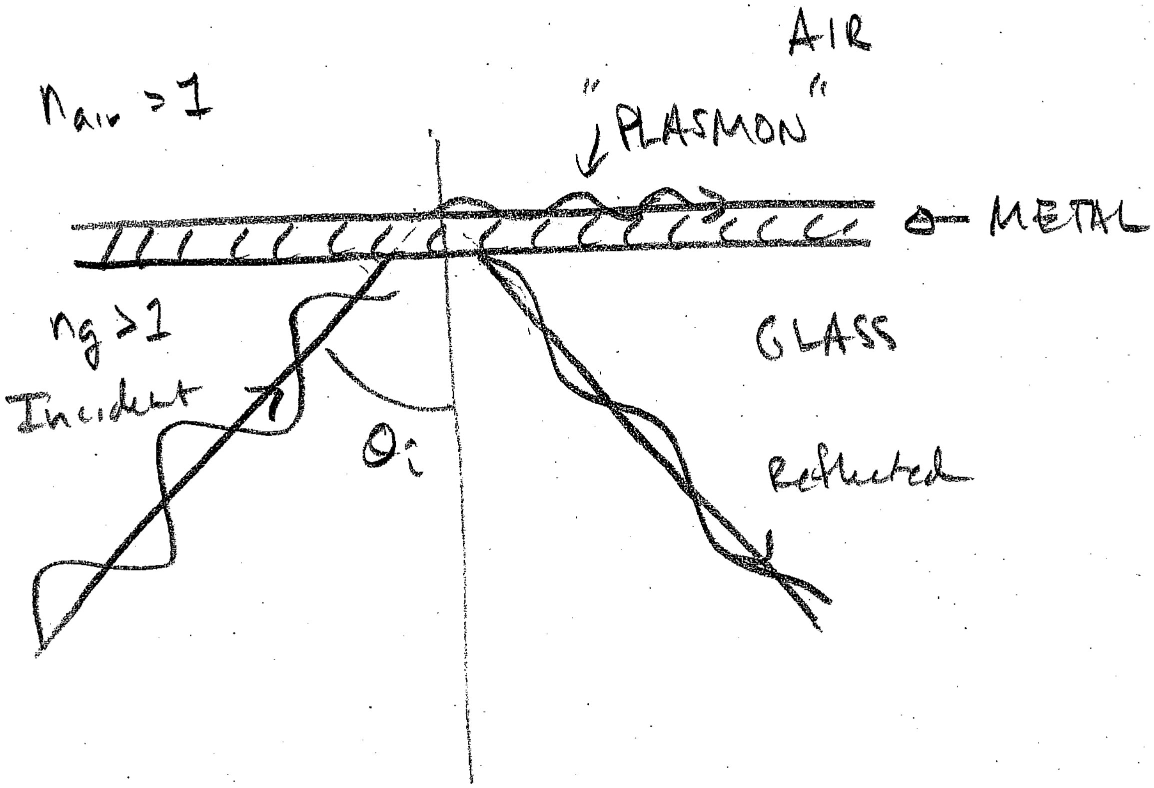
#1

SURFACE PLASMA OSCILLATIONS

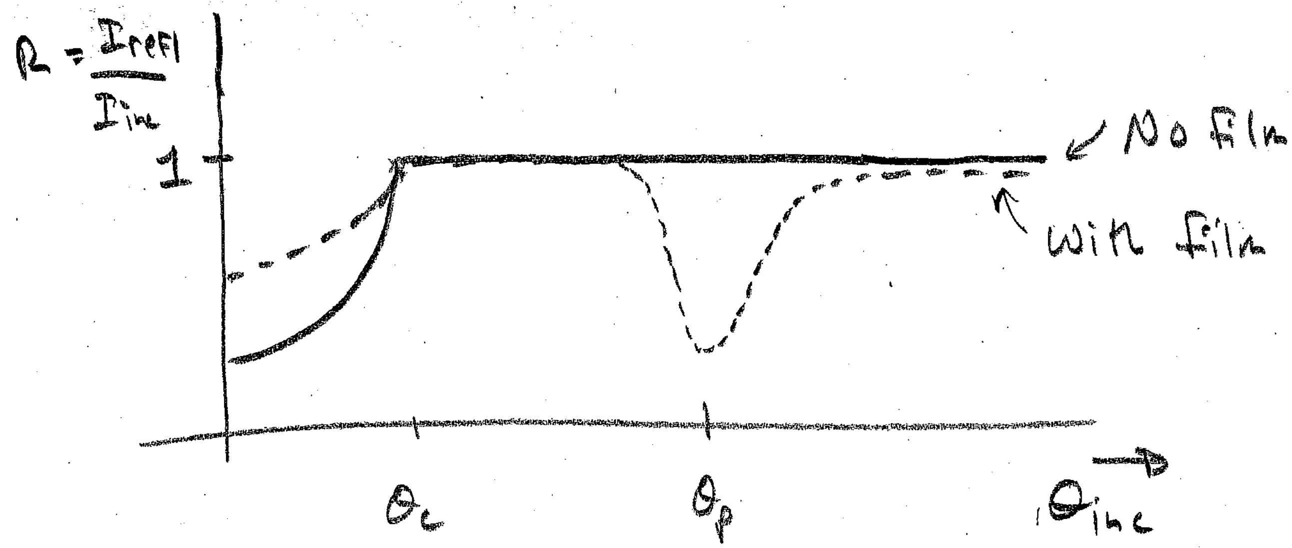


* FACT: At some $\theta_i > \theta_c = \sin^{-1}\frac{1}{n_g}$ we see a dip in reflected intensity for a given color of light

* FACT: Only works for "thin enough" films.

* FACT: Only works for one polarization.

#2



WANT TO SHOW

- WHAT IS FORM OF "PLASMON" WAVE?

