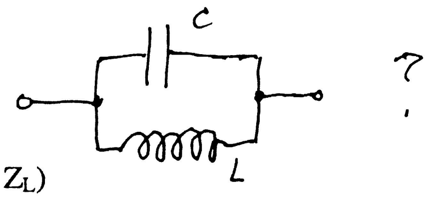
Phys. 334 **Day 3 Lecture Notes**

LC Resonant Circuits:

What is the impedance of:



Like resistors, $Z = Z_C \,||\, Z_L = Z_C Z_L/(Z_C + Z_L)$

With $Z_C = 1/j\omega C$ and $Z_L = j\omega L$:

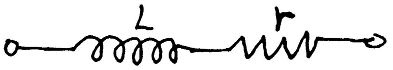
$$Z = \frac{j\omega L / j\omega C}{j\omega L + 1/j\omega C} = \underline{\frac{j\omega L}{1-\omega^2 LC}}$$

Note: at $\omega^2 = 1/LC$, $Z \to \infty$

while as $\omega \to 0$ and $\omega \to \infty$, $Z \to 0$. This parallel LC circuit is also called a resonant or tuned circuit because of its special behavior at $\omega = [1/LC]^{1/2}$ ($f_{res} = [1/LC]^{1/2}/2\pi$).

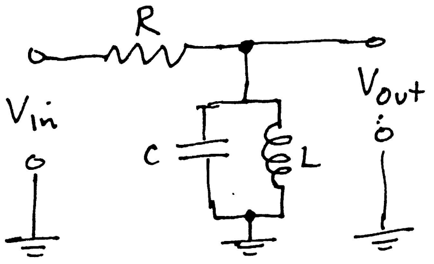
Infinities never occur in real circuits (ie $Z_{LC} \neq \infty$) because long before they do, the non-ideal characteristics of the circuit elements become important (in this case, the resistance of the wire that makes up an inductor limits the ultimate impedance).

A real inductor is better modeled as:



that is, an ideal inductor in series with an ideal resistor. As practice, you should compute the impedance of a real inductor in parallel to an ideal capacitor.

A typical use of a parallel LC circuit is to make a 'tuned filter', that is, a voltage divider that passes only a narrow range of frequencies. This is most common in RF circuits where you need to tune your receiver to the frequency of a particular radio or TV signal.



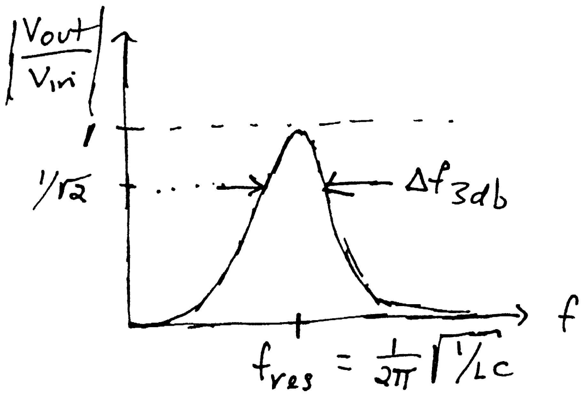
$V_{out}/V_{in} = Z_{LC}/(Z_R + Z_{LC})$ where $Z_R = R$, and

$Z_{LC} = j\omega L/(1 - \omega^2 LC)$, from above.

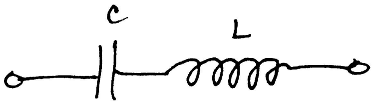
$V_{out}/V_{in} = j\omega L/(j\omega L + (1 - \omega^2 LC)R)$

So that $|V_{out}/V_{in}|$ looks like:

ie. this voltage passes a narrow band of frequencies centered at f_{res}. The range of frequencies between the points where $|V_{out}/V_{in}| = 1/\sqrt{2}$ is called Δf_{3db}, or the 3 db width of the filter. The Quality factor, $Q = f_{res}/\Delta f_{3db}$.



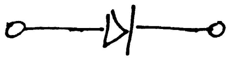
A "Notch Filter" is made by:



Here, $Z = Z_C + Z_L = 1/j\omega C + j\omega L = (1 - \omega^2 LC)/j\omega C$

So that at $\omega^2 = 1/LC$, $Z = 0$, while as $\omega \rightarrow 0$ and $\omega \rightarrow \infty$, $Z \rightarrow \infty$.

