Phys. 428, Lecture 3

LECTURE	DATE	INSTRUCTOR	TOPIC
1	March 29	PK	Overview: Imaging equation, inverse problem
2	April 5	PK	2D-LSI imaging systems, X-ray physics: formation and interaction
3	April 12	WH	X-ray detection and imaging systems
4	April 19	WH	X-ray computed tomography (CT) systems
5	April 26	PK	X-ray CT part 2. Contrast Agents
6	May 3	PK	Image reconstruction and image quality
7	May 10	PK	Nuclear decay schemes and isotopes
8	May 17	PK	Gamma cameras: components and systems
9	May 24	PK	Tomography in molecular imaging: SPECT scanners
10	May 31	PK	Positron emission tomography (PET) and hybrid PET/CT scanners

Each student should email at two questions on today's lecture to Professor Kinahan (kinahan@uw.edu) by Monday 6:30 PM

Please include "Phys 428 Lecture 3 Questions" in the subject line.

Why X-rays and not χ-rays (Chi-rays)?

Wilhelm Röntgen
1901 Nobel Prize:
Discovery and characterization of X-ray radiation
Nationality: Germany
Institution: Universities of Strassburg Hohenheim, Giessen, Würzburg, and Munich

Ernest Rutherford
1908 Nobel Prize: Disintegration of the elements
Discovered several nuclear radiations: a b g
Nationality: New Zealand, United Kingdom
Institution: McGill University
University of Manchester

Summary of Lecture 3: X-ray radiography

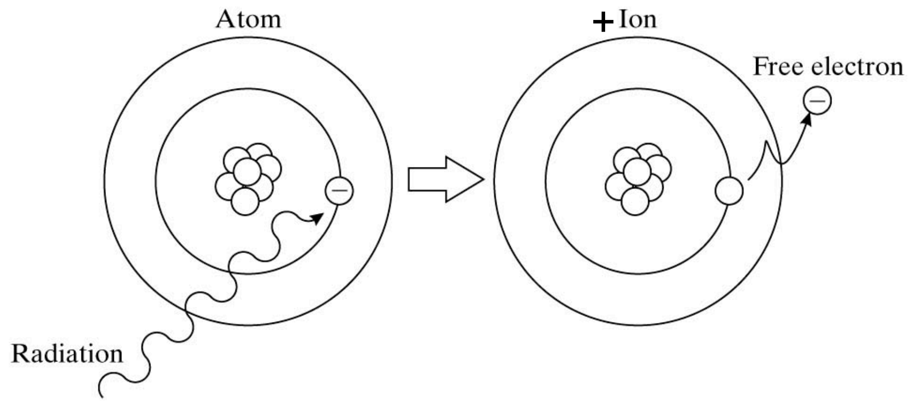
- Ionizing radiation: Properties & Processes
  - ✧Interaction types
  - ✧X-ray beam formation
  - ✧Spectral characteristics
  - ✧Attenuation and beam hardening
  - ✧Radiation dose
- X-ray Imaging system: Application
  - ✧X-ray tube
  - ✧Beam filtration
  - ✧Collimation and conditioning
  - ✧Scatter reduction
  - ✧Detectors
  - ✧Imaging Equation
  - ✧Signal-to-noise ratio
  - ✧Impact of Scatter
  - ✧Use of contrast agents
- Computed Tomography … time permitted

X-ray Radiograph

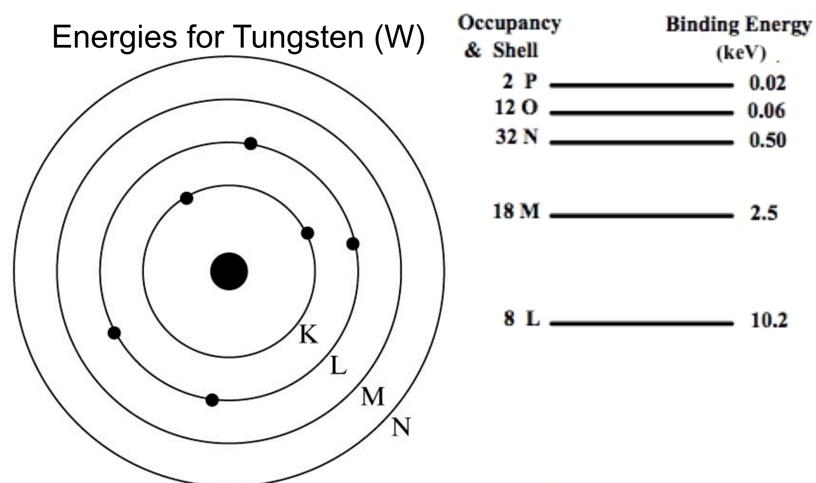
- Ion: Atom or molecule in which the total number of electrons is not equal to the total number of protons, giving it a net positive or negative electrical charge
- Radiation: Process in which energetic particles or energetic waves travel through a medium or space
- X-rays: Ionizing photon radiation arising from elecronic interactions. Can pass through a significant thickness of tissue before being absorbed.
- Radiograph: Shadow image related to the projection of material density and composition that can be made with radiation.

Ionizing Radiation

- Radiation (such as high energy electromagnetic photons behaving like particles) that is capable of ejecting orbital electrons from atoms
- Can also be particles (e.g. electrons)
- Ionizing energy required is the binding energy for that electron's shell
- Energy units are electron volts (eV or keV), the energy of an electron accelerated by 1 volt
- For Hydrogen K orbital electrons, *E*=13.6 eV
- For Tungsten K orbital electrons, *E*=69.5 keV
- In medical imaging we need photons with enough energy to transmit through tissue so are in range of 25 keV to 511 keV and is thus ionizing

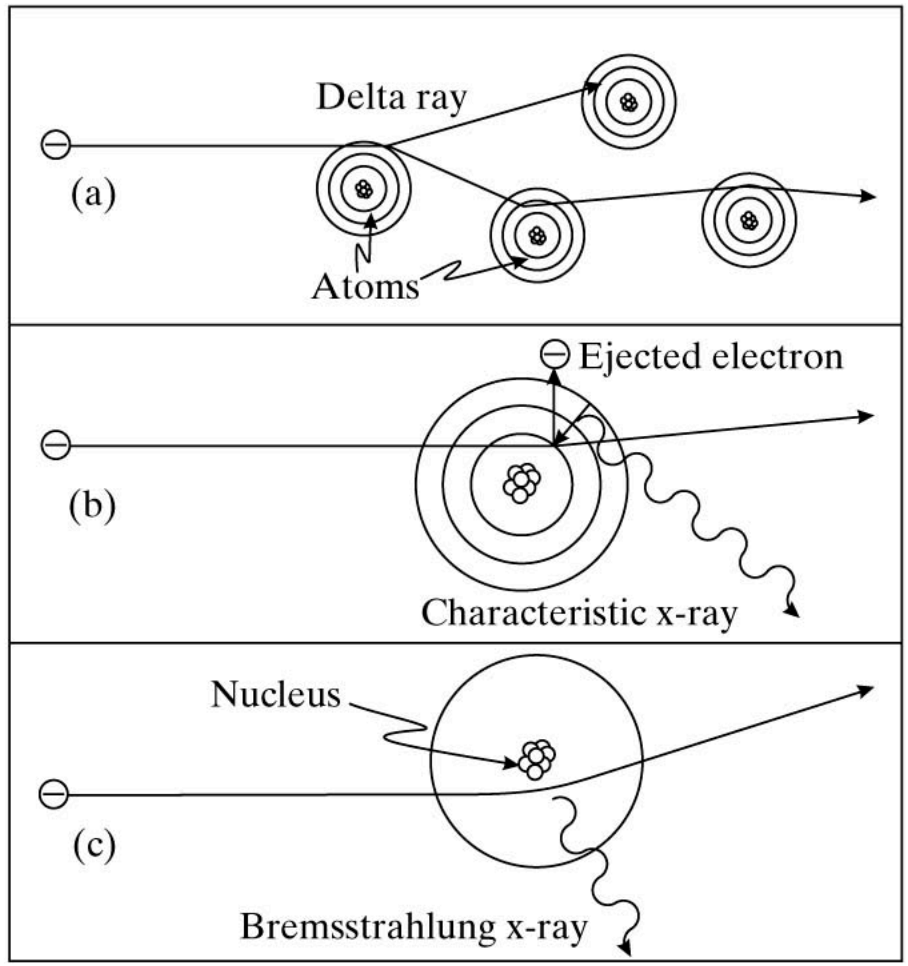


Energies for Tungsten (W)



Electrons as Ionizing Radiation

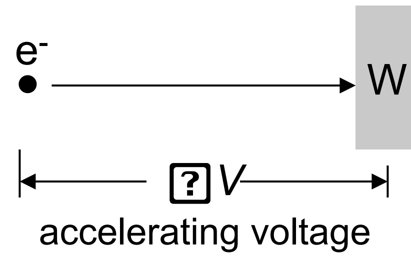
- Used in X-ray generation
- Electron kinetic energy $E = (mv^2)/2$
- Three main modes of interaction in the energy range we are considering
  a) Collision with other electrons and possible creation of delta-rays (high-energy electrons)
    - This is the most common mode and excited atoms lose energy by IR radiation (heat)
  b) Ejection of an inner orbital electron
    - This orbit is filled by an outer electron and the difference in energy is released as a 'characteristic x-ray'
  c) Bending of trajectory by nucleus
    - Since acceleration of a charged particle causes radiation, this causes 'braking radiation' or *bremsstrahlung*