- WHAT CONDITION(S) MUST BE SATISFIED TO SEE EFFECT?

- HOW DO PROPERTIES OF METAL GOVERN EFFECT?

#3)

BASIC THEORY — MAXWELL EQUATIONS and PLANE WAVES

IN MKS SYSTEM

Gauss Laws
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\cdot\vec{D} = \rho$$

Ampere's Law
$$\nabla\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Faraday's Law
$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Constitutive equations - Material properties

$$\vec{B} = \mu_0\vec{H} + \vec{M} = \mu_0(\vec{H} + \chi_m\vec{H}) = \mu_0\mu\vec{H}$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0(1+\chi_E)\vec{E} = \varepsilon_0\varepsilon\vec{E}$$

Assume that susceptibility is proportional to applied field and is a scalar (same direction).

#4

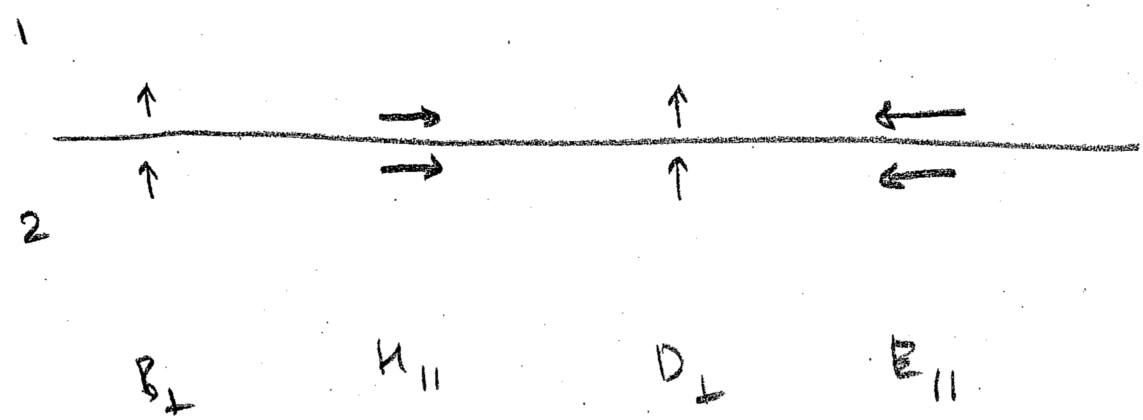
Basic Maxwell Equations require certain continuity at any boundary.

$$B_\perp^{(1)} = B_\perp^{(2)} \qquad H_\parallel^{(1)} = H_\parallel^{(2)}$$

(SEE ANY E&M TEXT)

$$D_\perp^{(1)} = D_\perp^{(2)} \qquad E_\parallel^{(1)} = E_\parallel^{(2)}$$

(IN ABSENCE OF SURFACE CURRENTS OR CHARGES.)

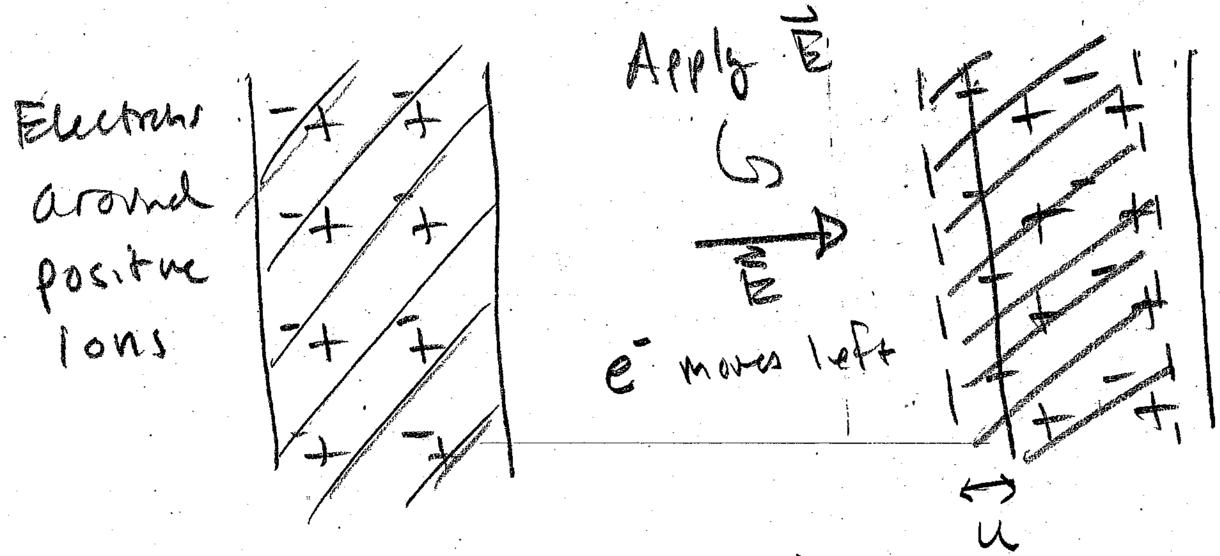


#5

PLASMA OSCILLATION

Oscillation of charged gas under influence of oscillating electric field.

Model: Assume free electrons (not in atomic orbitals) moving in a positively charged background.



