object.

A Simple View of Hollogram



Hecht pg 628, fig. 13.48

$$\overline{AB} = \frac{\lambda}{\sin\theta}$$

$$\frac{x}{\overline{\overline{AB}}} = \frac{\phi(x)}{2\pi}$$

$$\phi(x) = 2\pi x \sin(\theta) / \lambda$$

$$= kx\sin(\theta)$$

The intensity distribution on the plane

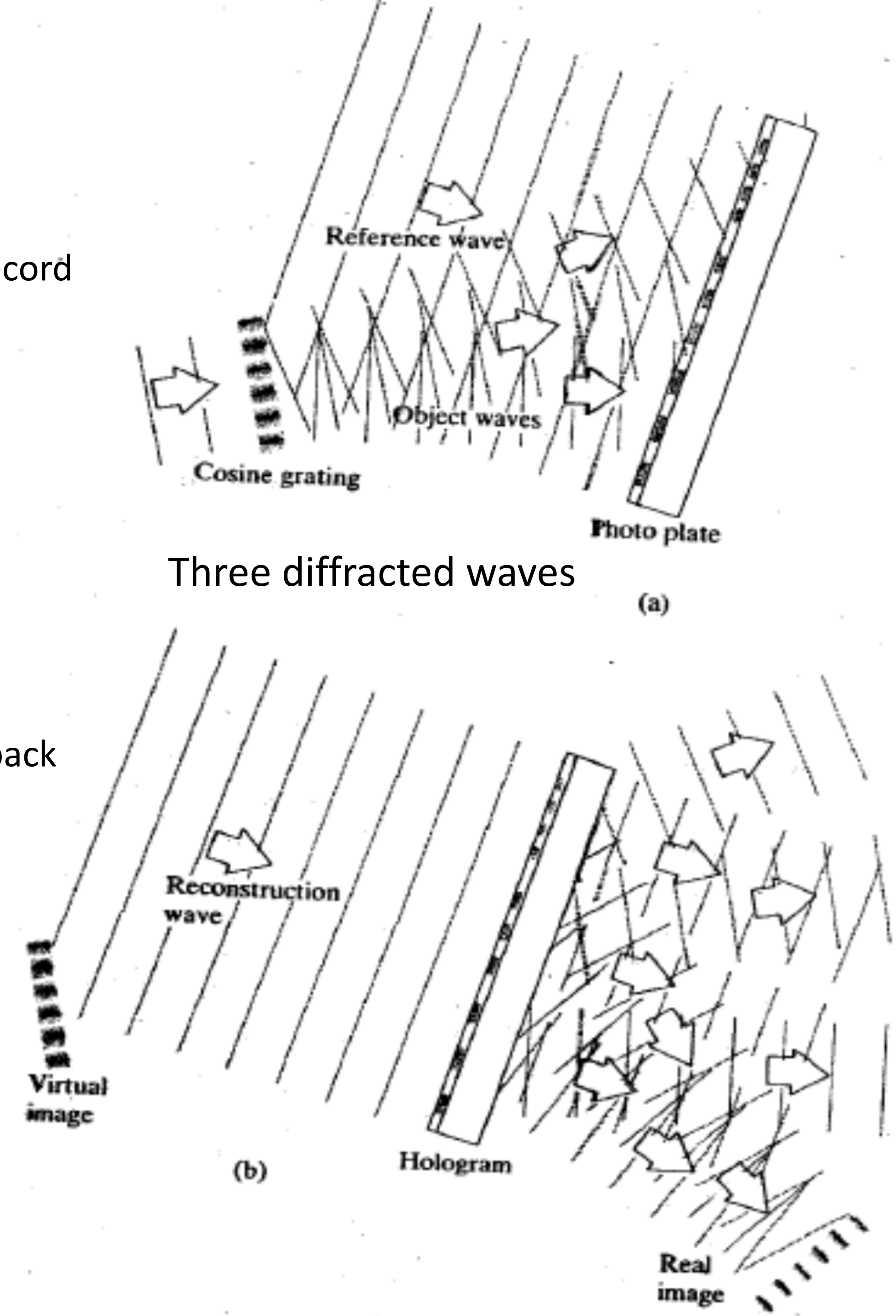
$$I(x) = I_o(1 + \cos\phi)$$

A cosine grating is created with spatial period of AB .