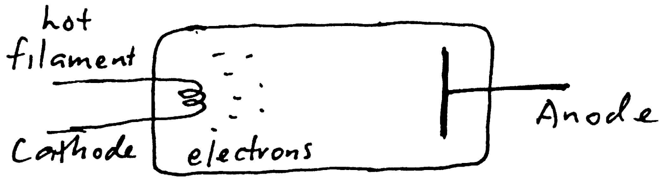
DIODES: Symbol:



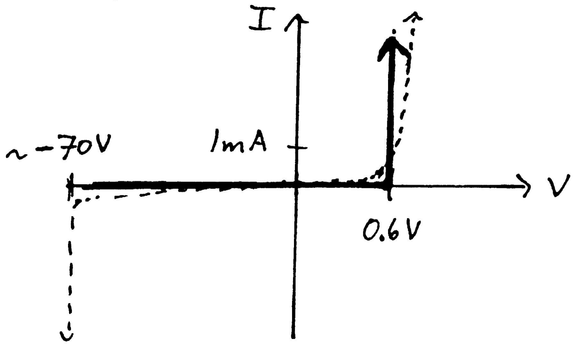
R's, C's, and L's are all linear devices in the sense that at a fixed frequency, $V \propto I$. Diodes are our first example of a non-linear, two terminal device. This means that there is no unique impedance of a diode, rather we must consider a 'dynamical resistance' that depends on the operating conditions of the diode. Similarly, there is no Thevenin equivalent of diode circuits, except in the same dynamical sense.

Diodes act like 'one-way' conductors. The direction of conduction (current flow) is represented by the arrow in the symbol for the diode.

Before semi-conductors, diodes were made by vacuum tubes. Below is a sketch of a thermionic vacuum tube diode. A hot filament (cathode) can boil off electrons. If a nearby plate (anode) has a positive voltage, the electrons will be accelerated to the plate and a steady current will flow. If the anode has a negative voltage, it will repel the electrons and no current will flow, hence the one-way conductor.



Semi-conductor diodes, which are used today, also act as one-way conductors but operate by a different mechanism (see the end of this week's notes). The I vs. V curve for an ideal (solid line) and real (dotted line) silicon diode is shown below:



For an ideal diode, which is what we will most often use as our model for a diode,

For $V < 0.6$ Volts, $I = 0$, ie. $R = \infty$.

For $V = 0.6$ Volts, $R = 0$, (any positive I can flow). Note, you cannot apply more than 0.6 Volts across an ideal diode.

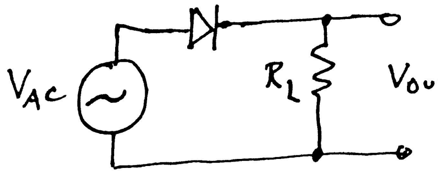
A real diode differs from our ideal diode in two ways: 1.) the current does not turn on suddenly at 0.6 Volts, rather it increases exponentially for voltages near 0.6 volts and reaches a current of $\approx$ 1 mA at 0.6 Volts, and 2) there is a maximum negative voltage that you can apply across a diode before it 'breaks down' ie. conducts in the reverse sense. This reverse breakdown voltage is typically around -75 Volts and diodes are not operated near the reverse breakdown voltage. Cousins of the diode, Zener diodes (that we saw briefly in Week 1), are designed to operate at the reverse breakdown voltage -- that is what we called V_Z, the voltage at which the Zener diode turned on.

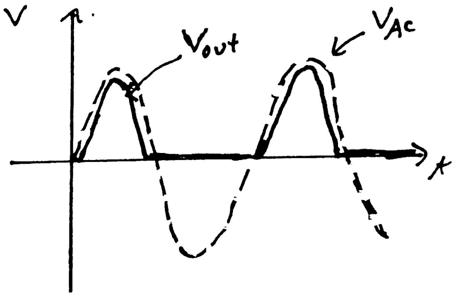
You cannot connect a voltage source with V > 0.6 Volts across a diode: too much current would try to flow and would 'fry' the diode.

Diode Uses:

Half-Wave Rectifier:

A common use for diodes is to turn a sinusoidal signal into a signal of one sign only, that is, to 'rectify' the signal. This allows DC voltages to be obtained from AC sources. The simplest example is the half-wave rectifier, shown below.