Because of applied $\vec{E}$, electrons get displaced, material becomes polarized:

$$\vec{P} = -n e \vec{u}$$

Polarization ($\vec{P}$), electron density (n), electron charge (e), displacement ($\vec{u}$)

#6

For free electrons, displacement follows Newton's Law:

$$\vec{F} = m\frac{d^2\vec{u}}{dt^2}$$

$$\downarrow$$

or

$$-en\vec{E} = mn\frac{d^2\vec{u}}{dt} = \frac{-mn}{ne}\frac{d^2\vec{P}}{dt^2}$$

or $$\vec{E} = \frac{m}{ne^2}\frac{d^2\vec{P}}{dt^2}$$

Let $\vec{E} = \vec{E}_0(\vec{r})e^{i\omega t}$ — oscillating $\vec{E}$

Then with $\vec{P} = \vec{P}_0(\vec{r})e^{i\omega t}$, $\frac{d^2\vec{P}}{dt^2} = -\omega^2\vec{P}_0(\vec{r})e^{i\omega t}$

so $$\vec{E}(\vec{r},\omega) = \frac{-\omega^2 m}{ne^2}\vec{P}(\vec{r},\omega)$$

or $$\boxed{\vec{P}(\vec{r},\omega) = -\frac{ne^2}{m\omega^2}\vec{E}(\vec{r},\omega)}$$

#7

Now, since $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \varepsilon \vec{E}$

we get $\vec{D}(\vec{r},\omega) = \varepsilon_0 \vec{E}(\vec{r},\omega) - \frac{ne^2}{m\omega^2} \vec{E}(\vec{r},\omega)$

$$= \varepsilon_0 E(\vec{r},\omega)\left[1 - \frac{ne^2}{m\omega^2\varepsilon_0}\right]$$

From which we obtain the frequency dependent dielectric permittivity:

$$\varepsilon(\omega) = 1 - \frac{ne^2}{m\varepsilon_0} \cdot \frac{1}{\omega^2}$$

We define $\omega_p^2 = \frac{ne^2}{m\varepsilon_0}$

called the plasma frequency

so $$\boxed{\varepsilon(\omega) = 1 - \omega_p^2/\omega^2}$$

is the free-electron dielectric permittivity.

#8

Note, if $\omega < \omega_p$ $\epsilon(\omega) < 0$

$\boxed{\text{This will be important}}$

Back to Maxwell...

* Maxwell equations "admit" plane wave solutions.

* Combining Maxwell equations, one can get wave equations.

  (You should know/Learn How to do this !!!)

* Operating on Plane waves gives handy form of Equations

#9

Example:

Let $\vec{B}(\vec{r},t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

↑ plane wave

$$\nabla\cdot\vec{B} = \nabla\cdot\vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(B_{0x}\hat{i} + B_{0y}\hat{j} + B_{0z}\hat{k}\right) e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$= i\left(B_{0x}k_x + B_{0y}k_y + B_{0z}k_z\right) e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= i\vec{k}\cdot\vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\boxed{\text{or} \quad \nabla\cdot\vec{B} = i\vec{k}\cdot\vec{B}}$$

Likewise, other Maxwell equations become

$$\nabla\cdot\vec{B} = 0 \qquad\qquad \vec{k}\cdot\vec{B} = 0$$

$$\nabla\cdot\vec{D} = 0 \qquad\qquad \vec{k}\cdot\vec{D} = 0$$

$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \quad\longrightarrow\quad \vec{k}\times\vec{H} = -\omega\vec{D}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad\qquad \vec{k}\times\vec{E} = \omega\vec{B}$$

Assuming no free charge ($\rho = 0$)
no free current ($\vec{J} = 0$)

#10

So for plane waves, we can combine equations to get

$$\vec{k} \times (\vec{k} \times H) = \vec{k} \times (-\omega \vec{D}) = \omega \varepsilon_0 \varepsilon (\vec{k} \times \vec{E})$$

$$= -\omega \varepsilon_0 \varepsilon (\omega \vec{B})$$

$$= -\omega^2 \varepsilon_0 \varepsilon \mu_0 \mu \vec{H}$$

$$\vec{k} \times (\vec{k} \times H) = \vec{k}(\vec{k} \cdot \vec{H}) - \vec{H}(\vec{k} \cdot \vec{k})$$

$$= 0 - k^2 \vec{H}$$

So $$\boxed{k^2 \vec{H} = \omega^2 \varepsilon_0 \varepsilon \mu_0 \mu \vec{H}}$$

$$= \varepsilon \mu \frac{\omega^2}{c^2} \vec{H}$$

since $c^2 = \frac{1}{\varepsilon_0 \mu_0}$

This gives plane wave Dispersion relation

$$\boxed{k^2 = \varepsilon \mu \frac{\omega^2}{c^2}}$$

#11

DISPERSION RELATION

Relationship between wavelength (or wave number) and frequency. Recall $f\lambda = v$ from 1st year (or high school). $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$ so $f\lambda = \frac{\omega}{k} = v$

or $k = \frac{\omega}{v}$ $\Longleftrightarrow$ $k = \sqrt{\varepsilon\mu}\,\frac{\omega}{c}$

compare

Phase speed in medium is $v = \frac{c}{\sqrt{\varepsilon\mu}}$

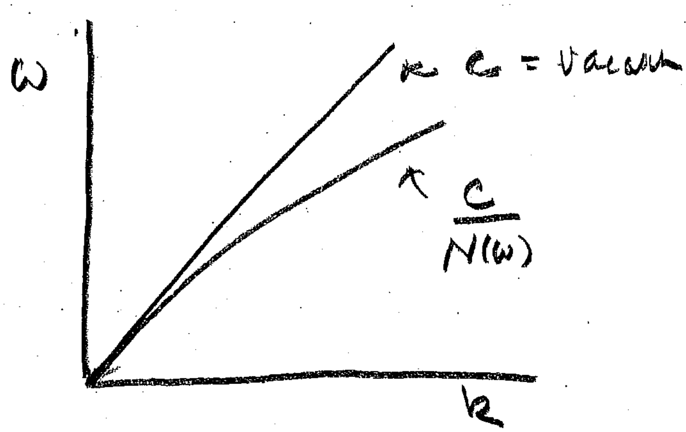
For non magnetic materials (e.g. glass) $\mu = 1$

so $v = \frac{c}{\sqrt{\varepsilon}} \equiv \frac{c}{N}$

N = refraction index, and $\boxed{\varepsilon \equiv N^2}$

If ε (or N) depends on ω, material "<u>disperses</u>" light

$N(\omega)$ typically <u>bigger</u> for <u>blue</u> than <u>red</u>.