If intensity is correct, a cosine (sine) grating is produced with diffraction peaks 0, +/- 1 only

Hologram of Cosine Grating

Record



Three diffracted waves

Playback

The amplitude of the object wave at every point on the film plane will be encoded in the visibility of the resulting fringes.

Complicated object -> complicated fringe pattern and contrast

Hologram: record both intensity and phase information.

Question: In the hologram, where is the phase information stored?
And where is the intensity information stored?

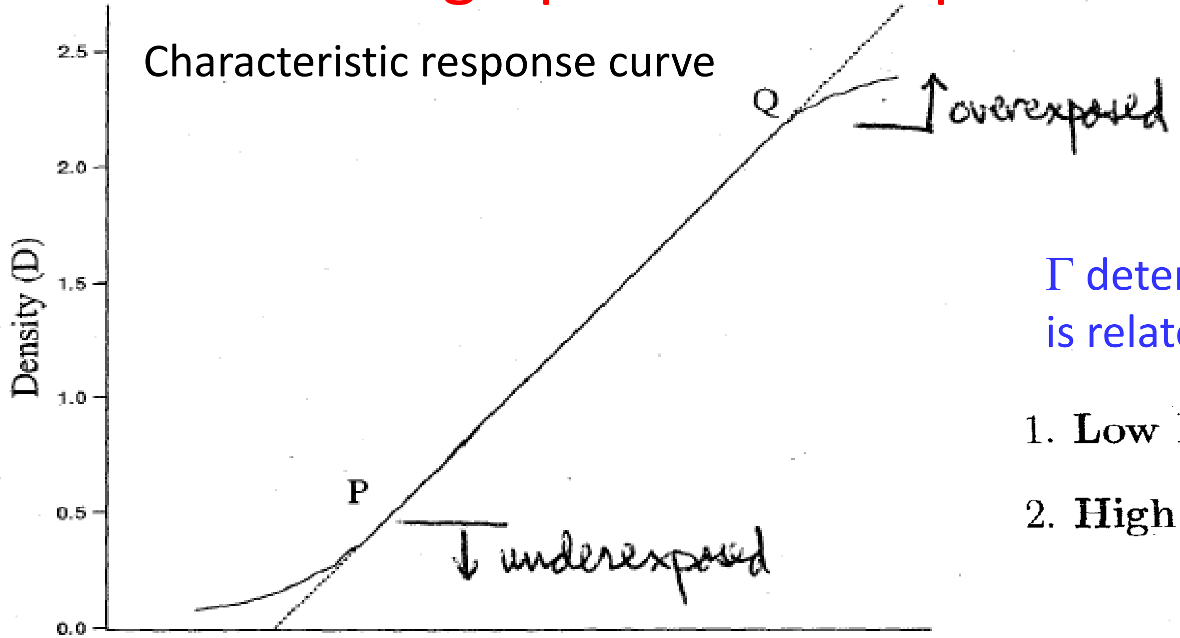
Question: In the hologram, where is the phase information stored? (fringe pattern)

And where is the intensity information stored? (fringe contrast)

If we block most of the hologram and only leave part of it illuminated by the reconstructed wave, can we observe the whole object and why?

If yes, what is the difference compared to illuminating the whole hologram?

Photographic Film Response



Γ determines contrast and is related to film speed

1. **Low** $\Gamma \approx 1 \longrightarrow$ **Low contrast**
2. **High** $\Gamma \approx 2-3 \longrightarrow$ **High contrast**

Above curve is called: **Hurter-Driffield curve of photographic response.**

Plot of D **(Density)** vs log of H **(Exposure)**:

$\boxed{D \equiv -\log_{10} T}$ T is **Intensity Transmission** of developed photograph

