Faraday Rotation + Geometrical Optics

- Faraday Rotation
  - Polarization of Light
  - Optical Activity (birefringence)
  - Zeeman Effect induced Faraday Rotation

- Geometrical Optics
  - Law of Reflection and Refraction
  - Brewster Angle

- Make-up labs for the next week

nature nanotechnology

LETTERS

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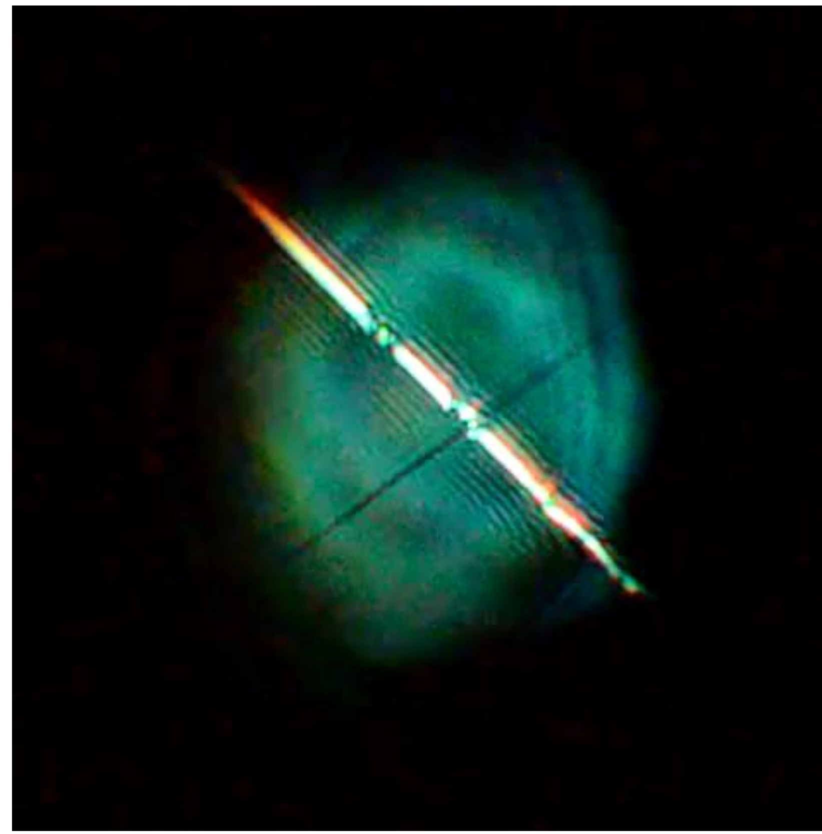
High-throughput optical imaging and spectroscopy of individual carbon nanotubes in devices