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Dispersion relation for free electron plasma:

$$k^2 = \varepsilon(\omega)\mu \frac{\omega^2}{c^2}$$

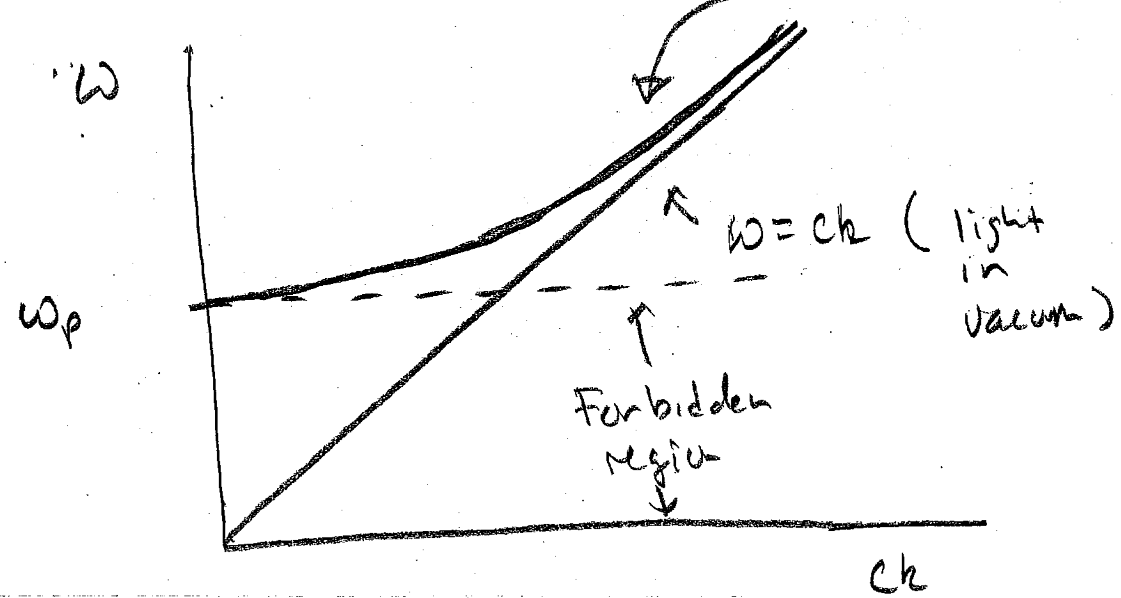
or $(ck)^2 = \mu\, \varepsilon(\omega)\, \omega^2$

$$= \mu \left(1 - \frac{\omega_p^2}{\omega^2}\right)\omega^2$$

$$= \mu\left(\omega^2 - \omega_p^2\right)$$

some math $\longrightarrow$ $\boxed{\omega = \left(\frac{(ck)^2}{\mu} + \omega_p^2\right)^{1/2}}$

Let $\mu = 1$ (non magnetic plasma)



#13

For $\omega < \omega_p$, there is no wave propagation through plasma — when $\varepsilon(\omega) < 0$, there can be no propagation.

$$k^2 = \varepsilon\mu\frac{\omega^2}{c^2} < 0 \quad \text{if} \quad \varepsilon < 0$$

k is thus imaginary, $\vec{k}\cdot\vec{r} = i\alpha(\vec{r})$

$$\vec{E}_0 e^{i(\vec{k}\cdot\vec{r})} e^{i\omega t} \rightarrow \underbrace{e^{-\alpha(\vec{r})}}_{\text{exponential damping}} \vec{E} e^{i\omega t}$$

SURFACE PLASMA OSCILLATIONS

Instead of wholly real or wholly imaginary $\vec{k}$ can be complex at a surface

Real in one direction

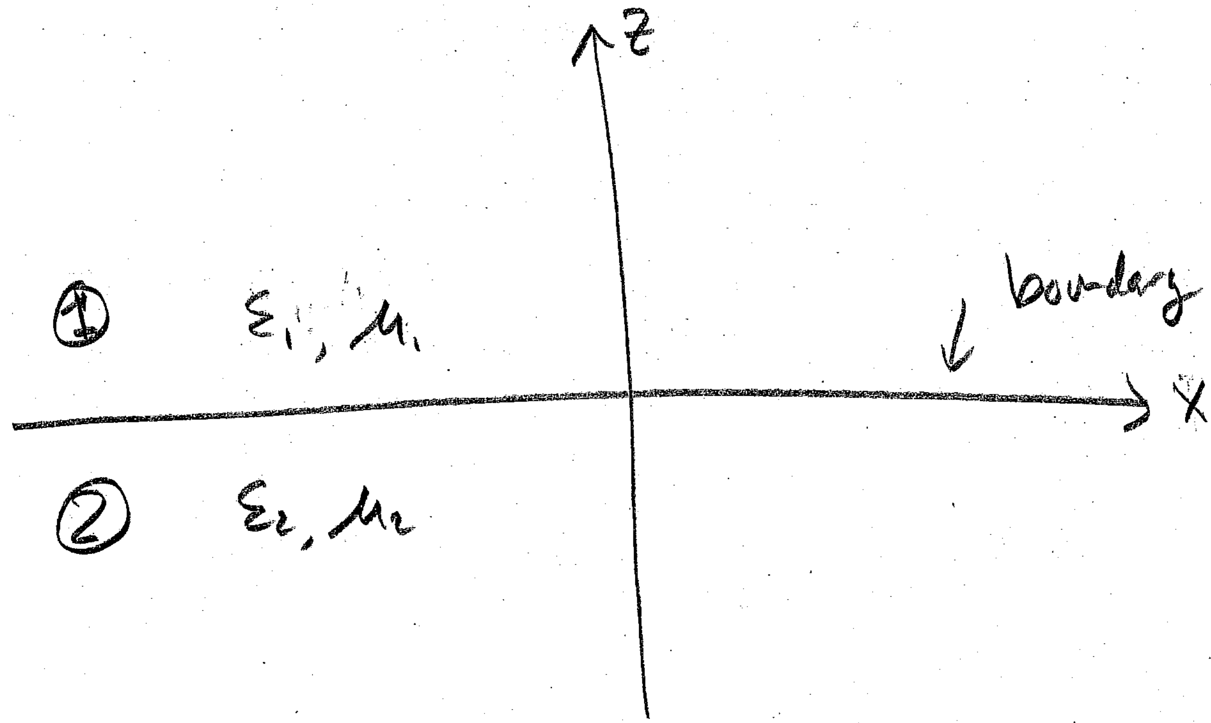
Imaginary in another direction.