$\boxed{H \equiv It}$ I is **Intensity of Light,** t is **Exposure time**

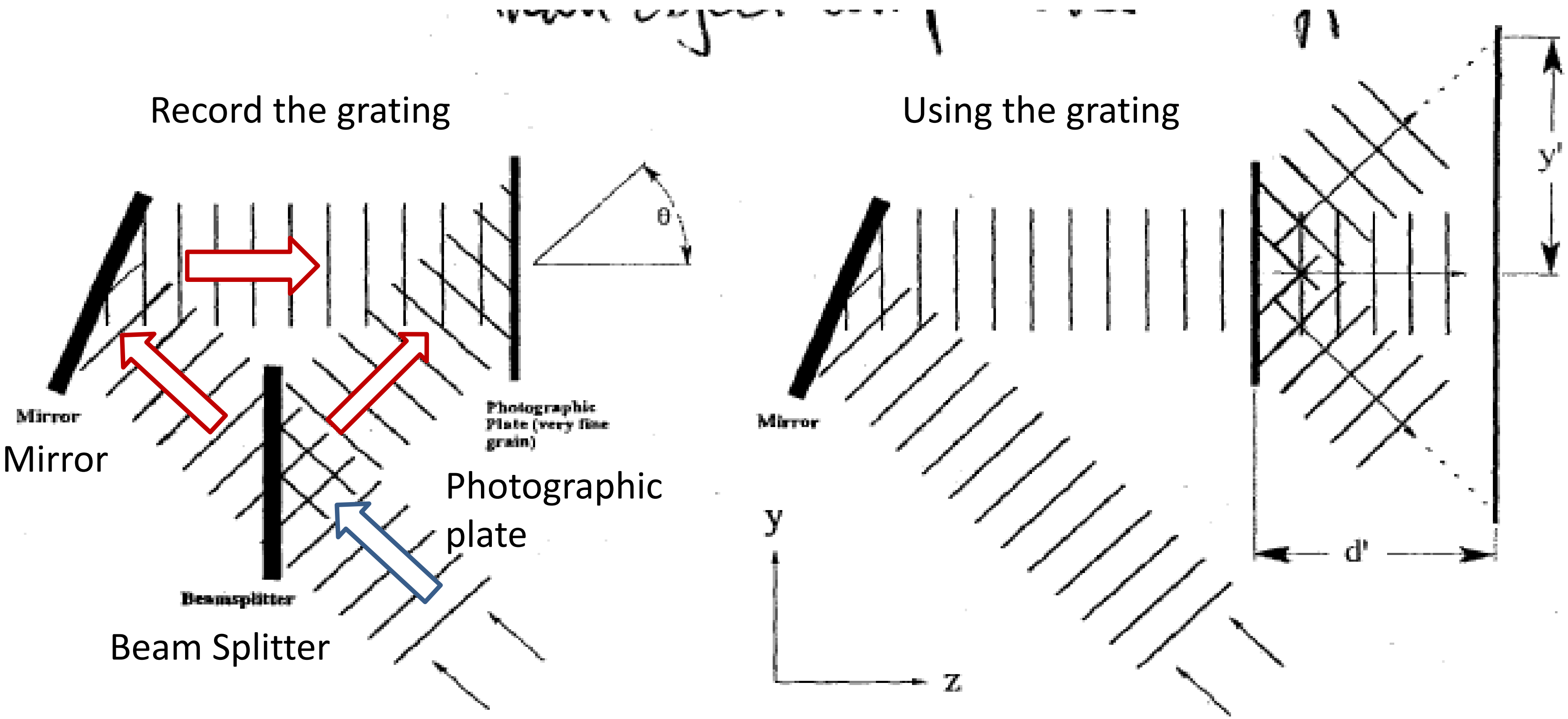
Slope of Linear region - between P **and** Q **is called:** Γ

Therefore: $D = D_0 + \Gamma \log_{10} \frac{H}{H_0}$ and $\boxed{T = T_0 \left(\frac{H}{H_0}\right)^{-\Gamma} = T_0 \left(\frac{I}{I_0}\right)^{-\Gamma}}$

Amplitude transmission $\equiv \boxed{t = t_0 \left(\frac{I}{I_0}\right)^{-\Gamma/2}}$