Carbon Nanotube – nanometer diameter



Polarization of Light

$$\overrightarrow{E}$$

Polarized electric field

$$\vec{k}$$

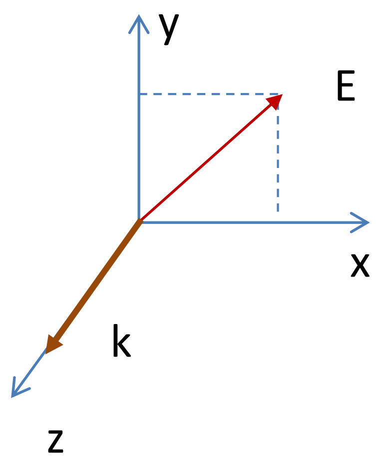
Wave-vector

$$\overrightarrow{E} \bullet \vec{k} = 0$$

Transverse wave

$$\overrightarrow{E} \bullet \vec{k} \neq 0$$

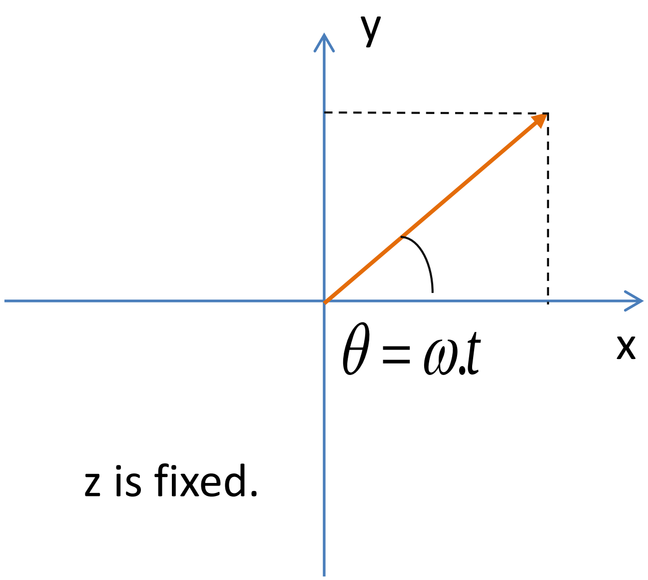
Longitudinal wave



$$\overrightarrow{E_x}(z,t) = \hat{x}E_x \cos(k.z - \omega t)$$

$$\overrightarrow{E_y}(z,t) = \hat{y}E_y \cos(k.z - \omega t + \xi)$$

Linearly Polarized Light



z is fixed.

The projection of a vector on x axis

$$\overrightarrow{E_x}(z,t) = \hat{x}E_x \cos(k.z - \omega t)$$

The projection of a vector on y axis

$$\overrightarrow{E_y}(z,t) = \hat{y}E_y \cos(k.z - \omega t + \xi)$$

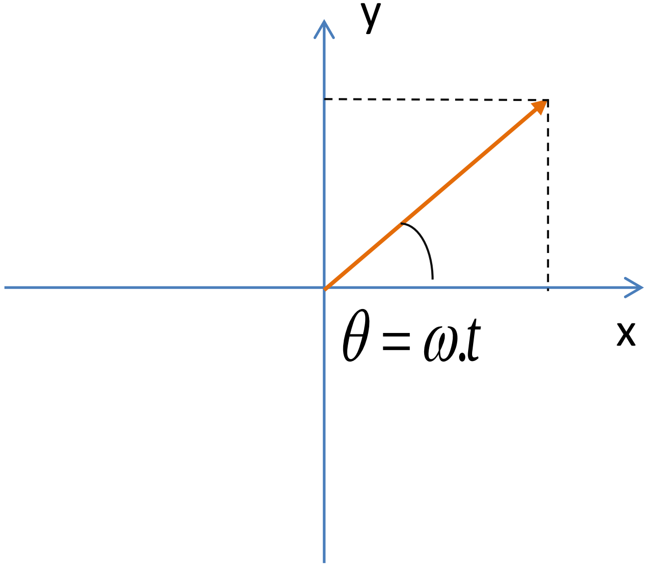
If $\xi = 0$

$$\vec{E}(z,t) = \left(xE_x + \hat{y}E_y\right)\cos(kz - \omega t)$$

Linearly polarized light
Electric field direction is fixed in one axis, and the amplitude oscillates.

3D movie glasses: one eyeglass is horizontally polarized.
The other one is vertically polarized.

Circularly Polarized Light



Circular polarized light: If $\xi = -\frac{\pi}{2}$

$$\overrightarrow{E_x}(z,t) = \hat{x}E_x \cos(k.z - \omega t)$$

$$\overrightarrow{E_y}(z,t) = \hat{y}E_y \sin(k.z - \omega t)$$

$$E_x = E_y = E_o$$

$$\vec{E}(z,t) = E_o\left(x\cos(kz-\omega t) + y\sin(kz-\omega t)\right)$$

Electric field amplitude is fixed, but the direction rotates

Clock wise: Right circular polarized light σ^-

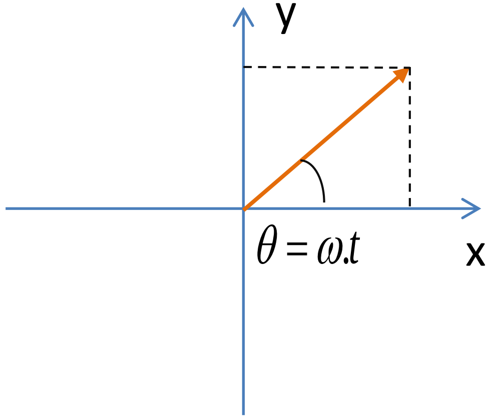
Anti-clock wise: Left circular polarized light σ^+

$$\vec{E}(z,t) = E_o\left(x\cos(kz-\omega t) + y\sin(kz-\omega t)\right)$$

Electric field amplitude is fixed, but the direction rotates

question: what type of circular polarization does this formula represent?