#14

Waves at surfaces can be "evanescent"

They do not radiate, and $\vec{E}, \vec{B}$ are not transverse to each other.

How does this work?



Let plane wave at boundary have k_x, ω be real (propagation along x direction)

Since $k_x^2 + k_z^2 = \varepsilon\mu \frac{\omega^2}{c^2}$

$$k_z^2 = \varepsilon\mu \frac{\omega^2}{c^2} - k_x^2 \Big\} \text{ real, but possibly } < 0$$

<u>AND</u> Boundary conditions:

$$E_{1\parallel} = E_{2\parallel}, \quad D_{1\perp} = D_{2\perp} \Rightarrow \varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$$

$$H_{1\parallel} = H_{2\parallel}, \quad B_{1\perp} = B_{2\perp} \Rightarrow \mu_1 H_{1\perp} = \mu_2 H_{2\perp}$$

What are possible plane waves at interface?

2 possibilities TE (transverse <u>Electric</u>)
TM (transverse magnetic)

TE means $\vec{E} \perp \vec{k}$

TM means $\vec{B} \perp \vec{k}$

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Choose TM first $\vec{B} \perp \vec{k}$ means

$\vec{B} = B(r,t)\,\hat{j}$ only (Field direction perp. to page..)

Let $\vec{B} = B_0 \hat{j}\, e^{i(k_x x + k_z z - \omega t)}$

with $\vec{k} \times \vec{H} = -\omega \vec{D} \Rightarrow \vec{k} \times \vec{B} = -\omega \mu \mu_0 \varepsilon_0 \varepsilon \vec{E}$

$= -\omega \frac{\mu\varepsilon}{c^2} \vec{E}$

So $\vec{k} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ k_x & 0 & k_z \\ 0 & B_0 & 0 \end{vmatrix} = -\hat{i}\, k_z B_0 + \hat{k}\, k_x B_0$

or $-\omega \frac{\mu\varepsilon}{c^2} \vec{E} = -k_z B_0 \hat{i} + k_x B_0 \hat{k}$

(Note, $\vec{E}$ has x, z directions)

Write $\vec{E} = \frac{c^2 k_z}{\mu\varepsilon\omega} B_0 \hat{i} - \frac{c^2 k_x}{\mu\varepsilon\omega} B_0 \hat{k}$

↑ Parallel to boundary

↑ Perpendicular to boundary

#17

From boundary conditions, for TM wave

$B_{\perp 1} = B_{2\perp} = 0$ (no perp. component at boundary)

$$H_{1\parallel} = H_{2\parallel} \Rightarrow \frac{B_{01}}{\mu_1} = \frac{B_{02}}{\mu_2}$$

Also

$$E_{1\parallel} = E_{2\parallel} \Rightarrow \frac{c^2 k_{z1}}{\mu_1 \varepsilon_1 \omega} B_{01} = \frac{c^2 k_{z2}}{\mu_2 \varepsilon_2 \omega} B_{02}$$

Combining, and eliminating constant factors

$$\frac{k_{z1}}{\varepsilon_1} = \frac{k_{z2}}{\varepsilon_2}$$

since $k_z = \pm\left(\varepsilon\mu \frac{\omega^2}{c^2} - k_x^2\right)^{1/2}$

$$\pm \varepsilon_2 \left(\varepsilon_1 \mu_1 \frac{\omega^2}{c^2} - k_x^2\right)^{1/2} = \pm \varepsilon_1 \left(\varepsilon_2 \mu_2 \frac{\omega^2}{c^2} - k_x^2\right)^{1/2}$$

or

$$\varepsilon_2^2 \left(\varepsilon_1 \mu_1 \frac{\omega^2}{c^2} - k_x^2\right) = \varepsilon_1^2 \left(\varepsilon_2 \mu_2 \frac{\omega^2}{c^2} - k_x^2\right)$$

#18

Solve for $\omega(k)$ to get dispersion relation for surface wave

$$\varepsilon_2^2\left(\varepsilon_1\mu_1\frac{\omega^2}{c^2}-k_x^2\right)=\varepsilon_1^2\left(\varepsilon_2\mu_2\frac{\omega^2}{c^2}-k_x^2\right)$$

$$\varepsilon_2^2\varepsilon_1\mu_1\frac{\omega^2}{c^2}-\varepsilon_2^2k_x^2=\varepsilon_1^2\varepsilon_2\mu_2\frac{\omega^2}{c^2}-\varepsilon_1^2k_x^2$$

$$k_x^2(\varepsilon_1^2-\varepsilon_2^2)=\varepsilon_2\varepsilon_1(\varepsilon_1\mu_2-\varepsilon_2\mu_1)\frac{\omega^2}{c^2}$$

$$\boxed{k_x^2=\frac{\varepsilon_2\varepsilon_1(\varepsilon_1\mu_2-\varepsilon_2\mu_1)}{\varepsilon_2^2-\varepsilon_1^2}\frac{\omega^2}{c^2}}$$

This is the most general form for dispersion relation of surface wave

When magnetic permeability μ can be ignored (insulators & many metals) get simpler

$$\boxed{k_x^2=\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)\frac{\omega^2}{c^2}}$$

#19

For metal-air interface, 1 = air
2 = metal, so let $\varepsilon_1 = 1$ and
$\varepsilon_2 = \varepsilon$ (Drop subscript)

Then $$\boxed{k_x = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{1/2} \frac{\omega}{c}}$$

This is the general form that must be obeyed at air-metal interface, regardless of ε form.