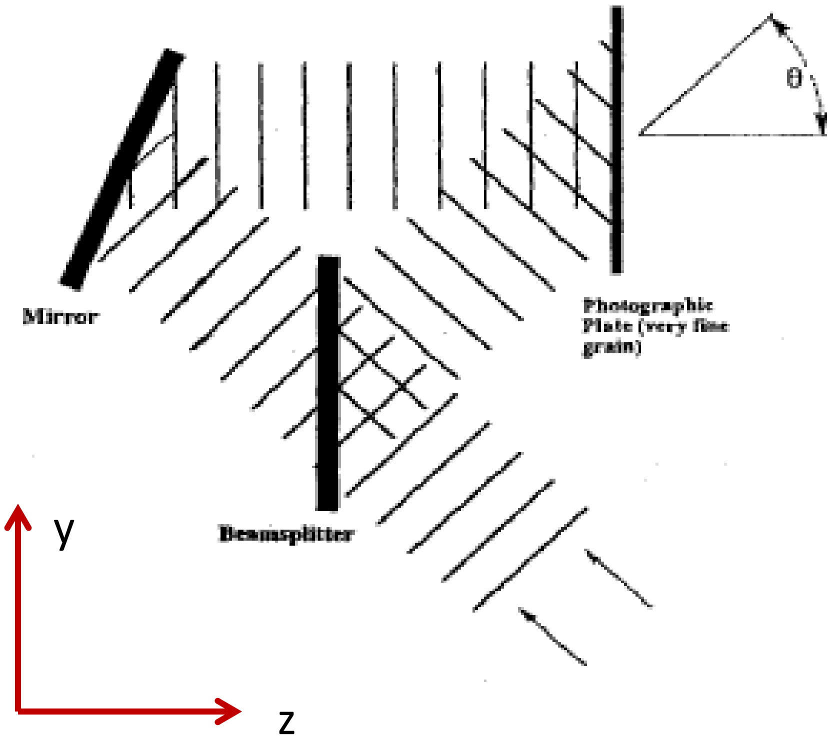
Γ **determines CONTRAST of file:**

Diffraction Grating generated using Holography



Because a holographic diffraction grating has no periodic errors or imperfections, it exhibits significantly lower stray light compared to a ruled grating

Diffraction Grating generated using Holography



Recording:

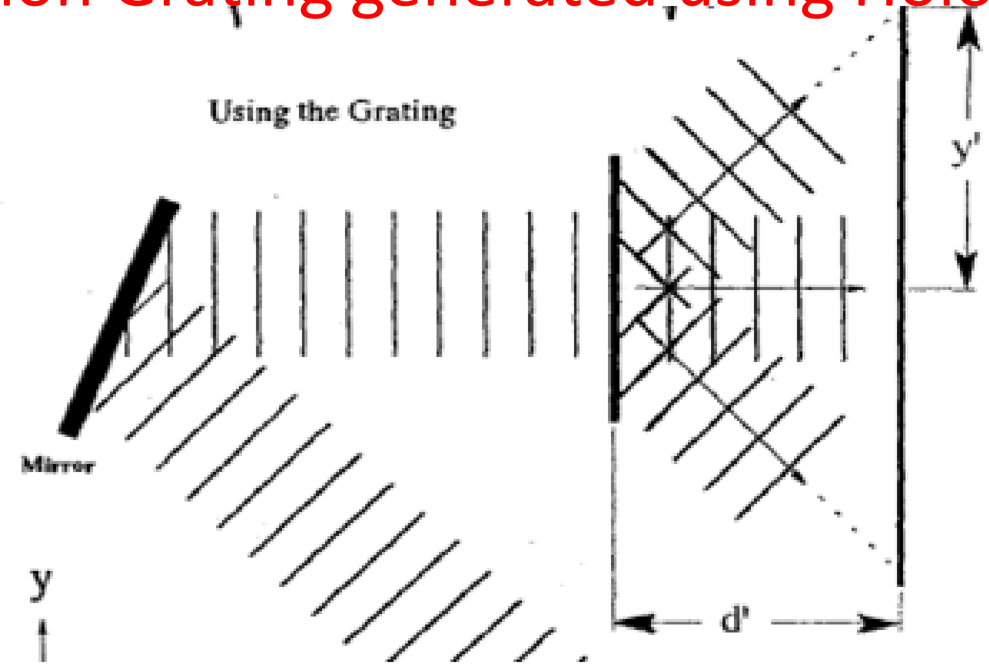
$$E = a_r e^{ikz} + a_s e^{i\mathbf{k}(z\cos\theta + y\sin\theta)}, \qquad a_r >> a_s$$

$$I \propto a_r^2 + a_s^2 + 2a_r a_s \cos\left(ky\sin\theta + \phi_0\right), \quad \phi_0 = kd(1-\cos\theta), \ \text{film at } z = d$$