$$\vec{E}(t) = E_o(\hat{x}\cos(\omega t) - \hat{y}\sin(\omega t))$$



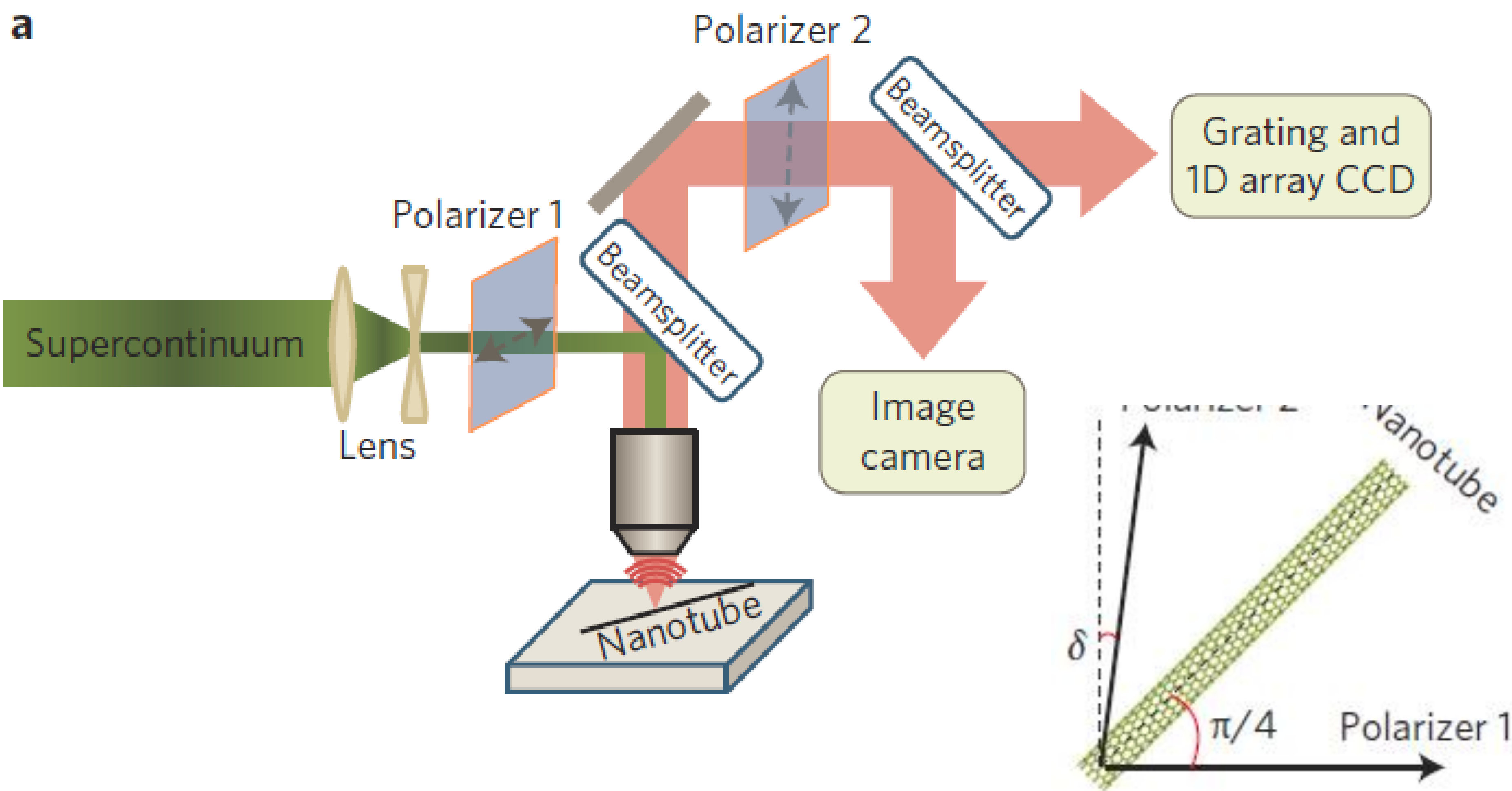
question: what type of polarization will it be if E_x is not equal to E_y ?

Elliptically polarized light

$$\vec{E}(z,t) = E_x\hat{x}\cos(kz-\omega t) + E_y\hat{y}\sin(kz-\omega t))$$

Optical Activity

Any material that causes the E field of an incident linear plane wave to rotate is said to be optically active. (sugar, quartz…)
Demo – the effect of sugar on light polarization