Note: the circuit again has the form of a voltage divider.

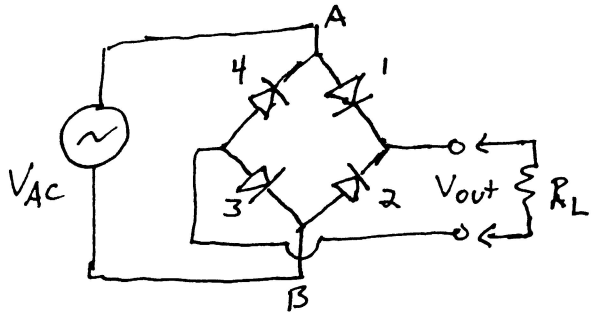
When $V_{AC} < 0.6$ Volts, the diode is off (no current flows through it) and so $V_{out} = 0$.

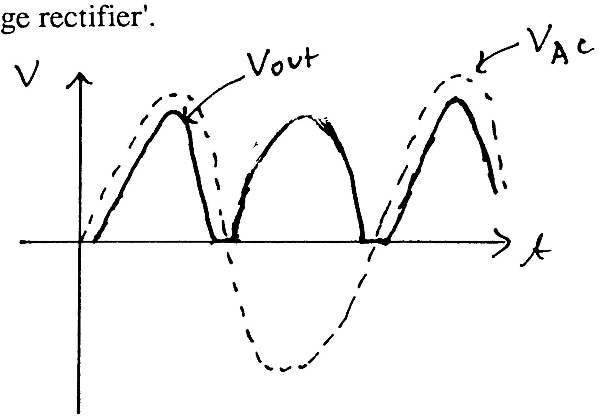
When $V_{AC} > 0.6$ Volts, the diode is on and $V_{out} = V_{AC} - 0.6$ V. (The diode cannot have more than a 0.6 V drop across it.)

This suggests a general way of thinking about diode circuits: treat the cases when the diode is off and on as separate circuits and find when it switches from one behavior to other.

The half-wave rectify isn't a very common circuit: it spends half of its time doing nothing and the output voltage is far from DC. We can do much better by using 4 diodes (which are inexpensive) in a configuration called a 'full-wave bridge rectifier'.

Full-Wave Bridge Rectifier (FWB):



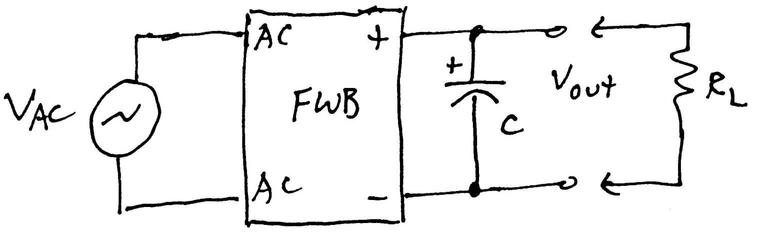


When $V_{AC} = V_A - V_B > 1.2$ Volts, diodes 1 and 3 turn on: current flows through diode 1, down through R_L, and through diode 3 back to the AC power source. Because the current goes through 2 diodes, $V_{out} = V_{AC} - 1.2$ Volts.

When $V_{AC} < -1.2$ Volts, $V_B - V_A > 1.2$ Volts and diodes 2 and 4 turn on: current flows through diode 2, down through R_L, and through diode 4 back to the AC power source. Again, $V_{out} = |V_{AC}| - 1.2$ Volts.

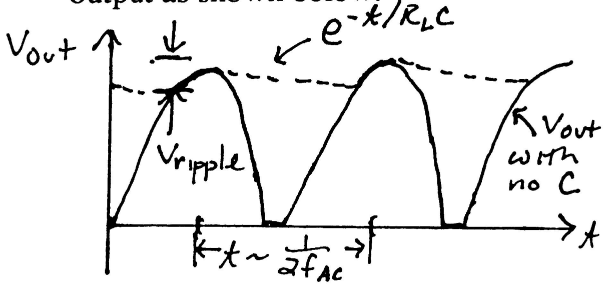
For the FWB, one set of diodes is always on (except when $|V_{AC}| < 1.2$ Volts) and the current always flows through R_L in the same direction, giving the same sign V_{out}.