This is a **cosine grating** since darkening has a **cosine dependence on** y

Plane Wave Illumination of Grating: This is a **Fraunhofer Diffraction Problem:** Integrate over **Aperture function** (film-observer distance = d')

Diffraction Grating generated using Holography



Aperture Function: $t(y) \propto I^{-\Gamma/2} \propto \left(a_r^2 + a_s^2 + 2a_r a_s \cos\left(ky\sin\theta + \phi_0\right)\right)^{-\Gamma/2}$

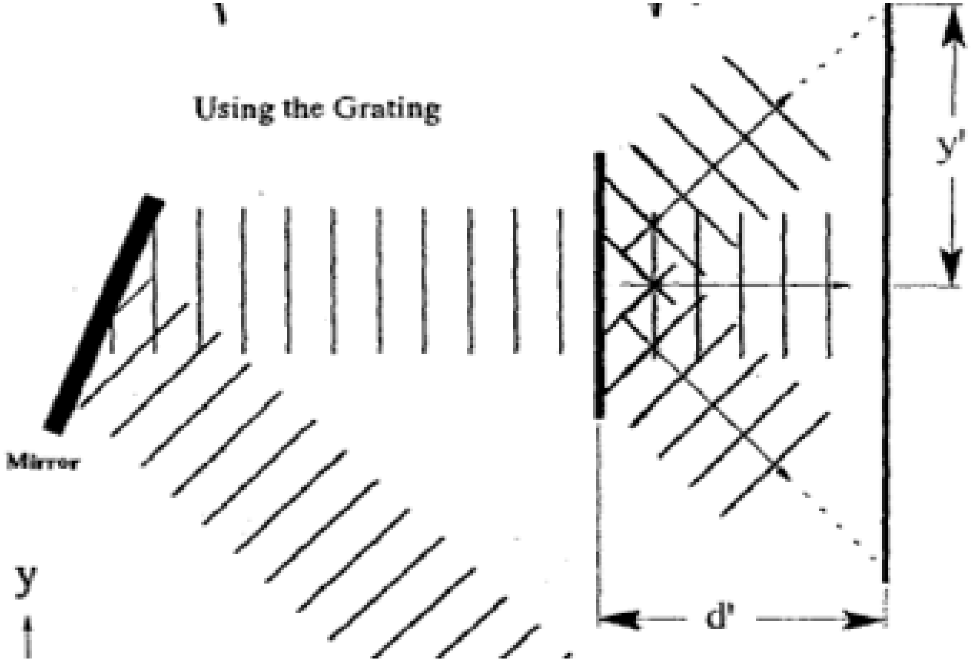
$$t(y) \propto a_r^2\left(1 - \Gamma\frac{a_s}{a_r}\cos\left(ky\sin\theta + \phi_0\right)\right) \quad \text{since} \quad a_r >> a_s$$

$$E_t(y', z' = d') \propto \int_{-\infty}^{\infty} (\text{aperture function}) e^{\frac{iky'y}{d'}} dy$$

$$\propto \int_{-\infty}^{\infty}\left(e^{\frac{iky'y}{d'}} - \Gamma\frac{a_s}{2a_r}e^{iky(y'/d' + \sin\theta)} - \Gamma\frac{a_s}{2a_r}e^{iky(y'/d' - \sin\theta)}\right)dy, \quad (\phi_0 = 0)$$

$$\boxed{E(y') \propto d'\delta(y') - \Gamma\frac{a_s}{2a_r}\delta(y'/d' - \sin\theta) - \Gamma\frac{a_s}{2a_r}\delta(y'/d' + \sin\theta)}$$

Diffraction Grating generated using Holography



Result: 3 waves, one undeflected, 2 deflected at angle $\sin\theta$ (note: same as recording angle for small θ!)

$$E(y') \propto d'\delta(y') - \Gamma\frac{a_s}{2a_r}\delta(y'/d' - \sin\theta) - \Gamma\frac{a_s}{2a_r}\delta(y'/d' + \sin\theta)$$