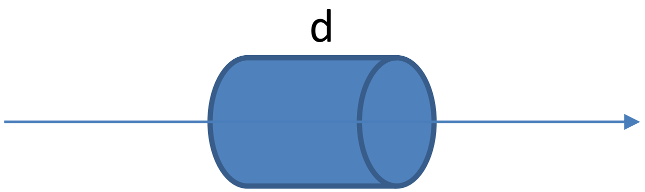
Optical Birefringence

$$\overrightarrow{E_x}(z,t) = \hat{x}E_x \cos(k.z - \omega t)$$

$\xi = 0 \rightarrow \textit{linearly polarzed light}$

$$\overrightarrow{E_y}(z,t) = \hat{y}E_y \cos(k.z - \omega t + \xi)$$

$\xi = \pm\frac{\pi}{2} \rightarrow \textit{circularly polarzed light}$



Refractive index , n, has polarization dependence ($n(\hat{x}) \neq n(\hat{y})$)

Assuming the incident light is linearly polarized ($\xi = 0$)

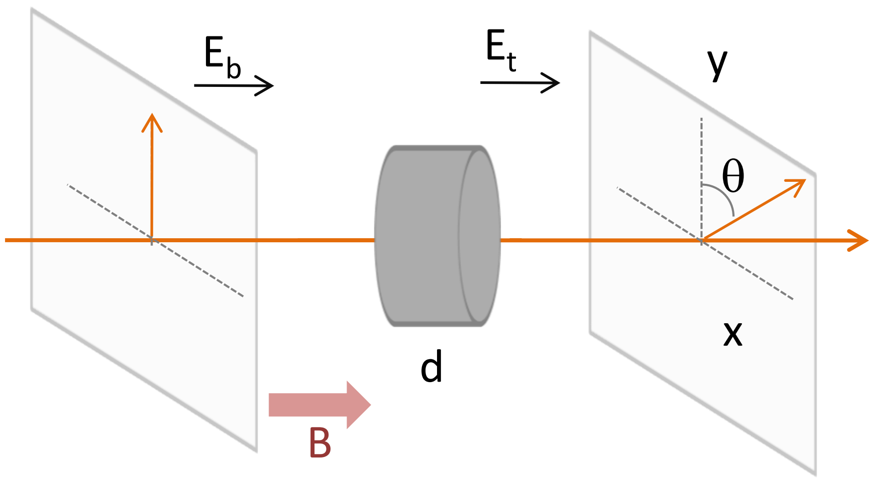
After travelling through the material

$$\xi = (n(x) - n(y))kd$$

Birefringence can be induced by many physical mechanisms: stress, magnetic field

Faraday Rotation

Faraday Effect: The polarization of linearly polarized light rotates when it passes through a material with a magnetic field applied along the propagation direction.



$$\theta = VBd$$

v: Verdet constant

v>0 : left rotation k||B

right rotation k is anti || B

Reversal of handedness

Natural optical activity:
no reversal of handedness

Faraday Rotation

$$\sigma^{\text{L}} = E_o\left(x\cos(kz-\omega t)+\hat{y}\sin(kz-\omega t)\right)$$

$$\sigma^{\text{R}} = E_o\left(x\cos(kz-\omega t)-\hat{y}\sin(kz-\omega t)\right)$$

$$\overrightarrow{E}_x = \frac{\left(\sigma^R+\sigma^L\right)}{2}$$

$$\overrightarrow{E}_y = \frac{-\left(\sigma^R-\sigma^L\right)}{2}$$

The material is circular birefringence in the presence of magnetic field

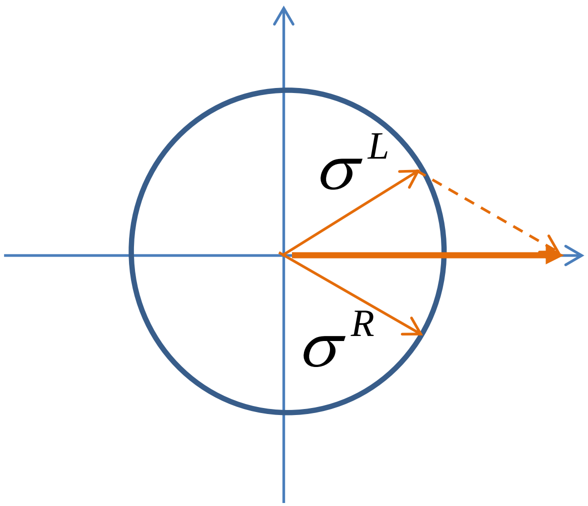
$$\sigma^R \rightarrow n_R \qquad \sigma^L \rightarrow n_L$$