To convert the FWB output to a DC (constant) voltage, we need only add a capacitor. Below, the FWB is shown as a box: in fact, you normally use a package called a FWB which contains four identical diodes wired together like the circuit on the previous page. The capacitor is usually a large one, often electrolytic, which has the circuit symbol shown below. (The symbol represents the fact that certain capacitors, like electrolytic ones are polar -- they have a preferred side for positive vs. negative voltages.)



For $R_L = 0$, (ie. nothing connected to the output) V_{out} = constant = $V_{AC,max} - 1.2$ V.
The reason for this is because the capacitor stores charge, $Q = CV$, so that $Q_{max} = CV_{out,max} = C(V_{AC,max} - 1.2V)$.
Now when the AC voltage begins to fall from its maximum value, all four diodes in the bridge prevent current from flowing out of the capacitor. (Looking at the circuit on the previous page, diodes 1 and 2 block any current from flowing back to V_{AC}.) The result is that the capacitor is left with its maximum charge and V_{out} is constant.

The only way charge flows off the capacitor is when you connect something to the output, such as R_L: then the capacitor discharges exponentially with a time constant $\tau = R_LC$: $V_{out} = V_{out,max} \exp(-t/R_LC)$ until the next time (one half cycle later) when $|V_{AC}|$ again reaches its peak value and recharges the capacitor. You therefore see a 'ripple voltage', V_{ripple} on the output as shown below.



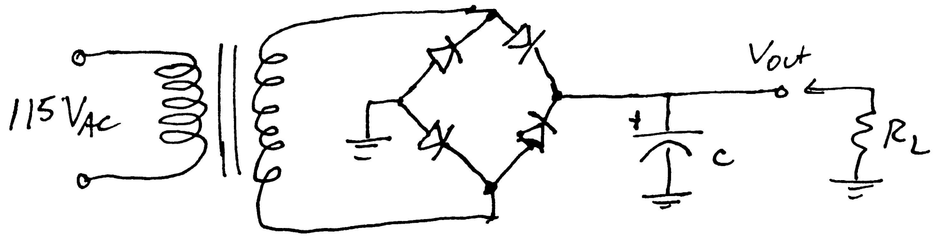
$$V_{ripple} = V_{out,max} - V_{out,min}$$

$$= V_{out,max}(1 - \exp(-t/R_LC))$$

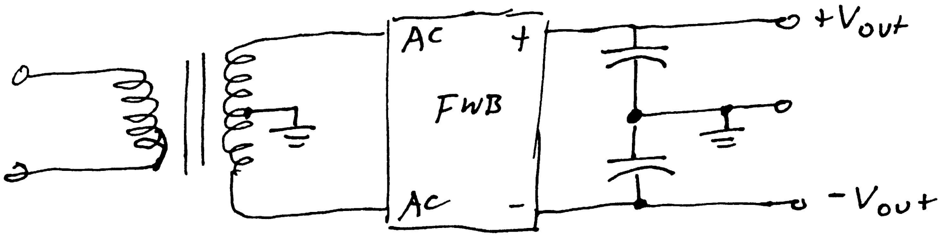
where $t = 1/2f_{AC}$ (f_{AC} = freq. of V_{AC})

For small V_{ripple}, we can expand the exponential and find $V_{ripple} \approx V_{out,max}/(2f_{AC}R_LC)$
To make the ripple small, we use as large a capacitor as necessary: $R_LC >> 1/2f_{AC}$.

DC Power Supply: Almost any electronic box you build will need a source of DC voltage, that is, a DC power supply. You normally build the DC supply as part of the circuit as shown below. The variables you have at your disposal are the turns ratio of the transformer ($N_{secondary}/N_{primary}$ which determines V_{AC} coming out of the transformer) and the size of the capacitor: you know how much current your circuit will need and if you take $R_L = V_{out}/I$, you use our result above to select C to keep the ripple voltage as small as you need it.