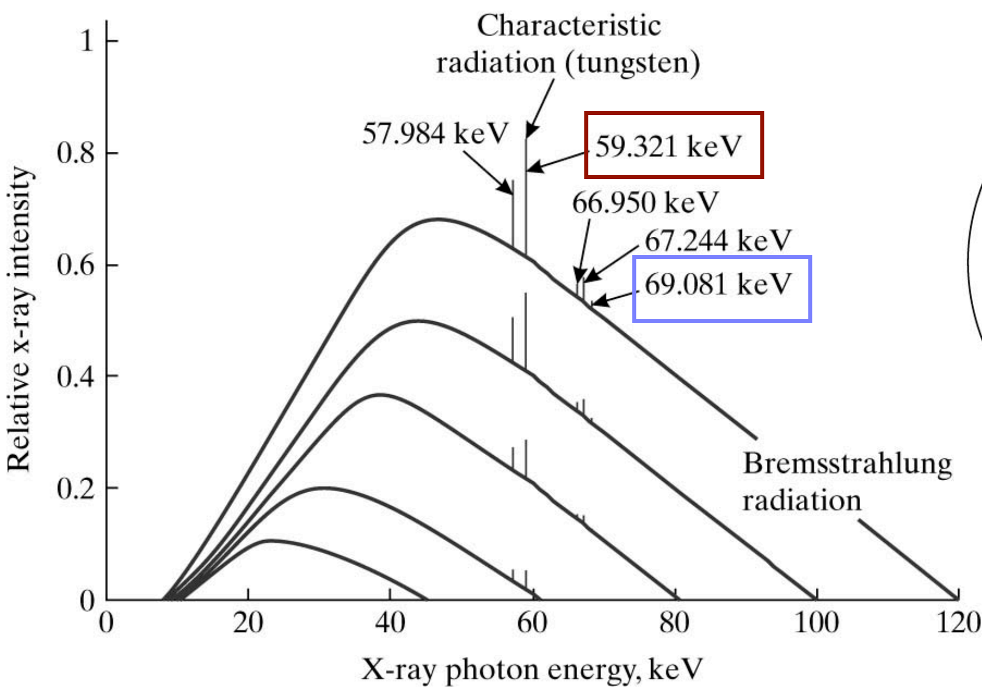


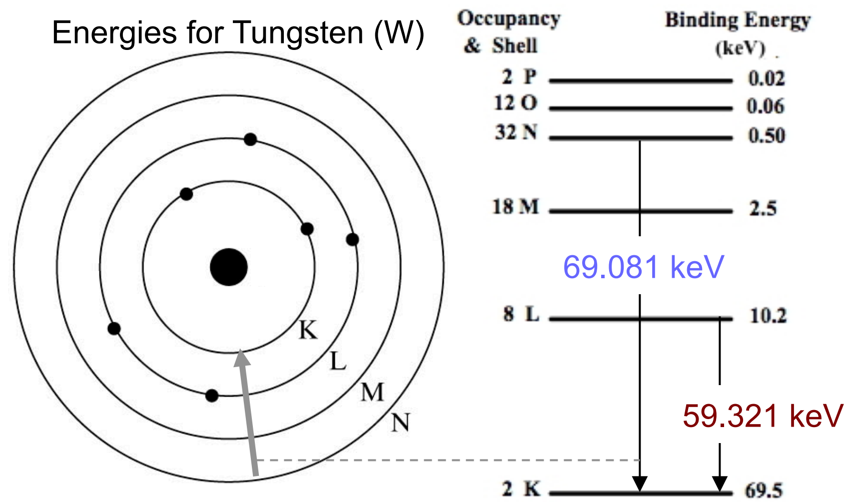
X-ray Spectrum from Electron Bombardment

When high energy electrons hit tungsten (symbol W), three effects occur

1. Heat (> 99.9% of the energy)
2. Characteristic x-rays
3. Bremsstrahlung x-rays







Electromagnetic (EM) radiation and Photons

- The energy of EM radiation is given by $E = \hbar$???? where ??? = c/? is the wavelength and $h = 6.626 \times 10^{-34}$ J-s is Plank's constant
- At low energies EM radiation behaves like a wave and is non-ionizing
- At high energies EM radiation behaves like a particle (localized wave packet) and can ionize (i.e. > 13.6 eV)

Source	Type
bremsstrahlung	x-rays
trar	

The Electromagnetic Spectrum

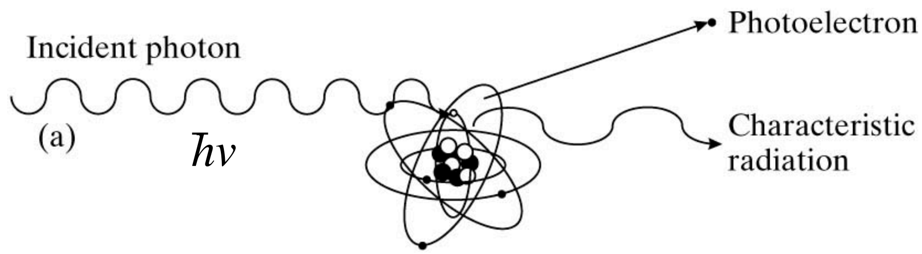
Frequency Range	Wavelengths	Photon Energies	Description
1.0×10^{5}–3.0×10^{10} Hz	3 km–0.01 m	413 peV–124 μeV	Radio waves
3.0×10^{12}–3.0×10^{14} Hz	100–1 μm	12.4 meV–1.24 eV	Infrared radiation
4.3×10^{14}–7.5×10^{14} Hz	700–400 nm	1.77–3.1 eV	Visible light
7.5×10^{14}–3.0×10^{16} Hz	400–10 nm	3.1–124 eV	Ultraviolet light
3.0×10^{16}–3.0×10^{18} Hz	10 nm–100 pm	124 eV–12.4 keV	Soft x-rays
3.0×10^{18}–3.0×10^{19} Hz	100–10 pm	12.4–124 keV	Diagnostic x-rays
3.0×10^{19}–3.0×10^{20} Hz	10–1 pm	124 keV–1.24 MeV	Gamma rays

ionizing

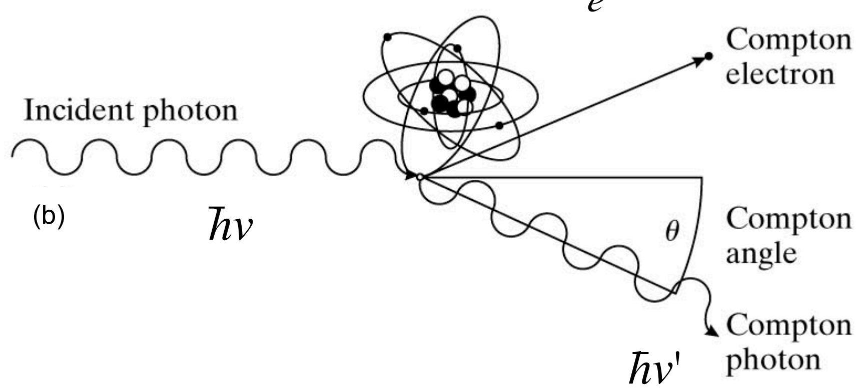
Photons as Ionizing Radiation

- Two main modes of interaction in the energy range we are considering
  a) Photoelectric effect
     - Causes ejection of an inner orbital electron and thus also characteristic radiation as orbital hole is filled
     - Energy of ejected photoelectron is E_e = h[?][?][?][?]E_B
  b) Compton scattering
     - A photon ejects an outer (valence) electron and the photon scatters off with energy given by:

$$h\nu' = \frac{h\nu}{1+(1-\cos\theta)h\nu/(m_0c^2)}$$



$$E_e = h\nu - h\nu'$$

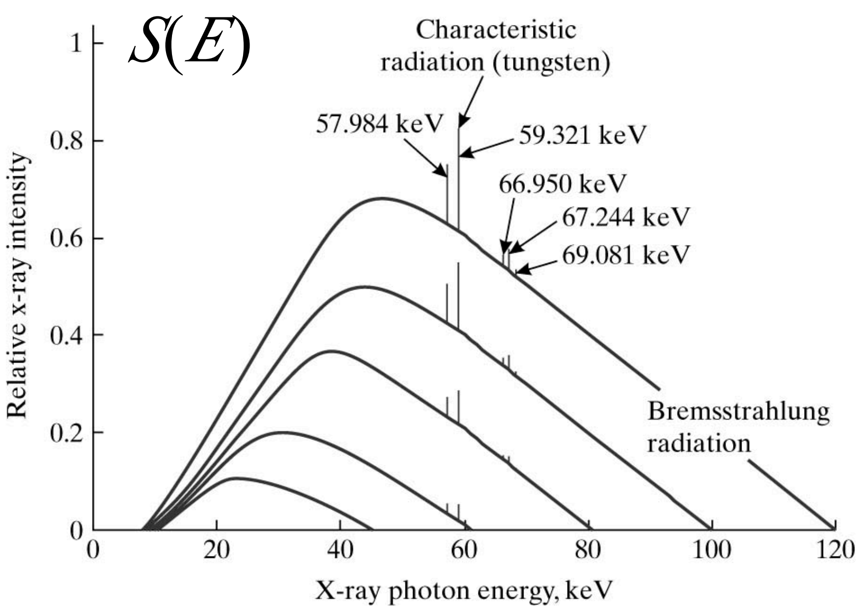


max E_e?