$$\varphi = knd \qquad n_R = n_L$$

No net phase difference

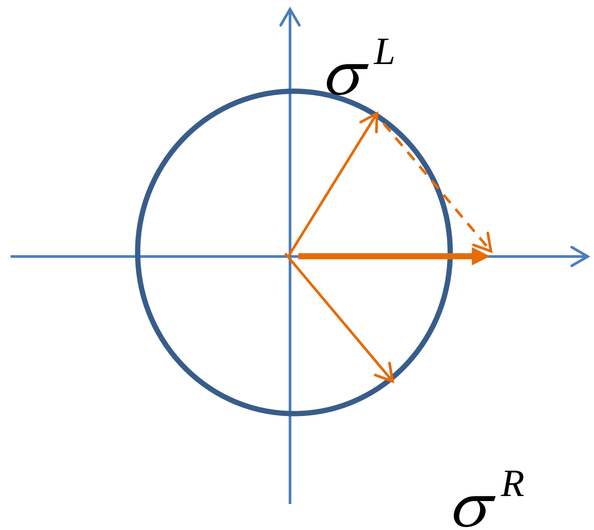
Circular birefringence $\quad n_R \neq n_L \quad \Delta\varphi = k(n_R - n_L)d$

start

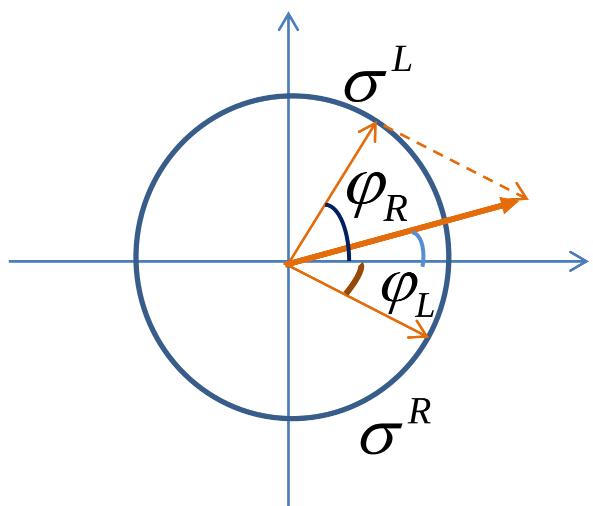


end

$$n_R = n_L$$



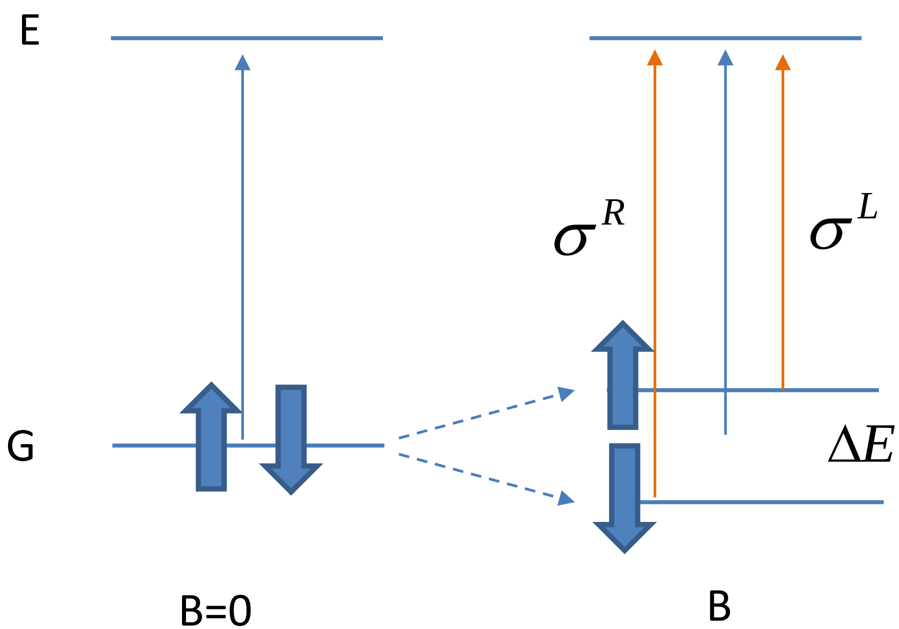
$$n_R \neq n_L$$



$$\Delta\varphi = k(n_R - n_L)d$$

$$\theta = \varphi_R - \varphi_L = k\Delta n d$$

In the lab write up -> Zeeman splitting + polarization selection rules-> Faraday rotation