Assume free-electron metal,

Then $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$

$$k_x = \left(\frac{1 - \omega_p^2/\omega^2}{1 + (1 - \omega_p^2/\omega^2)}\right)^{1/2} \frac{\omega}{c}$$

or $$c k_x = \left(\frac{\omega^2 - \omega_p^2}{2\omega^2 - \omega_p^2}\right)^{1/2} \omega$$

#20

$$(ck_x)^2(2\omega^2-\omega_p^2)=\omega^2(\omega^2-\omega_p^2)$$

some math

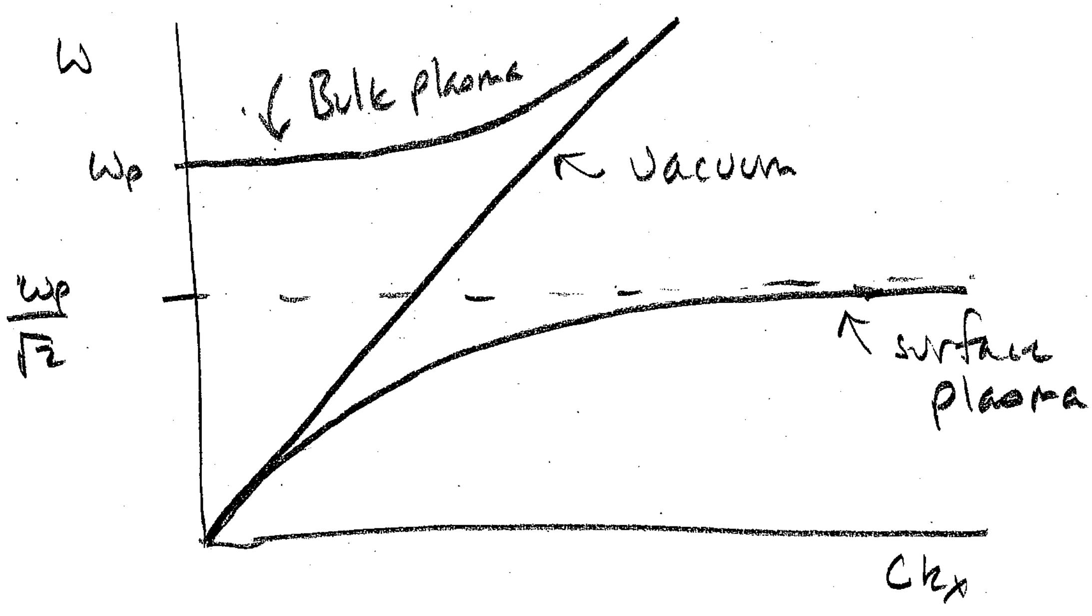
$$\omega^4-\left(2(ck_x)^2+\omega_p^2\right)\omega^2+\omega_p^2(ck)^2=0$$

quadratic in ω^2

$$\omega^2=\frac{\omega_p^2}{2}+(ck_x)^2-\sqrt{(ck_x)^4+\frac{\omega_p^4}{4}}$$

Note, for $ck_x\to 0$, $\omega^2\to 0$

for $(ck_x)\to\infty$ $\omega^2\to\frac{\omega_p^2}{2}$



#21

Look again at k_z relation:

$$k_z = \pm\left(\varepsilon\mu \frac{\omega^2}{c^2} - k_x^2\right)^{1/2}$$

On air side, $\mu = 1$ $\varepsilon_a = 1$, $\varepsilon_m = \varepsilon$

$$k_z = \pm\left(\frac{\omega^2}{c^2} - \left(\frac{\varepsilon}{1+\varepsilon}\right)\frac{\omega^2}{c^2}\right)^{1/2}$$

$$= \pm\frac{\omega}{c}\left(1 - \frac{\varepsilon}{1+\varepsilon}\right)^{1/2}$$

$$= \pm\frac{\omega}{c}\left(\frac{1+\varepsilon-\varepsilon}{1+\varepsilon}\right)^{1/2} = \pm\frac{\omega}{c}\left(\frac{1}{1+\varepsilon}\right)^{1/2}$$

On metal side $\mu = 1$ $\varepsilon_m = \varepsilon$

$$k_z = \pm\left(\varepsilon\frac{\omega^2}{c^2} - \left(\frac{\varepsilon}{1+\varepsilon}\right)\frac{\omega^2}{c^2}\right)^{1/2}$$

$$= \pm\frac{\omega}{c}\left(\varepsilon - \frac{\varepsilon}{1+\varepsilon}\right)^{1/2}$$

$$= \pm\frac{\omega}{c}\left(\frac{\varepsilon+\varepsilon^2-\varepsilon}{1+\varepsilon}\right)^{1/2}$$

$$= \pm\frac{\omega}{c}\left(\frac{\varepsilon^2}{1+\varepsilon}\right)^{1/2}$$

Now consider these in light of $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$.