X-ray Fluence, Energy, and Intensity

- Fluence rate: number of photons/unit area/time $\phi = N / (A \cdot \Delta t)$
- Beam intensity is the energy fluence rate: $\psi = N h\nu / (A \cdot \Delta t)$

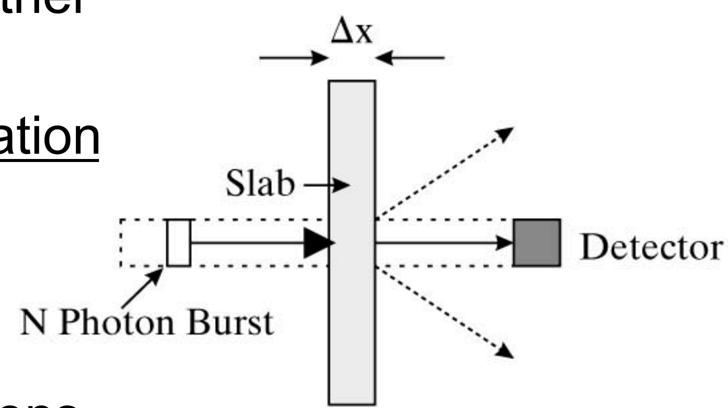
- In X-ray imaging we typically have a continuous range of energies due to bremsstrahlung radiation, so the intensity is the energy weighted integral of the fluence rate



$$I = \int_{E=0}^{E_{max}} E' S(E')\, dE'$$

Attenuation

- Photons can be lost due to photoelectric absorption and Compton scatter (and a few other very rare effects)
- We lump these losses together as the <u>attenuation</u> of the x-ray beam
- To analyze, we start with the 'narrow beam' attenuation approximation



- For very thin slabs we expect number of photons lost ?N to be proportional to ?x and to N, with a proportionality constant, ?, called the linear attenuation coefficient, which is a property of the material at x

$$\Delta N = -\mu N \Delta x$$

- Solving the differential equation

$$\frac{d}{dx} N(x) = -\mu(x) N(x)$$

gives Beer's law:

$$N(x) = N(0) e^{-\int \mu(x)\, dx}$$

fluence

$$I(x) = I_0 e^{-\int \mu(x)\, dx}$$

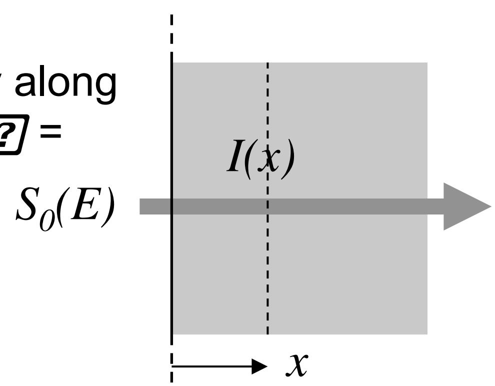
intensity

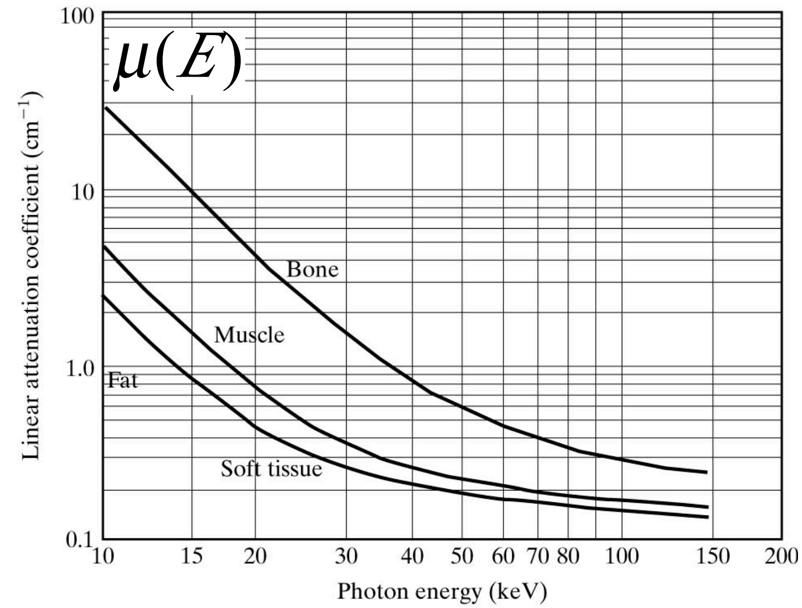
Narrow-beam Polyenergetic Attenuation

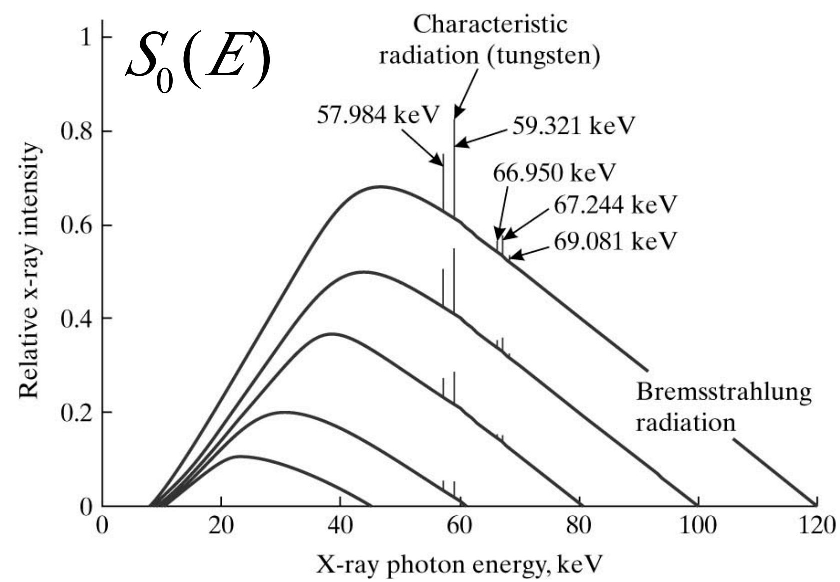
- The attenuation depends on material (thus position of material) and energy
- With bremsstrahlung radiation, there is a weighted distribution of energies
- We combine previous results to get the imaging equation

$$I(x) = \int_{E=0}^{E_{\max}} E' S_0(E') e^{-\int_0^x \mu(x', E')\,dx'}\, dE'$$

beam intensity along a line with ?? = ??(x)

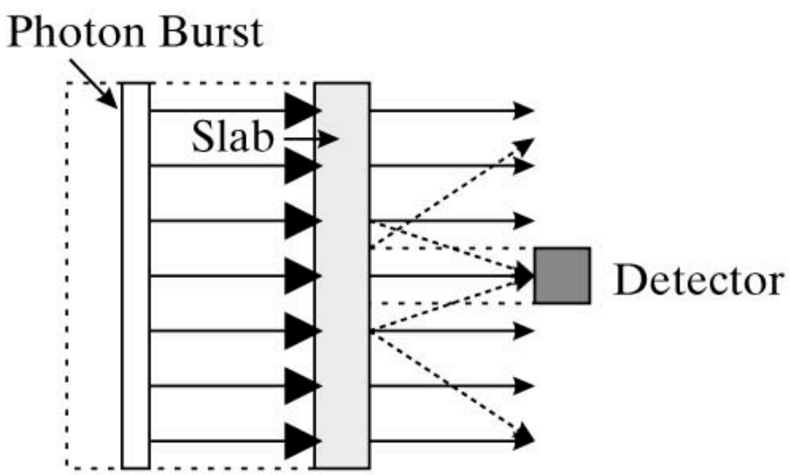






Broad-beam Attenuation

- Photons from outside the detector's line-of-sight geometry get scattered toward the detector by Compton interactions
- Further complications come from the energy dependent scatter
- This violates narrow-beam model and should be compensated for by scatter rejection methods, physical or mathematical, if our imaging equation is to remain valid



Radiation Dose from Photons

- Loss of photon energy means some is being transferred to tissue
- Basic concepts:
  - Exposure: number of ion pairs produced in a specific volume of air by radiation
    - Units are coulombs per kilogram of air (C/kg)
    - Useful in medical imaging is the roentgen (R) 2.58 × 10−4 C/kg
    - Can easily be measured with an ionization chamber
  - Absorbed dose: amount of absorbed energy per mass
    - Note this a implicity a concentration, not a total
    - Units are J/kg, with a special unit of *gray* (Gy)
    - Useful in medical imaging is the rad, which is the absorption of 100 ergs per gram of material
    - 1 roentgen of yields one 1 rad of absorbed dose in soft tissue
  - Equivalent dose: Takes into account type of radiation for tissue T
    - w_R = 1 for photons, 2 for protons, 20 for nuclear fragments

$$H_T = \sum_R w_R D_{T,R}$$

  - Effective dose: Takes into account cumulative effect over all tissues
    - meant to compare relative risks between different procedures
    - wildly inaccurate
    - Units are also J/kg, with a special unit of *sievert* (Sv)
    - 1 Gy give 1 Sv for x-rays in soft tissue

$$E = \sum_T w_T H_T$$

X-ray Imaging Systems

Projection Radiography

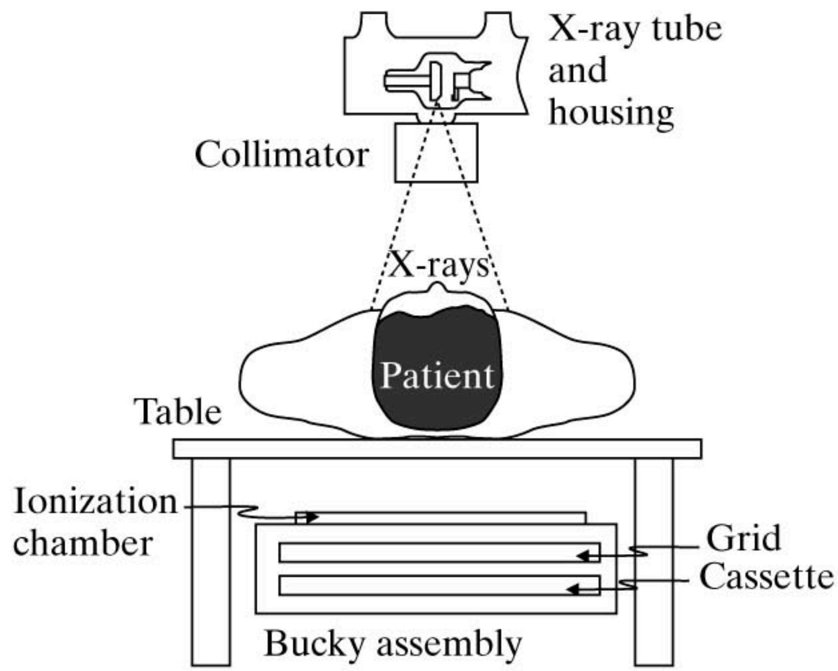
- Projection radiography produces radiographs, which are 2-D projections of a 3-D object
- A projection radiography system consists of an x-ray tube, devices for beam filtration and restriction, compensation filters, grids, and (usually) a film-screen detector
- The basic imaging equation describes the energy- and material-dependent attenuation of the x-ray beam produced by the system as it passes through the patient