σ^R σ^L

$\Delta E = \mu_B B$

$$n(\nu) = n_o(\nu_o) + \frac{dn}{d\nu}\Delta\nu$$

$$n_R = n_o(\nu_o) + \frac{dn}{d\nu}\left(-\frac{\mu_B B}{2h}\right)$$

$$n_L = n_o(\nu_o) + \frac{dn}{d\nu}\left(\frac{\mu_B B}{2h}\right)$$

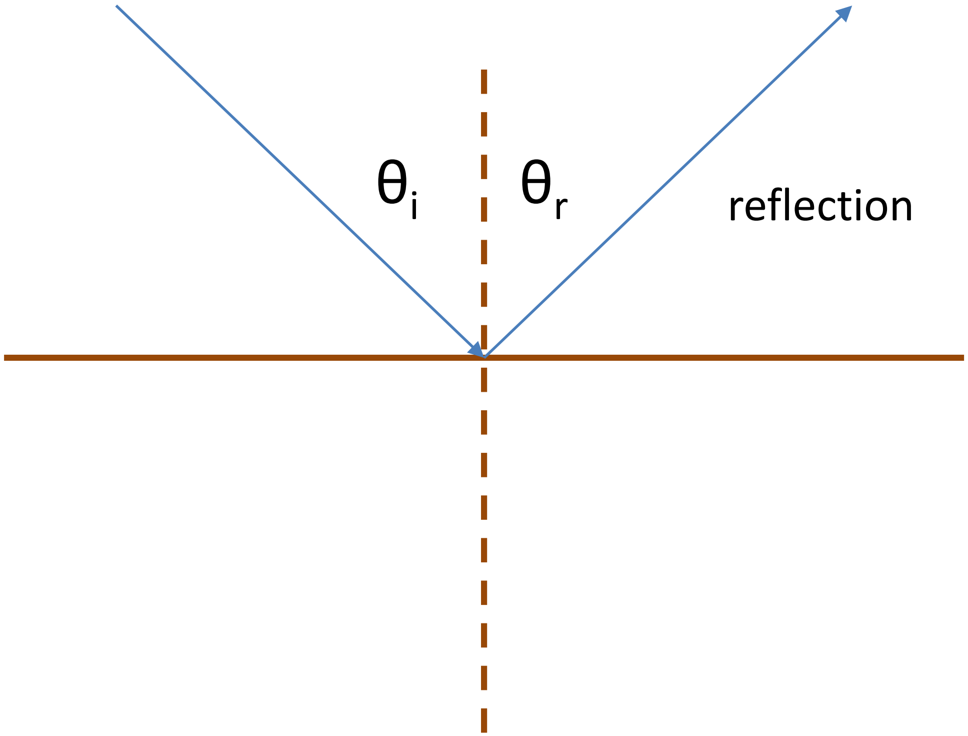
$$n_R - n_L = \frac{dn}{d\nu}\Big|_{\nu=\nu_o} \times \frac{\mu_B B}{h}$$

$$\theta = k(n_R - n_L)d$$

$$= k\frac{dn}{d\nu}\Big|_{\nu=\nu_o} \times \frac{\mu_B B d}{h}$$

$$\theta = VBd \rightarrow V = \frac{k\mu_B}{h}\frac{dn}{d\nu}\Big|_{\nu=\nu_o}$$