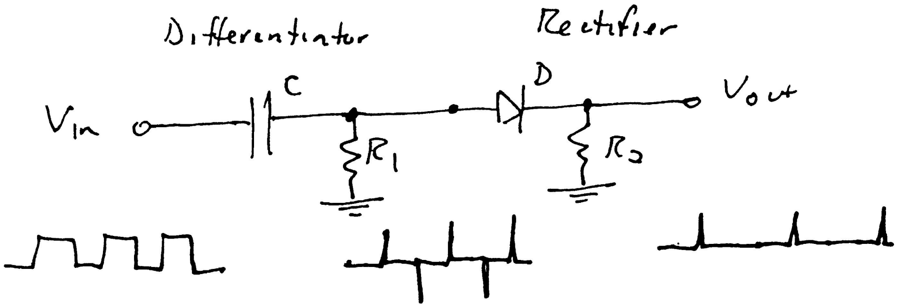


Split Supply: You can make a DC power supply that provides both positive and negative voltages (a 'split' supply) with a single FWB if you use a 'center tapped' transformer. This is a transformer where the center point of the secondary windings is brought out as an additional wire. The FWB acts just as before, only we've taken a different point as our ground.

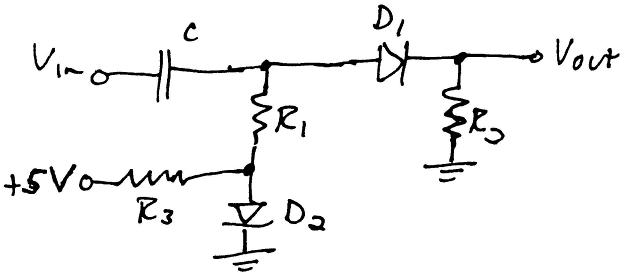


In reality, it is difficult to find capacitors large enough to eliminate the ripple voltage for circuits that need Amps of current. In such cases, one begins with a power supply as shown above and then uses a 'Voltage Regulator' chip. This is an integrated circuit that uses feedback to take a nearly constant input voltage and turn it into a more constant output voltage.

Signal Rectifier: In addition to AC power sources, diodes can be used to rectify any signal. For example, if you wanted to make a sequence of pulses that occurred at the positive going edge of a square wave input, you might differentiate the square wave (with our differentiator circuit) and then rectify the output of the differentiator, as shown below.

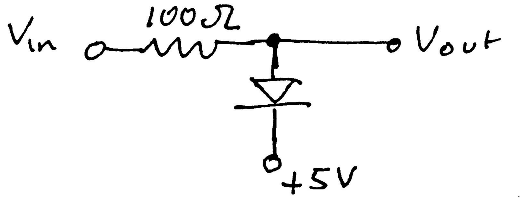


One drawback of the circuit above is that the output is 0.6 Volts smaller than the input to the diode. There is a neat trick, however, to eliminate this 'diode drop'. The trick is to use another similar diode to raise the voltage at the input of the rectifier diode to 0.6 Volts. This makes the rectifier diode ready to conduct for any additional voltage increase at its input.



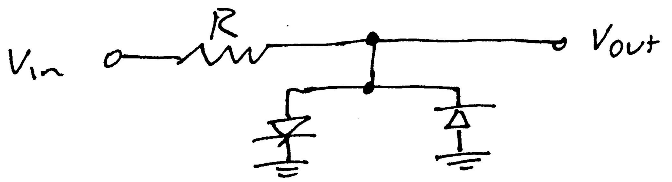
D_2 is always on (powered by the a 5 V power supply in this example). This makes the voltage at the top of D_2 equal to 0.6 Volts and the voltage at the input side of D_1 very near to 0.6V. (Very little current flows through R_1 because if it did, the voltage drop across R_1 would turn off D_1.)

Diode Clamp: This is a circuit that's prevents the input voltage from exceeding a set value (to protect subsequent components in the circuit). The voltage source that you use (+5 V in the example below) plus 0.6 Volts (one diode drop) determines the 'clamping' voltage: V_{out} will be less than 5.6 V. If V_{in} exceeds 5.6 V, the diode turns on and the voltage drop across R will keep $V_{out} < 5.6$ V.



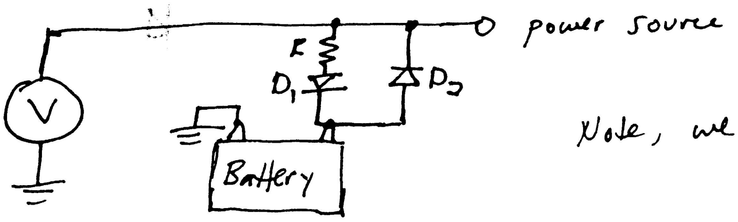
: V_{out} is held to ≤ 5.6 V.