X-ray Imaging Systems

- X-ray source
- attenuation by object
- detectors

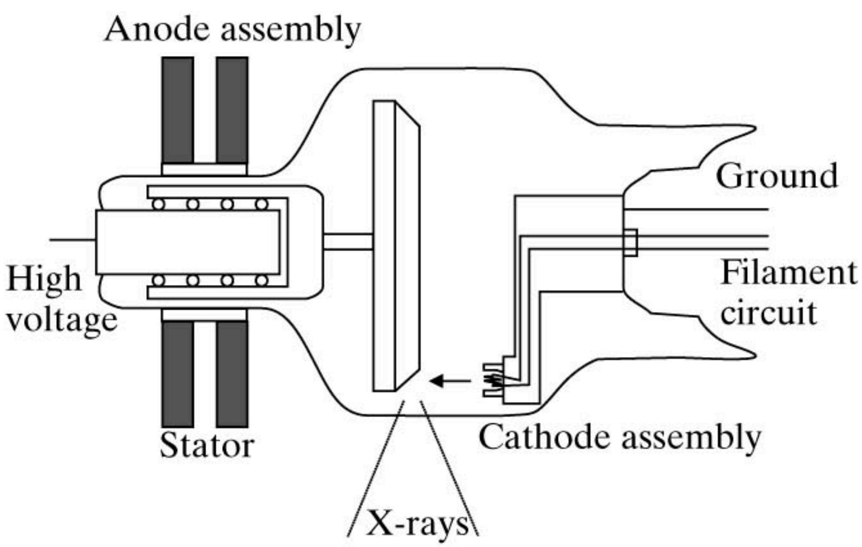
need to be described in the imaging equation

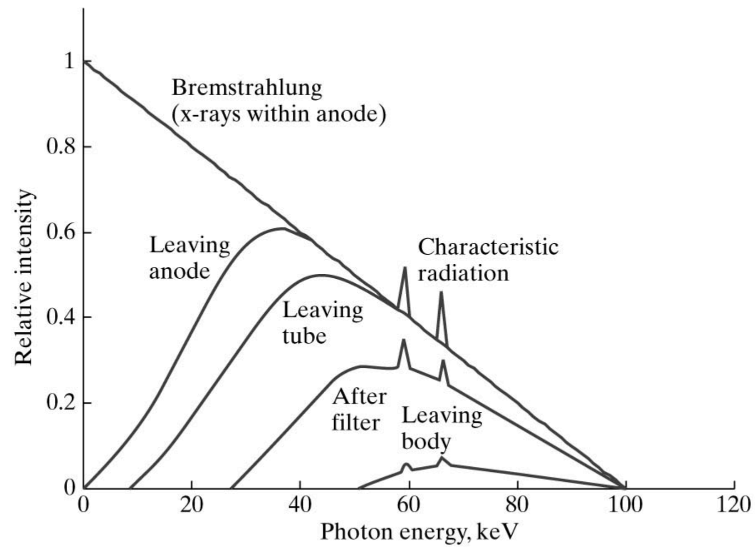
- We want *partial* attenuation



X-ray tubes

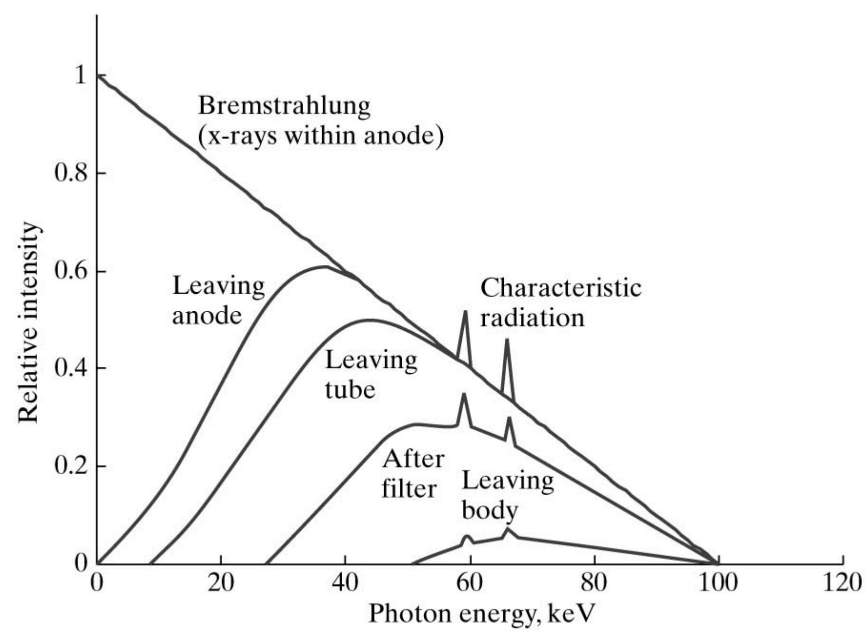
- In a vacuum assembly
- A resistive filament is used to 'boil off' electrons in the cathode with a carefully controlled current (10 to 500 mA)
- Free electrons are accelerated by the high voltage towards the anode
- Voltage determines maximum and x-ray energy, so is called the kVp (i.e. kilo-voltage potential), typically 25 kVp to 150 kVp
- High-energy electrons smash into the anode
  - More than 99% energy goes into heat, so anode is rotated for cooling (3000+ RPM)
  - Bremmstrahlung then produces polyenergetic x-ray spectrum





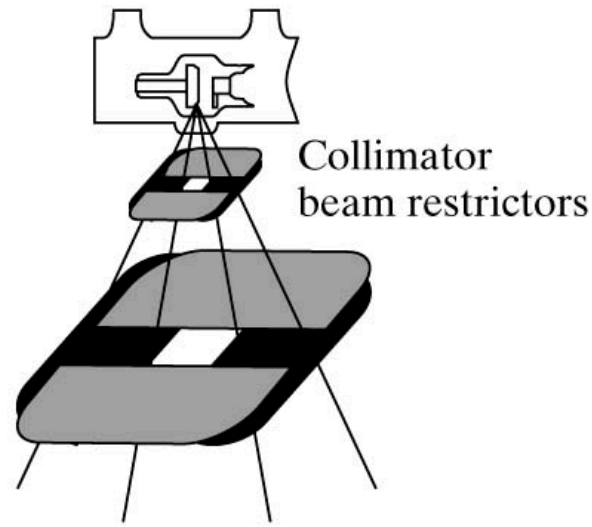
X-ray Beam Filtration

- Lower energy photons are undesirable, as they contribute to radiation dose but not the image quality (if they are all absorbed)
- Some filtering of the low energy photons is provided by the X-ray tube assembly
- A thin (1-2mm) plate of material (Al or Cu) is used to further filter out low energy photons that could not make it through the body
- Note that the average energy increases, a phenomena called *beam hardening*

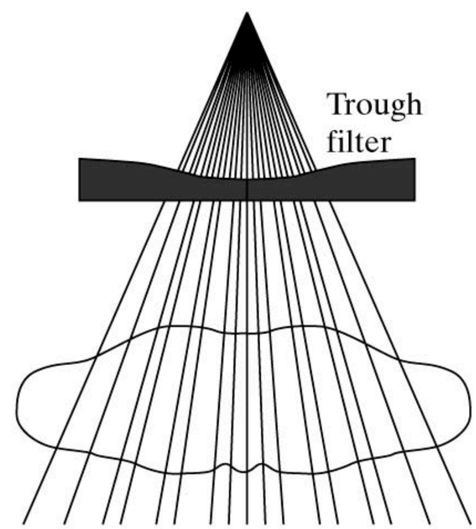


X-ray Beam Collimation and Conditioning

- Many photons are not aimed at the detector, so we can remove them by collimation (also called restriction)
  - high Z materials are used

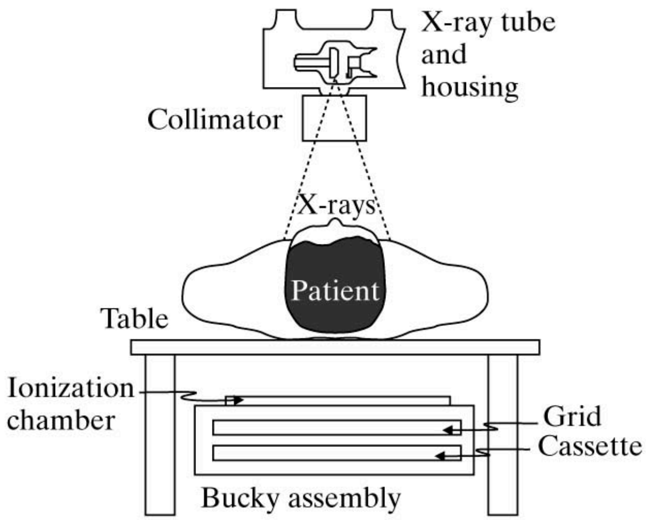


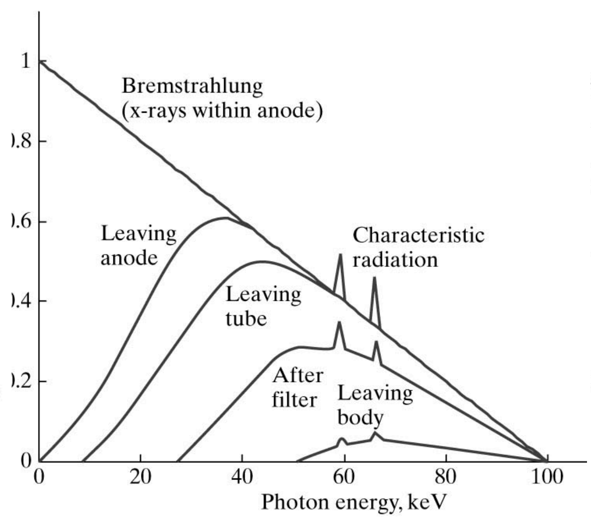
- It is also possible to adjust the relative fluence to improve the ratio of image SNR to patient radiation dose
  - equalizes the *transmitted* fluence
  - typically use plastic or leaded plastic