The Law of Reflection

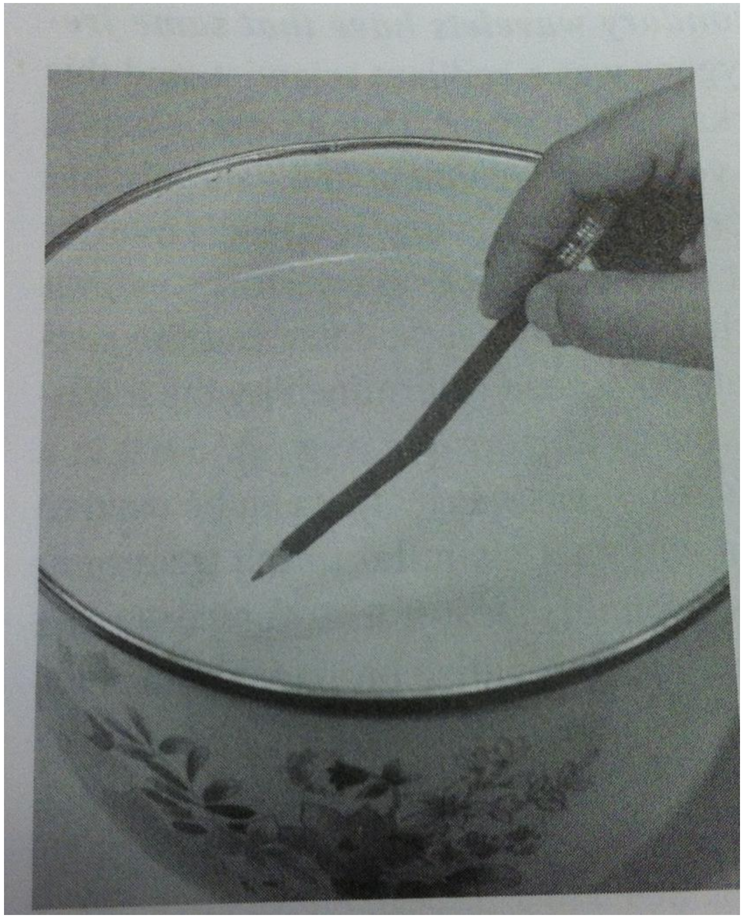


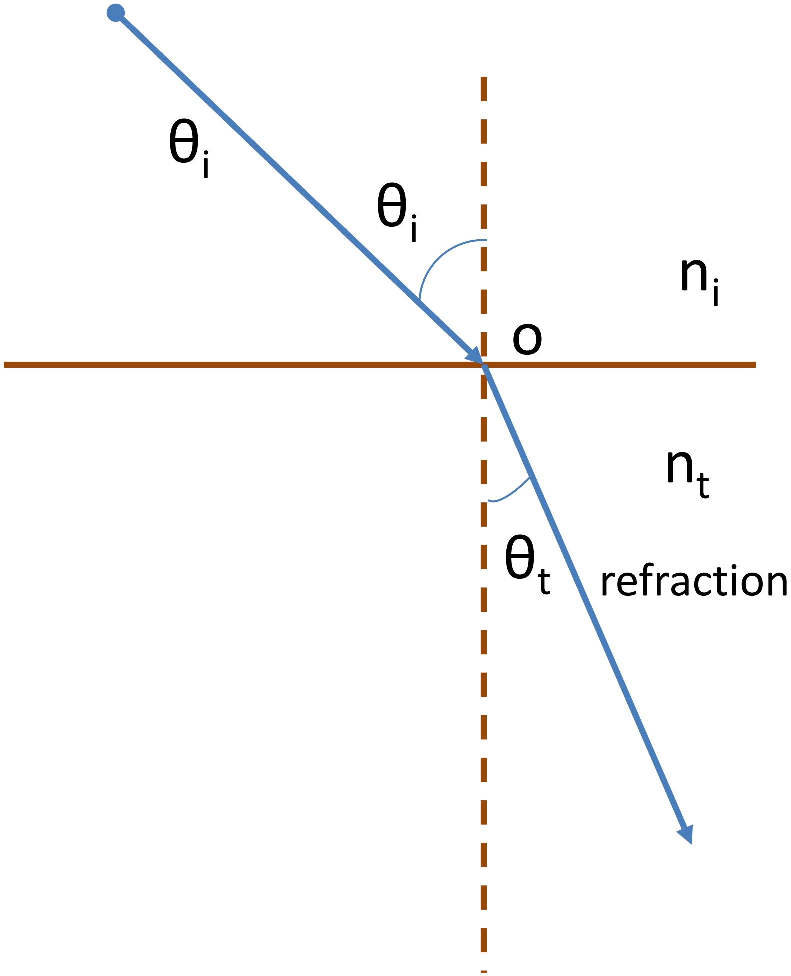
$\theta_i = \theta_r$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$\theta_i = \theta_r$$