For low frequencies, $\omega < \omega_p$ $\varepsilon(\omega) < 0$

In fact we know that for all values of k_x we must have $\omega < \frac{\omega_p}{\sqrt{2}}$

This means $\sqrt{2} < \frac{\omega_p}{\omega}$, $2 < \frac{\omega_p^2}{\omega^2}$

and $\varepsilon(\omega) < 1 - 2 = -1$

so $\frac{\varepsilon}{1+\varepsilon} > 0$, and k_x is real (by construction)

BUT $\frac{\varepsilon^2}{1+\varepsilon} < 0$ and $\frac{1}{1+\varepsilon} < 0$

so $\boxed{k_z \text{ is } \underline{\text{imaginary}}}$

#23

Look back at wave-plane solutions at boundary (TM)

$$\vec{B}(\vec{r},t) = B_0\hat{j}\, e^{i(k_x x + k_z z - \omega t)}$$

Write $k_z = \pm\left(\epsilon\mu \frac{\omega^2}{c^2} - k_x^2\right)^{1/2} = \pm i\left(k_x^2 - \epsilon\mu\frac{\omega^2}{c^2}\right)^{1/2}$

↑ Real

Then $$\boxed{\vec{B}(\vec{r},t) = B_0\hat{j}\, e^{i(k_x x - \omega t)}\, e^{\mp\left(k_x^2 - \epsilon\mu\omega^2/c^2\right)^{1/2} z}}$$

Likewise

$$\vec{E}(\vec{r},t) = \left[\frac{c^2 k_z}{\mu\epsilon\omega}B_0\hat{i} - \frac{c^2 k_x}{\mu\epsilon\omega}B_0\hat{k}\right] e^{i(k_x x + k_z z - \omega t)}$$

Make $\vec{E}$ look like write-up in Lab.

Want $\vec{E} = E_0\hat{i} + \pm i k_x\left(k_x^2 - \epsilon\mu\frac{\omega^2}{c^2}\right)^{-1/2} E_0\hat{k}$

so Let $E_0 = \frac{c^2 k_z}{\mu\epsilon\omega}B_0$

Then $B_0 = \frac{\mu\epsilon\omega}{c^2 k_z}E_0$

And
$$\frac{c^2 k_x}{\mu\varepsilon\omega} B_0 = \left(\frac{c^2 k_x}{\mu\varepsilon\omega}\right)\frac{\mu\varepsilon\omega}{c^2 k_z} E_0 = \frac{k_x}{k_z} E_0$$

So Now
$$\vec{E}(\vec{r},t) = \left(E_0\hat{\imath} + \pm i k_x\left(k_x^2 - \varepsilon\mu\frac{\omega^2}{c^2}\right)^{-1/2} E_0\hat{k}\right)$$
$$\times\, e^{\pm(k_x^2 - \varepsilon\mu\,\omega^2/c^2)^{1/2} z}\, e^{i(k_x x - \omega t)}$$

And
$$\vec{B}(\vec{r},t) = \pm i\frac{\varepsilon\mu\omega}{c^2}\left(k_x^2 - \varepsilon\mu\frac{\omega^2}{c^2}\right)^{-1/2} E_0\hat{\jmath}\ \Big)\, e^{\pm(k_x^2 - \varepsilon\mu\frac{\omega^2}{c^2}) z}\, e^{i(k_x x - \omega t)}$$

LAB WRITE-UP EQUATIONS

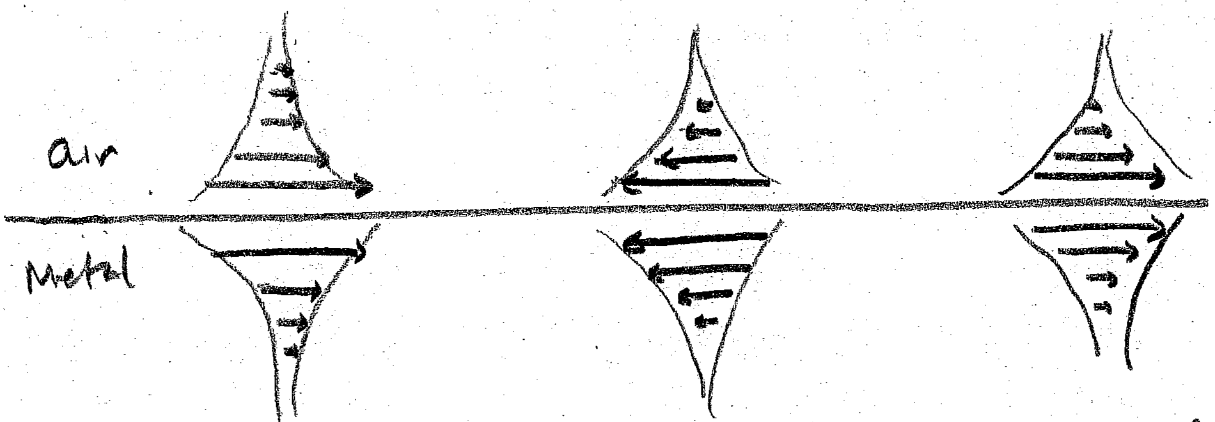
Some points — or what is <u>forest</u>?
I see lots of TREES!!

- Must choose sign of k_z to make exponentials <u>decay</u> ~~over distance~~.

- Imaginary numbers in constant parts indicate a <u>phase</u> shift.