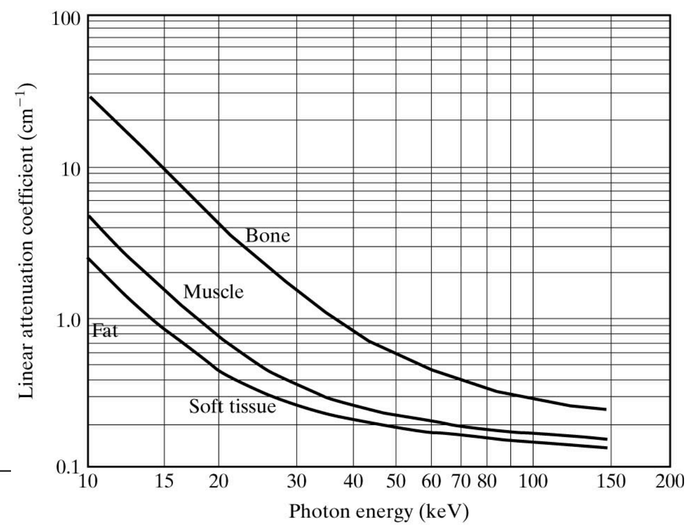


Interaction of X-rays in the Body

- At this point we have a heavily *conditioned* beam of x-rays entering the body



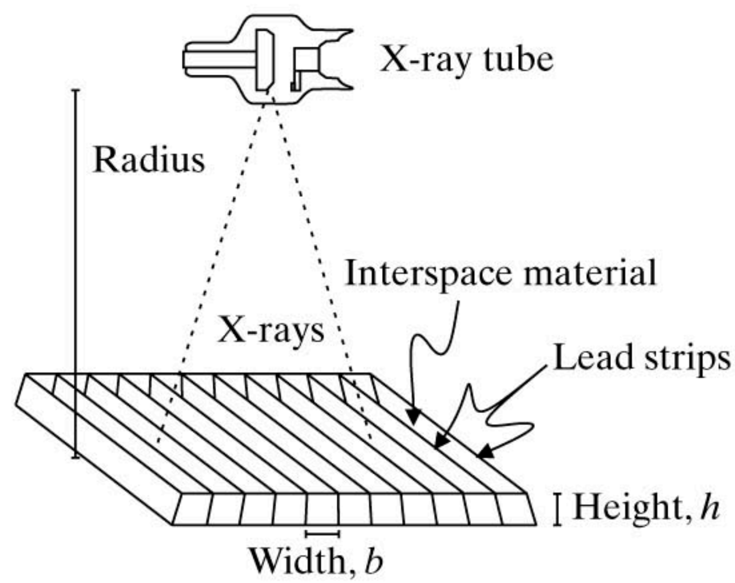




- The attenuation of x-rays in the body depends on material and energy
- We can enhance attenuation by using 'contrast agents', typically iodine (injected) or barium (ingested)

Scatter Reduction

- Grids can be placed between the patient and the detectors to reduce the amount of scattered radiation detected
- Grids will also reduce the number of un-scattered photons, which is not desirable, so optimization of grid parameters is used improve the ratio of image SNR to patient radiation dose
- Grids are typically moved during the imaging to reduce artifacts



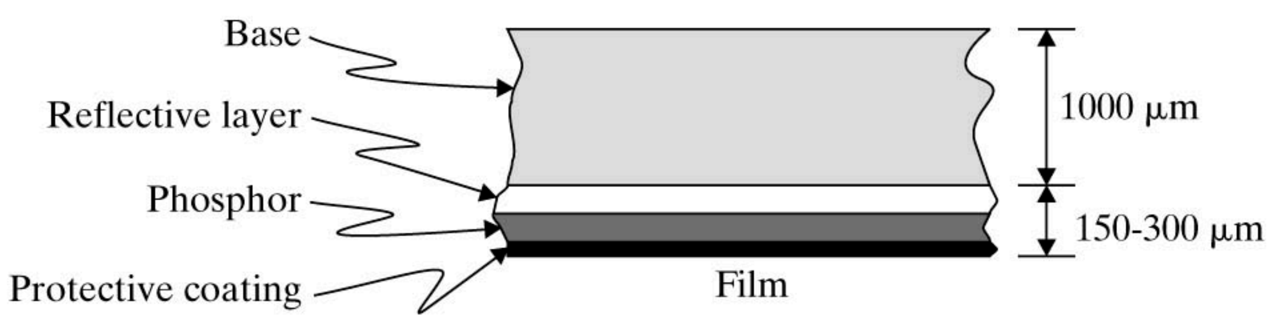
X-ray Detectors

- Several types, including
  - phosphor screen + film systems
  - storage phosphors (also called computed radiography (CR))
  - digital detectors (also called digital radiography (DR))
  - image intensifier + camera systems

- We will only discuss screen+film and DR systems

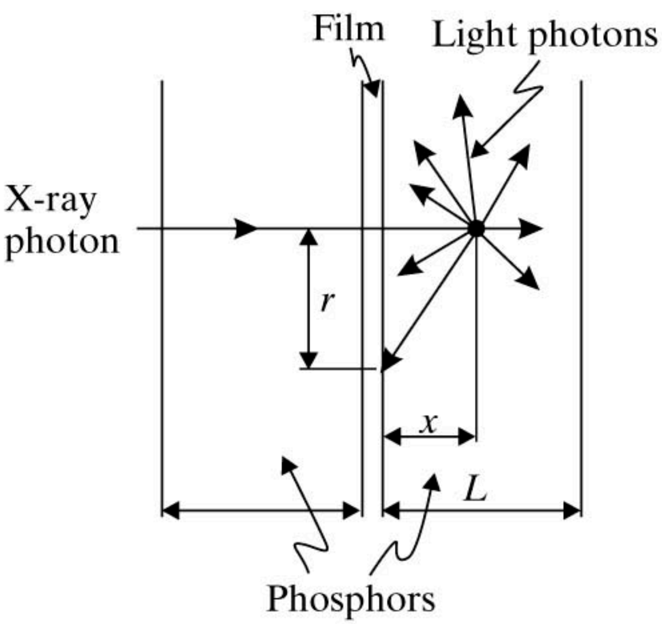
Film-Screen Detectors

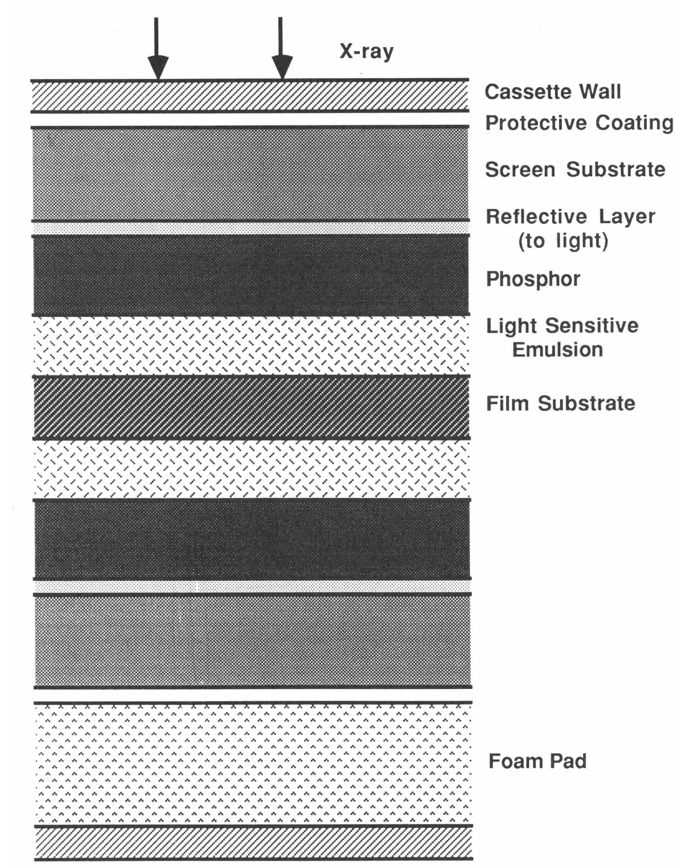
- X-rays can expose film directly
  - used by Roentgen in 1895 discovery of mysterious 'x-rays'
  - very inefficient as most rays pass directly though film
- Sensitivity, or 'stopping power' of detectors is a critical feature
  - we want thick and/or dense material
  - another reason why choice of photon energy is important
- Sensitivity of screen film systems can be enhanced by using a luminescent *intensifying screen* on both sides of the radiographic film
- A good intensifying screen phosphor should be highly x-ray attenuating and should emit many light photons for every x-ray photon that is stopped
- In the 1970s rare earth phosphors were introduced with ~10^3 light photons per incident 50 keV x-ray photon
- Higher conversion efficiencies lead to faster exams and less radiation dose



Film Characteristics

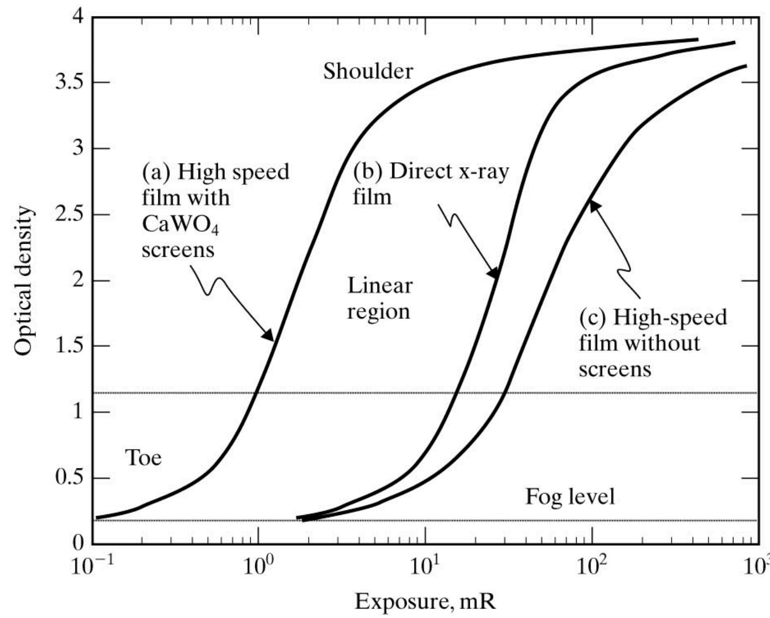
- Film cassettes contain double-sided film emulsion and two phosphors to increase sensitivity to x-rays and improve contrast