Diode Limiter: Like the diode clamp, but now $|V_{out}| < 0.6$ Volts.



: $|V_{out}| \leq 0.6$ Volts

Battery Backup: A use of diodes to allow a battery to provide power if a voltage source fails. D_1 allows the voltage source to keep the battery charged. If the voltage source fails, the battery provides current through D_2. Otherwise D_2 is off.

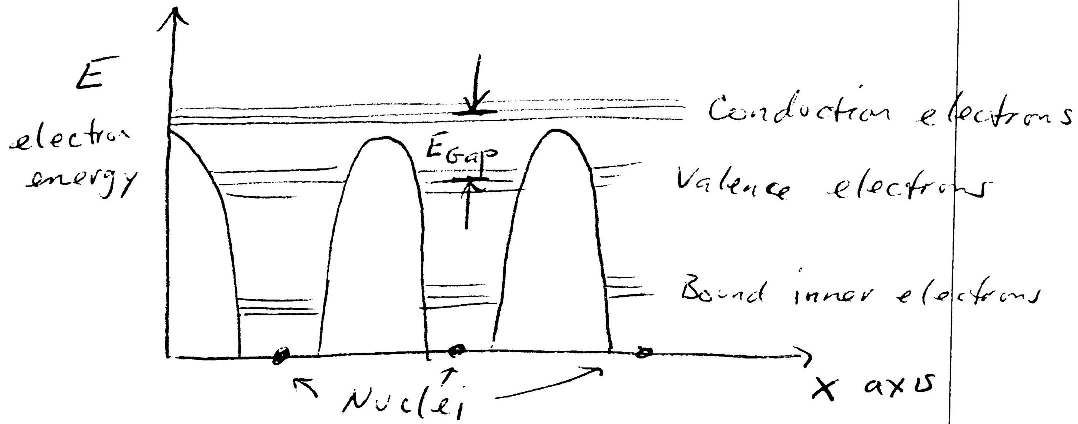


Note, we need $V_{battery} \leq V - 0.6V$

In reality, the current through a diode is given by: $I = I_S [\exp(V/V_T) - 1]$ where I_S is called the saturation current ≈ 1 nA (reverse leakage current) and $V_T = k_BT/e \approx 25$ mV at room temperature. This is a consequence of the properties of semi-conductors.

Semi-Conductor Review

Pure Crystal:



At T = 0°K, Valence Band is full, Conduction Band empty
ie every electron bound to a nucleus

Conductor: $E_{Gap} = 0$, ie Valance Band ≡ Conduction Band
ie They overlap & electrons are free to move

Semi-Conductor: $E_{Gap} = 1eV$ (Si), $E_{Gap} = 0.7eV$ (Ge)

Insulator: $E_{Gap} = 3$ to 5 eV.

$KT_{Room} \approx .025eV$

$\#$ of electrons in conduction band = $N_{cb} = AT^{3/2} e^{-E_{Gap}/KT \times 2}$

At $T_{Room} \approx 300°K$, $N_{cb}(Si) \approx 10^{10}$ elec/cm^3

$\Rightarrow I \approx 1nA$ to $10nA$ ≡ Reverse bias current.

For more current to flow, add "donor" impurities to Si that have 5 electrons per atom (n-type semi-conductor). 4 of the electrons bind with neighboring Si atoms, leaving 1 weakly bound extra electron ($E_{Gap} \approx .05eV$)