The Law of Refraction

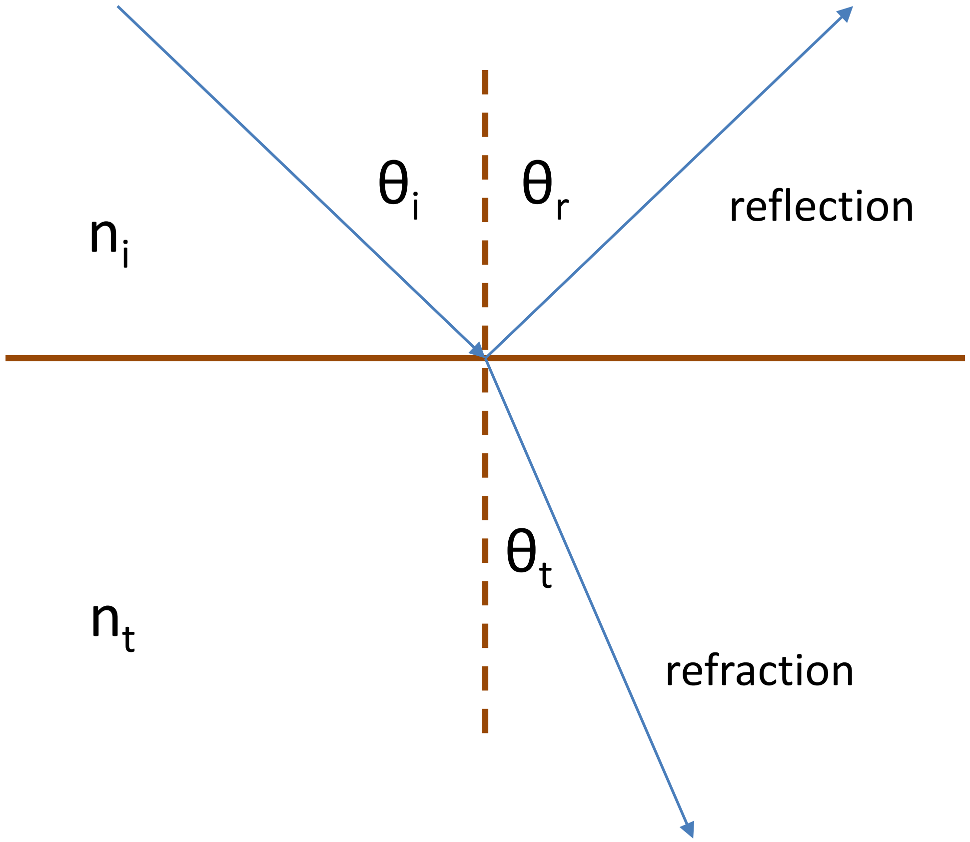




$$n_i \sin\theta_i = n_t \sin\theta_t$$

Snell's Law

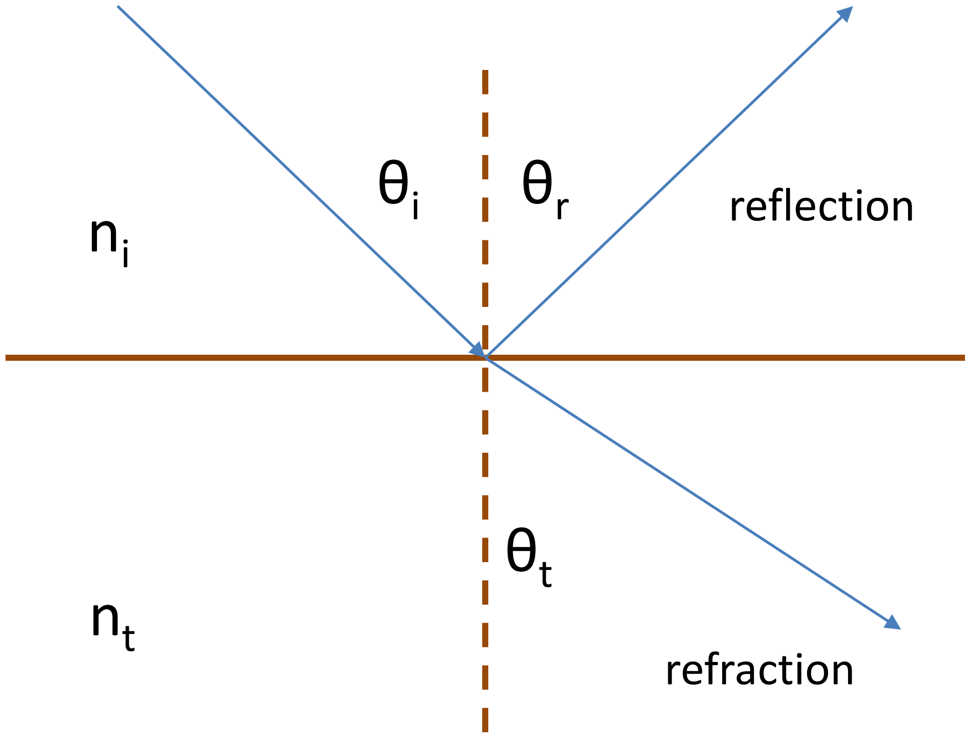
Law of Reflection and Refraction



$\theta_i = \theta_r$

$$n_i \sin \theta_i = n_t \sin \theta_t$$

$n_i < n_t$ The ray entering a higher-index medium bends toward the normal



$$\theta_i = \theta_r$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i > n_t$$

$$\theta_t = 90;$$

$$\theta_{ith} = Arc\sin\left(\frac{n_t}{n_i}\right);$$

Total Internal Reflection