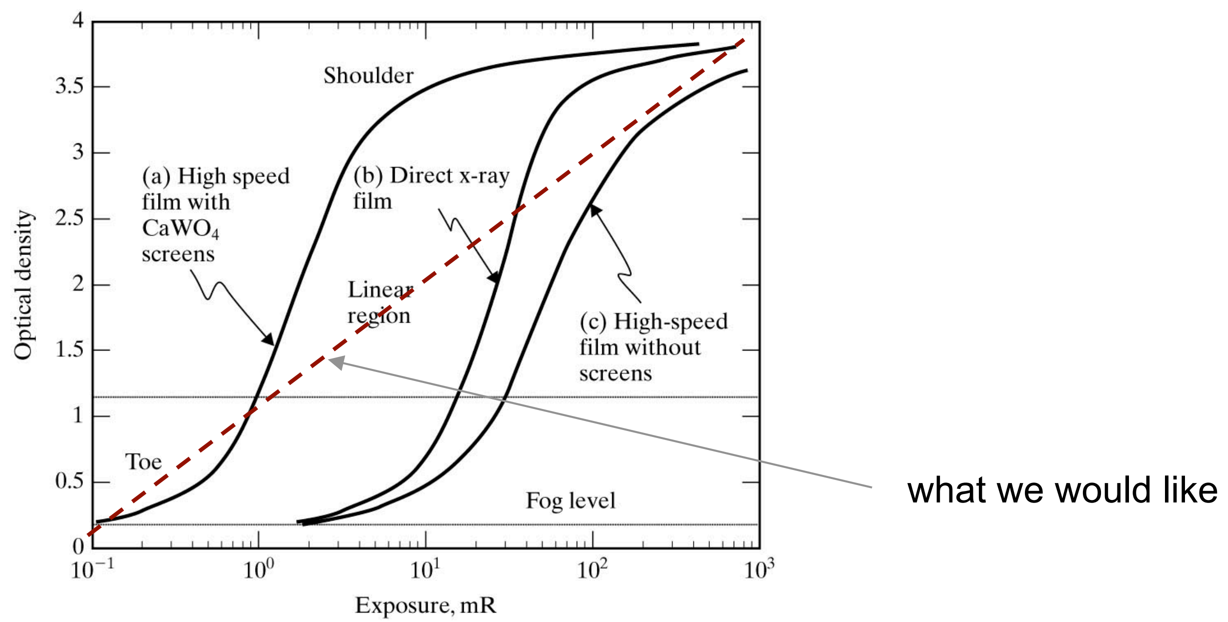
Film-screen H&D Curves

- The overall response of the optical density of the developed film to the x-ray exposure is not linear, but there is a linear region that can be used
- H&D curve is named after Hurter and Driffield
- Many trade-offs in using film, e.g. increasing the slope in the linear region improves contrast, but decreases usable range



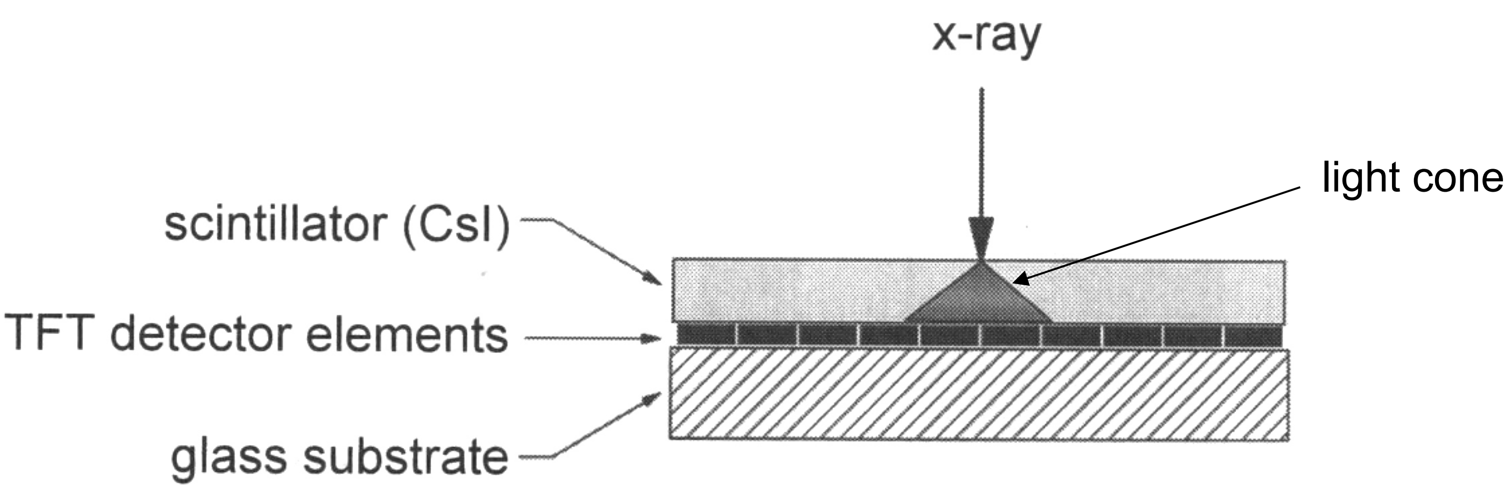
Digital X-ray Detectors

- Screen-film systems are falling out of use for three reasons
  - cost of film (silver based emulsions)
  - digital detectors have a wider dynamic range of linear operation
  - cost of digital detector systems is gradually reducing



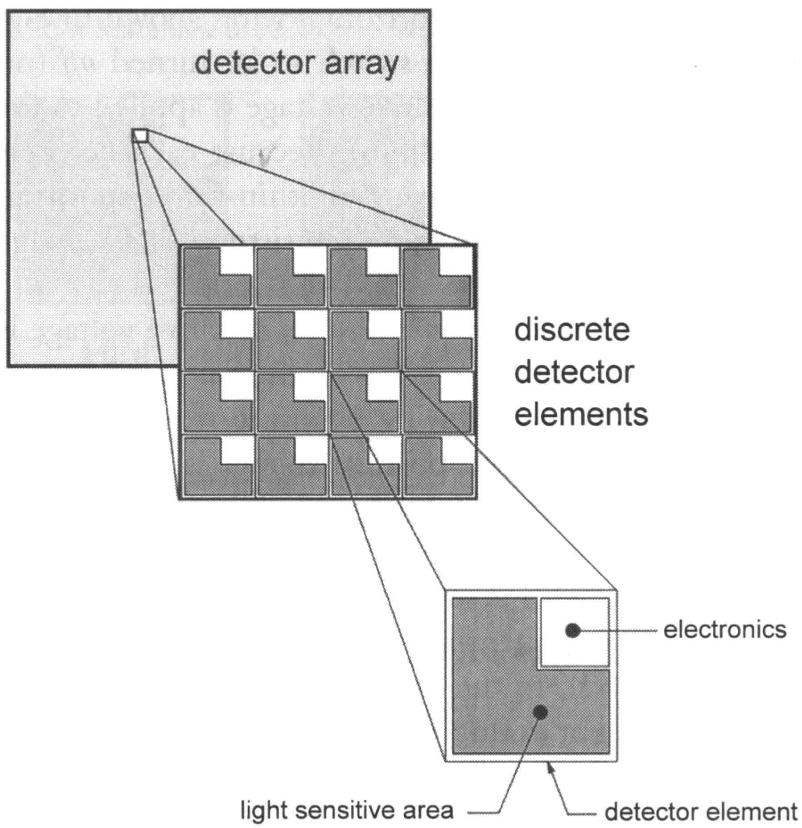
Digital X-ray Detectors

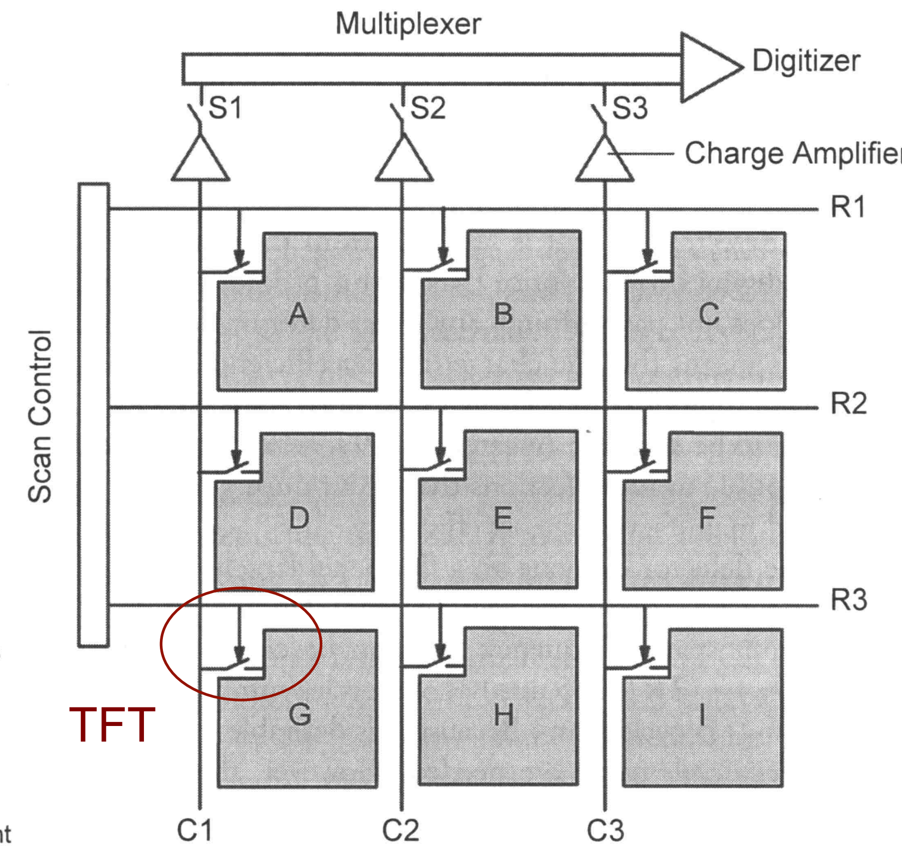
- Digital x-ray detector systems
  - an inorganic 'scintillator', such as CsI, converts x-ray photon to a small flash of optical photons
  - the optical photons hit the surface of a fine 2-D grid of low-noise photodiodes, which can be read out to determine scintillation location