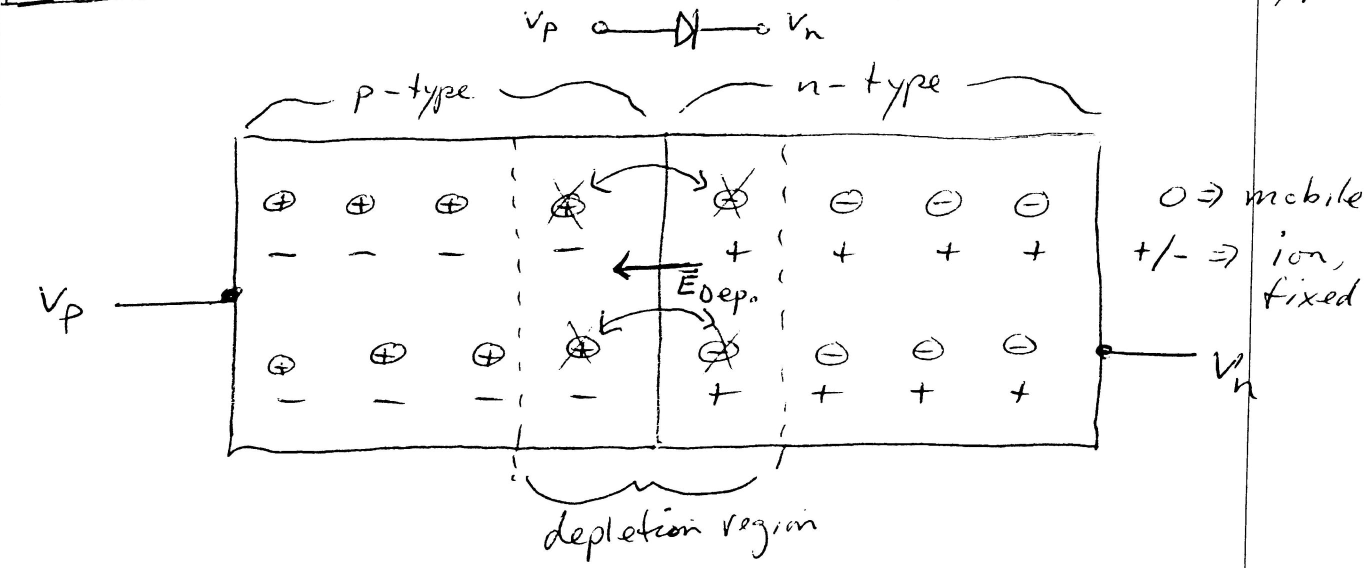
OR: add "acceptor" impurities to Si with 3 electrons per atom (p-type semi-conductor). The 3 electrons bind to neighboring Si atoms, leaving one vacant bond (hole) ≡ acts like a positive electron. ($E_{Gap} \approx .05eV$)

∴ n-type: have mobile (conduction) electrons & stationary positive lattice ions.

p-type: have mobile (conduction) holes & stationary negative lattice ions.

Then $N_{cb} \approx 10^{16} - 10^{18}$ elec/cm^3 $\Rightarrow$ $I \approx 1$ Amp.

<u>p-n junction ≡ Diode</u> : Now put n-type next to p-type



In <u>depletion region</u> at n-p junction, <u>mobile</u> electrons + holes recombine (ie cancel), leaving only the charged lattice ions (+ on n side + - on p side). These lattice ions generate an electric field, $\vec{E}_{Dep}$

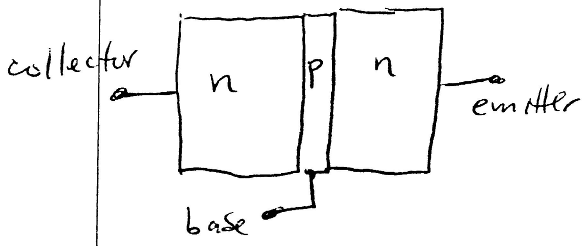
For $\underline{V_n > V_p}$, the external electric field increase $\vec{E}_{Gap}$, ie. adds to $\vec{E}_{gap}$ (by increasing the size of the depletion region) ⇒ <u>No current flows</u>

(except for reverse bias current due to N_{cb} (Si))

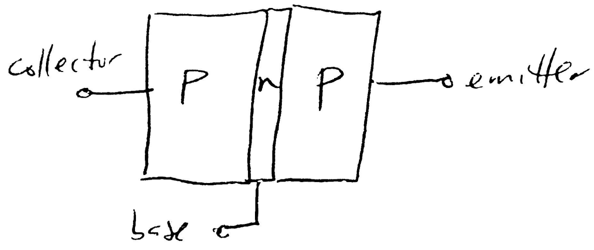
For $\underline{V_p - V_n \gtrsim 0.6V}$, external electric field cancels $\vec{E}_{gap}$ and now holes move to the right, electrons to the left (⇒ current to the right) and they recombine at the junction.
⇒ Current flows from p to n.

p-n junction ≡ Diode.

<u>Transistors</u> : npn junctions + pnp junctions make transistors



npn transistor



pnp transistor