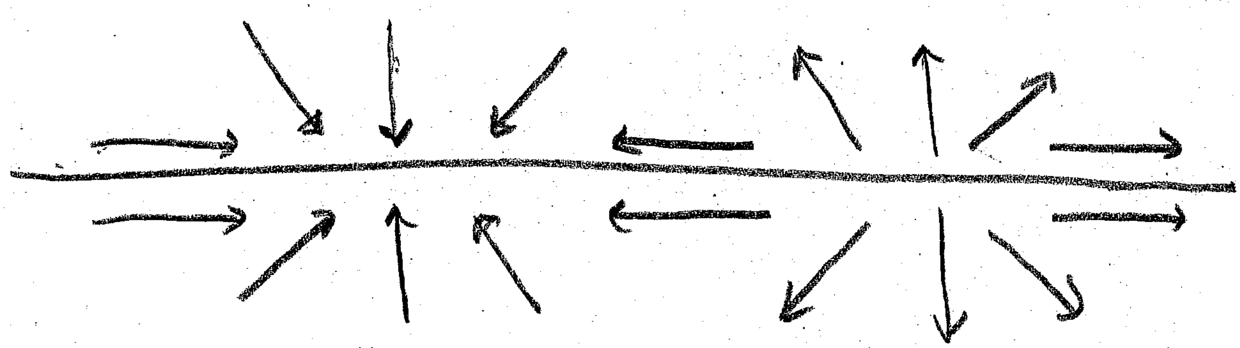
#25

- In z-direction – away from boundary – field amplitudes die off exponentially



x-component of $\vec{E}$ on either side of boundary

- Along x-direction x, z components of $\vec{E}$ are 90° out of phase, and E_z points in opposite directions across boundary



$\vec{E}_z$ as function of x at boundary

#26

Using simplified forms of k_z and letting $\mu = 1$

we have, above border (air) $\varepsilon_{air} = 1$, $\varepsilon_{metal} = \varepsilon$

In air

$$\left.\begin{aligned} \vec{E}_a(\vec{r},t) &= E_0 \hat{\imath} + i k_x \left(k_x^2 - \omega^2/c^2\right)^{-1/2} E_0 \hat{k} \\ \vec{B}_a(\vec{r},t) &= + i \frac{\omega}{c}\left(k_x^2 - \omega^2/c^2\right)^{-1/2} E_0 \hat{\jmath} \end{aligned}\right\} e^{-(k_x^2 - \omega^2/c^2)^{1/2} z} e^{i(k_x x - \omega t)}$$

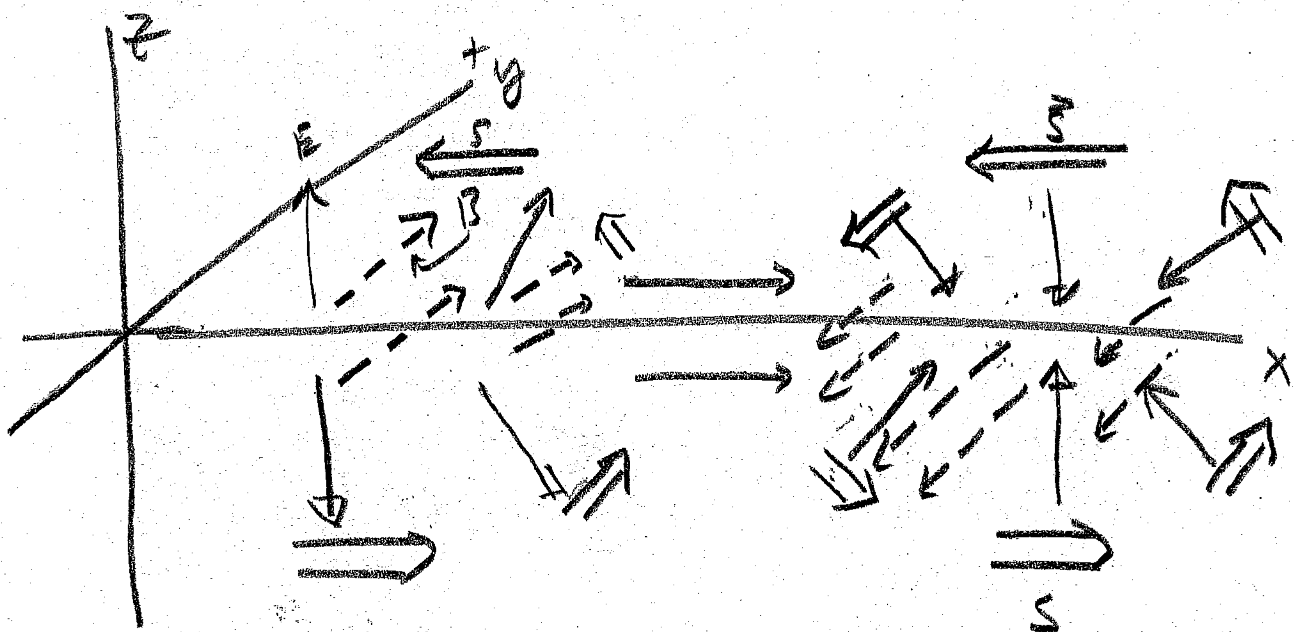
In metal

$$\left.\begin{aligned} \vec{E}_m(\vec{r},t) &= E_0 \hat{\imath} - i k_x \left(k_x^2 - \varepsilon \frac{\omega^2}{c^2}\right)^{-1/2} E_0 \hat{k} \\ \vec{B}_m(\vec{r},t) &= - i \frac{\varepsilon \omega}{c^2}\left(k_x^2 - \varepsilon \frac{\omega^2}{c^2}\right)^{-1/2} E_0 \hat{\jmath} \end{aligned}\right\} e^{+(k_x^2 - \varepsilon \frac{\omega^2}{c^2})^{1/2} z} e^{i(k_x x - \omega t)}$$

(Note $\vec{B}$ is continuous across border by virtue of dispersion relation required by $\left(k_x^2 - \frac{\omega^2}{c^2}\right)^{-1/2} = -\varepsilon\left(k_x^2 - \varepsilon \omega^2/c^2\right)^{-1/2} \Rightarrow k_x^2 = \left(\frac{\varepsilon}{1+\varepsilon}\right)\frac{\omega^2}{c^2}$)

$\vec{B}$ is <u>in phase</u> with E_z in air and <u>opposite phase</u> with E_z in metal.

So at interface



$\vec{B}$ - dotted $\dashrightarrow$

$\vec{E}$ - solid $\longrightarrow$

$\vec{S}$ - double $\Longrightarrow$

$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ gives energy flow