Flat panel digital detectors

- Uses same technology as flat-panel displays

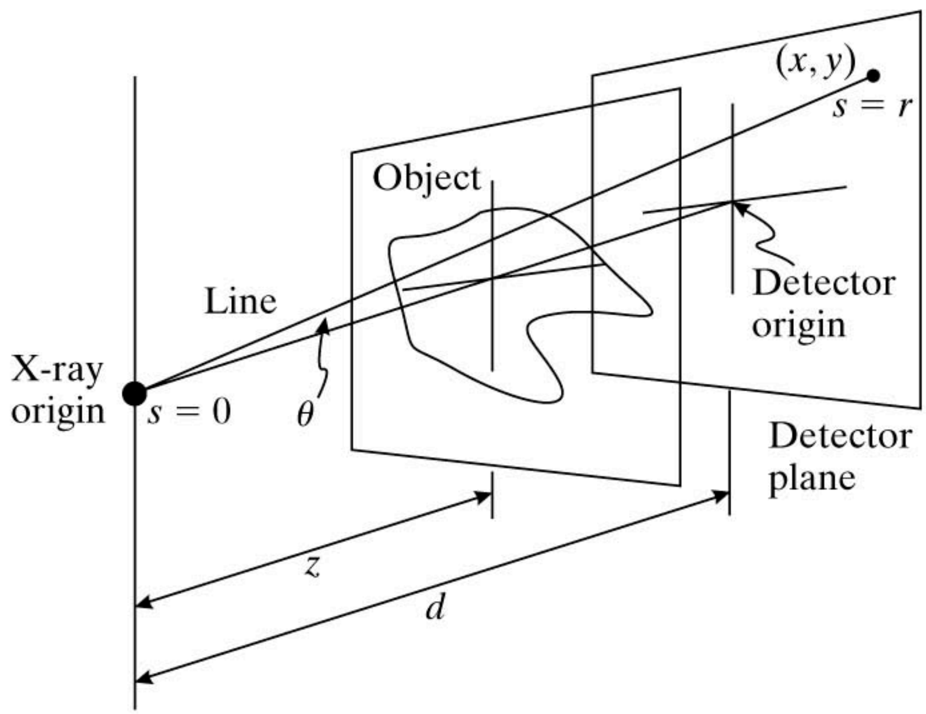




X-ray Imaging Equation

- Recall imaging equation $I(x,y) = \int_0^{E_{max}} E' S_0(E') e^{-\int_0^{r(x,y)} \mu(s,E')\, ds} dE'$

- This equation must be modified by several effects
  - inverse square law
  - obliquity
  - beam divergence
  - anode heel effect
  - path length
  - depth-dependent magnification
  - X-ray source size
  - afterglow
  - crosstalk
  - scatter
- We will outline a few effects



Inverse square law

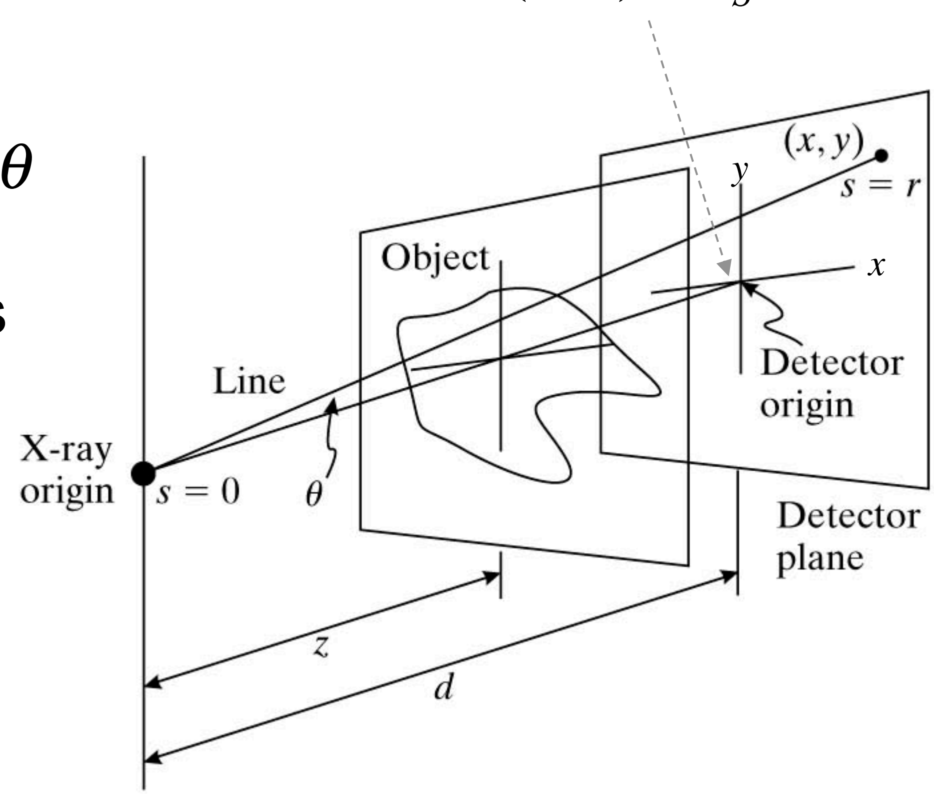
- the net flux of photons (i.e. photons per unit area) decreases as $1/r^2$, where r is the distance from the x-ray origin

- In general we can modify the imaging equation to

$$I(x,y) = I(0,0)\frac{d^2}{r^2} = I(0,0)\cos^2\theta$$

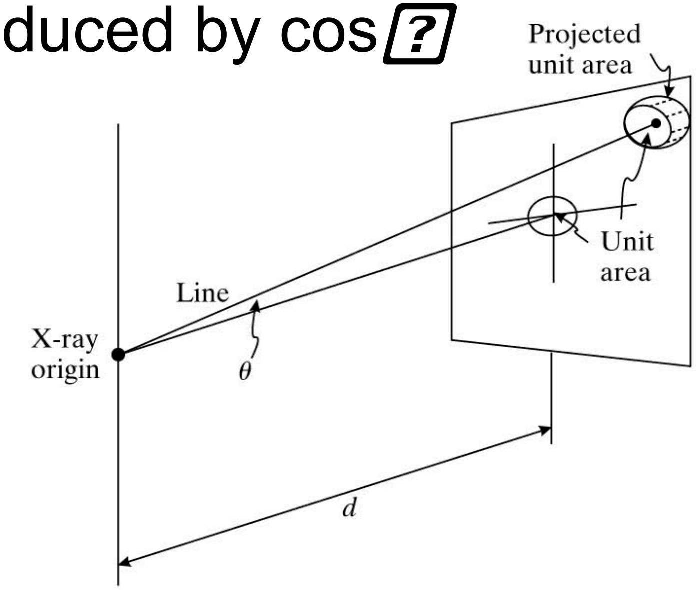
- if not taken into account, this effect can lead to incorrect estimate of attenuation

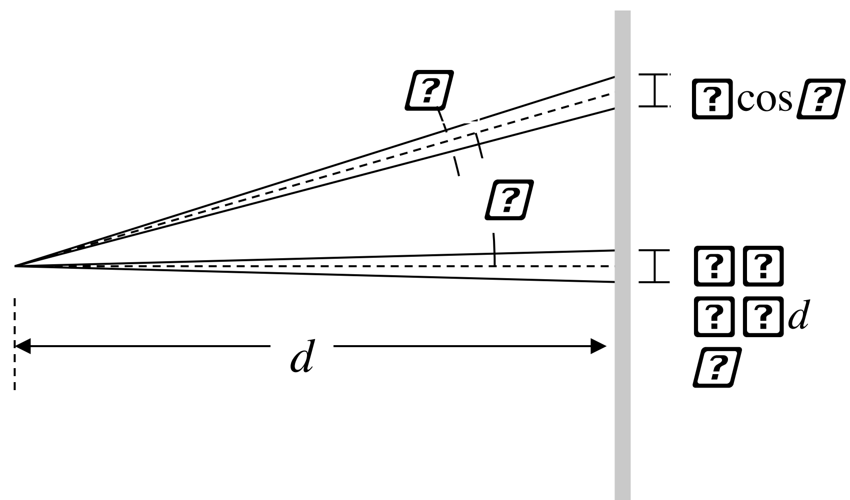
$$I(0,0) = I_S / 4\pi d^2$$



Obliquity

- Away from the origin on the detector plane, the distance from the source is greater ($1/r^2$ effect)
- In addition the orientation of the detector is no longer perpendicular, so the fluence and intensity per unit area is reduced by cos ?

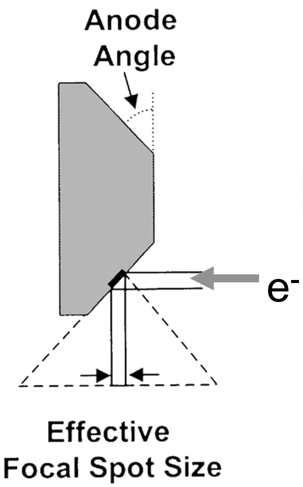




- Combined effect with $1/r^2$ effect is $I(x,y) = I(0,0)\cos^3\theta$
- If ? is small, this effect can be neglected

Effects of extended x-ray source

- The extended source or 'spot' on anode for x-ray generation

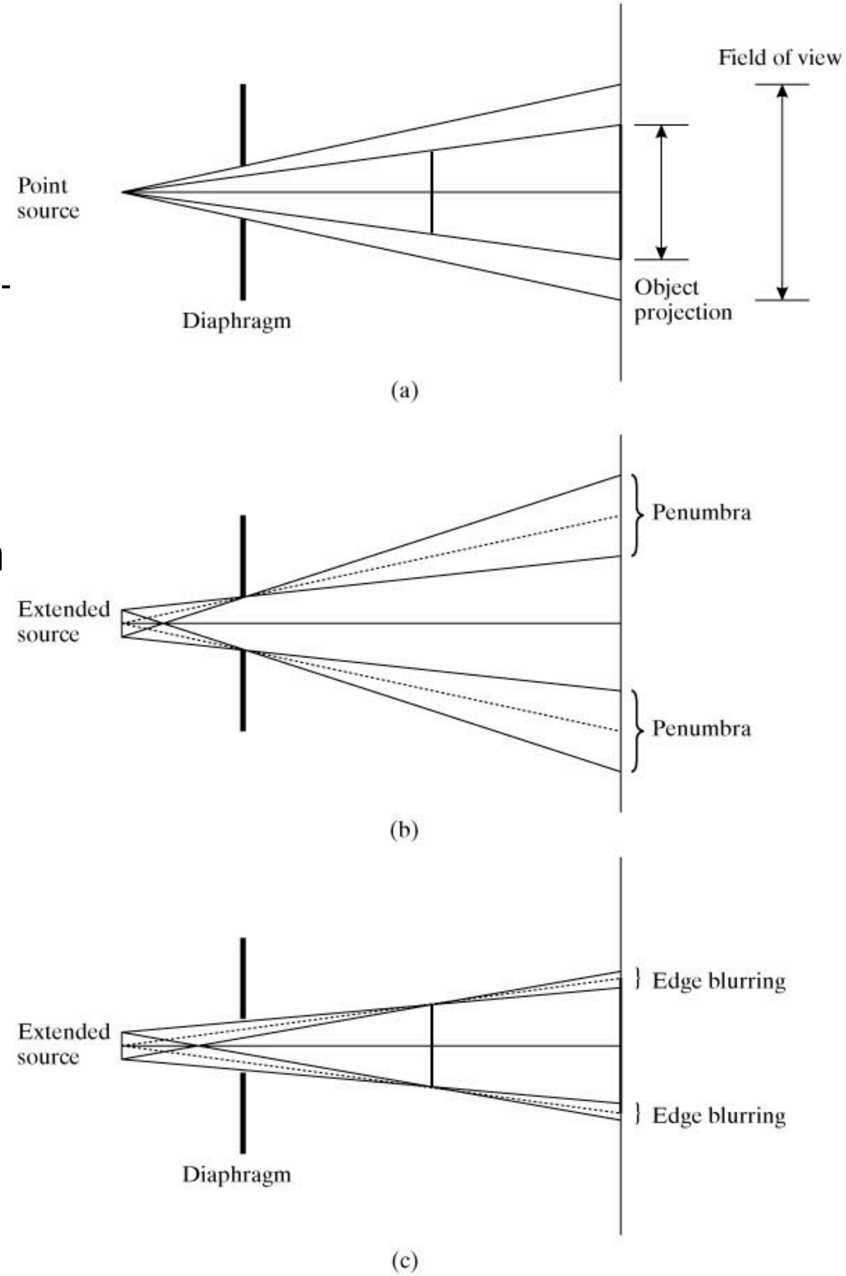


- This leads to several effects

a) Ideal field of view and object projection (with magnification)

b) Penumbra at edges of field of view due to extended source

c) Blurred object edges due to extended source



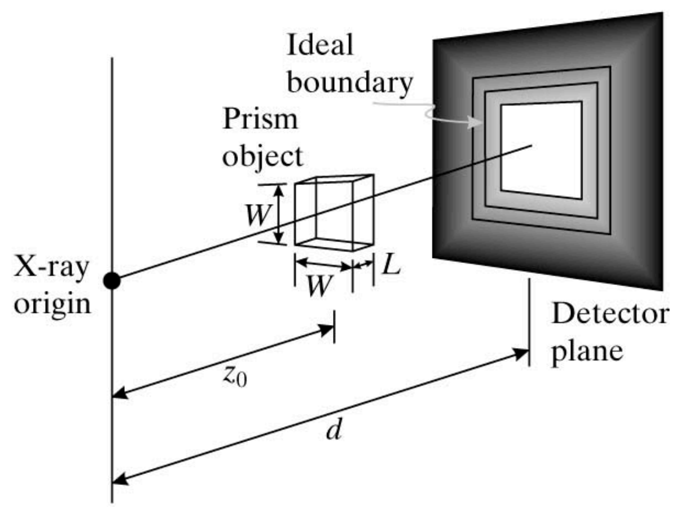
- The extended source effect is a convolution of the source shape with the object shape (with magnification effects also included)

Impact on Imaging Equation

- The imaging equation with the following effects (only)
  - inverse square law
  - obliquity
  - X-ray source size

$$I(x,y) = \overbrace{h(x,y)} * \cos^3\theta \int_0^{E_{\max}} E' S_0(E') e^{-\int_0^{r(x,y)} \mu(s,E')\,ds}\, dE'$$

- The imaging equation should be modified to account for all physics effects
- Some effects can be compensated



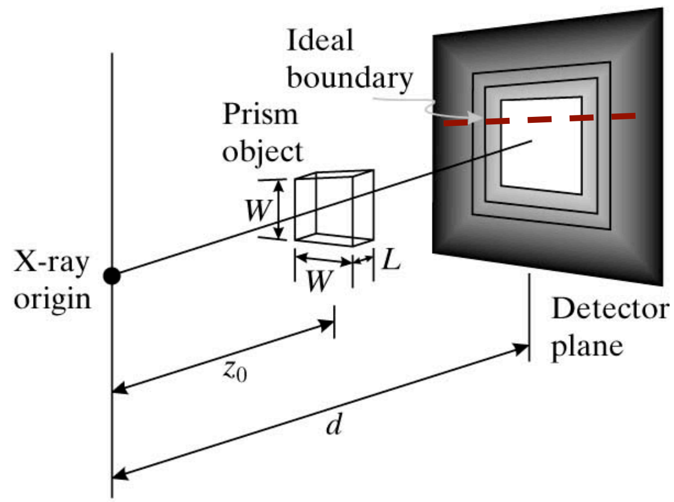
Signal to Noise Ratio

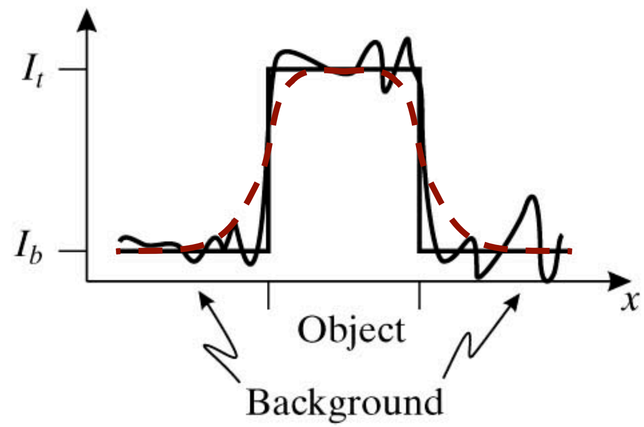
- The imaging equation tells us the measured values, i.e. on a profile through the image
- Physical effects cause distortion of the profile (dashed red curve)
- For now assume a rect() profile, with contrast

$$C = \frac{I_t - I_b}{I_b}$$

- There is also *quantum noise* from the finite # photons for each detector in the background with a signal to noise ratio (SNR) of

$$\mathrm{SNR} = \frac{I_t - I_b}{\sigma_b}$$





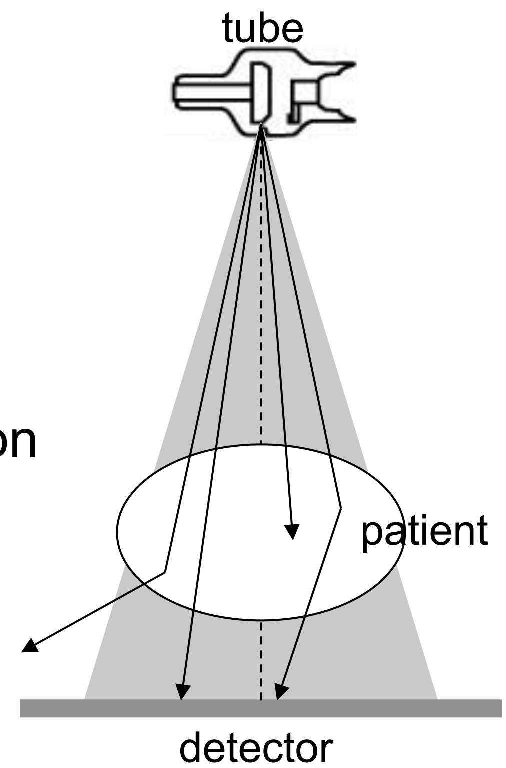
Signal to Noise Ratio for X-ray Imaging

- For x-rays with mean energy $h\nu$, the background intensity per detector element is $$I_b = \frac{N_b h\nu}{A\Delta t}$$
- Recall variance for a Poisson counting process is equal to the mean $$\sigma_b^2 = N_b \left(\frac{h\nu}{A\Delta t} \right)^2$$
- So SNR is given by $\mathrm{SNR} = C\sqrt{N_b}$
  - TO improve SNR, we need to increase the contrast and/or the # photons
  - As # photons goes up, so does radiation dose
  - Reducing energy of photons increases contrast, noise (fewer transmitted photons), and dose (more absorbed photons)
  - Impact on SNR and SNR/dose ratio is not always clear
- We can group detector elements together to reduce noise, but at some point contrast degrades

Impact of Scatter

- Types of events
  - detected true, or primary, photon
  - detected Compton scattered photon
  - undetected Compton scattered photon
  - undetected photon due to photoelectric absorption

  } attenuation



- Due to random directions, Compton scatter can be considered a constant

$$C' = \frac{(I_t + I_s) - (I_b + I_s)}{(I_b + I_s)} = \frac{I_t - I_b}{I_b} \frac{I_b}{I_b + I_s} = \frac{C}{1 + I_s / I_b}$$

- So scatter reduces contrast depending on the scatter/true ratio
- In addition scatter increases noise, so there is a further reduction in SNR

Contrast Agents

- Above 40 keV provide a significant enhancement
- There is a very small risk of serious medical complications in the kidney that excretes the contrast agents
- Air can be used to expand the the lungs or GI tract, providing 'negative' or contrast
- Example of an *intravenous pyelogram* used to look for damage to the urinary system, including the kidneys, ureters, and bladder