Surface Plasmon Resonance in a Thin Metal Film

1 Background

In this experiment you will observe and record the surface plasmon resonance (SPR) curve for a thin metal film. This is done by shining monochromatic light through a glass prism, onto the film, and recording the intensity of the reflected light vs. angle of incidence (see Fig. 3). After acquiring the resonance curve using the computer based system, you will use analysis and fitting programs to investigate how this curve depends on the real and imaginary parts of the dielectric constant of the metal film, and on the film thickness. Instructions for use of all programs are attached.

Surface plasmons are a quantum phenomenon—they are excited by photons incident on the metal-air interface, and can decay back into photons as evidenced by the increased intensity of light radiated from the film as the reflected intensity decreases. Surface plasmons are, however, well described by classical electromagnetic theory, and that is the approach taken here.

Part 2 of this write-up derives the dispersion relation (ω vs. k) for surface plasmons. This relation shows that conservation of energy and momentum in the plasmon excitation process can only be satisfied by having the light pass through the glass prism before it is incident on the metal film, as opposed to shining the light directly onto the film.

The actual expression for the reflected intensity as a function of the angle of incidence is quite messy given that the dielectric constant of the metal film has both real and imaginary parts. This expression is derived in the paper by Simon, Mitchell and Watson (1975), and is stated in equation 11 of their paper. The analysis and fitting programs take care of the messy math; you only need to vary the film parameters to find the best analytical fit to your data using this expression.

When surface plasmons were first observed nearly four decades ago, they were treated as an interesting optical phenomenon. Eventually it was recognized that SPR could be a very sensitive probe into the conditions at the surface of the metal film. As the dielectric constant of the medium adjacent the metal film changes, the angle of incidence at which the minimum in the resonance curve occurs also changes. SPR has been used as a probe to measure such diverse phenomena as thickness of the film adsorbed on the metal surface (SPR can detect less than 1/10 of a layer of atoms or molecules), concentration of various chemicals in solution, and protein-receptor binding.

2 Theory: surface plasmon dispersion relation

In a homogeneous and isotropic (what is the difference?) medium with charge density $\rho = 0$ and current density $\vec{J} = 0$, the dielectric permittivity ε and the magnetic permeability μ can be assumed to be scalar functions. This means that the constitutive relationships between a material's magnetization and an associated magnetic field, and the material's polarization and an associated electric field can be written $\vec{B} = \mu\mu_0\vec{H}$ and $\vec{D} = \varepsilon\varepsilon_0\vec{E}$. (See any text on electromagnetic theory for a full discussion.)

Under these constraints, Maxwell's equations take on a nice symmetric form involving the electric and magnetic fields, $\vec{E}$ and $\vec{B}$:

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{1}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\vec{\nabla} \times \vec{B} = \frac{\mu\varepsilon}{c^2}\frac{\partial \vec{E}}{\partial t} \tag{4}$$

where we have used the fact that $\mu_0\varepsilon_0 = 1/c^2$. These equations can be combined to yield wave equations in both $\vec{B}$ and $\vec{E}$, the solutions to which are plane waves most easily expressed in complex notation. For example, for the electric field

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{r} - \omega t\right)} \tag{5}$$

is a solution to the wave equation for $\vec{E}$ which describes an electric field polarized in the $\hat{E}$ direction that propagates in the $\hat{k}$ direction with angular frequency ω. The wave equation imposes a constraint between the magnitude of $\vec{k}$ and ω, namely,

$$k^2 = \frac{\omega^2}{c^2}\mu\varepsilon \,. \tag{6}$$

This relationship between ω and k is known as a "dispersion relation". In nonmagnetic materials, the relative permeability $\mu = 1$. In a dielectric material, such as glass, the relative permittivity ε usually varies with the frequency ω; as a consequence, the light which enters the material is refracted by different angles depending on the color—the light is dispersed.

Now consider a more complicated situation than that in simple homogeneous and isotropic material, namely, a plane boundary between two such regions, as is shown in Fig. 1. The dielectric constant of medium 1, ε_1 is assumed real and positive. For the time being assume ε_2 to be real, but make no assumption about its sign.
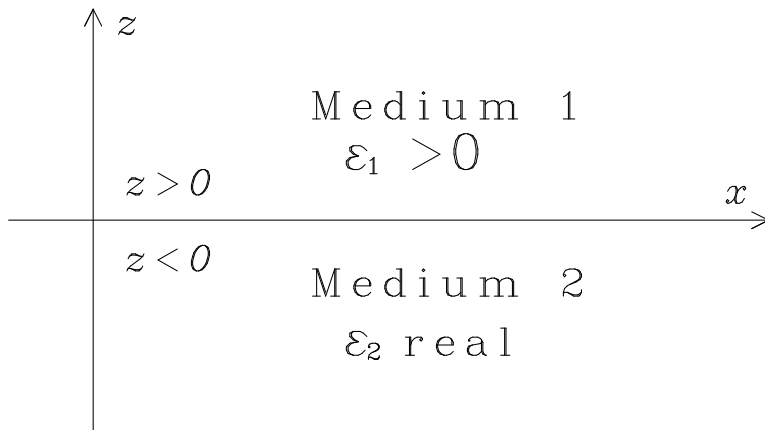

Figure 1: Interface between dielectric media.

There are a set of solutions called "surface waves" which exist at the boundary between medium 1 and 2. These waves propagate along the surface $z = 0$. We can choose any x and y orientation

for our coordinate system, so for a set of waves chosen that has no y component, we have for non-magnetic materials the following constraint from the dispersion relation:

$$k_z^2 = \varepsilon \frac{\omega^2}{c^2} - k_x^2 . \tag{7}$$

Notice that k_z^2 must be real, but possibly negative, and this means that k_z itself is either real (implying wave propagation in the z direction) or is imaginary (implying an exponential decay of the field in the z direction). One set of such waves for which k_z is imaginary may be written in complex form as follows:

For medium 1, $z \geq 0$,

$$\left.\begin{array}{rcl} E_x & = & E_0 \\ E_y & = & 0 \\ E_z & = & E_0 \dfrac{ik_x}{\sqrt{k_x^2 - \varepsilon_1 \omega^2/c^2}} \end{array}\right\} e^{-z\sqrt{k_x^2 - \varepsilon_1 \omega^2/c^2}} e^{i(k_x x - \omega t)} . \tag{8}$$

For medium 2, $z \leq 0$,

$$\left.\begin{array}{rcl} E_x & = & E_0 \\ E_y & = & 0 \\ E_z & = & E_0 \dfrac{-ik_x}{\sqrt{k_x^2 - \varepsilon_2 \omega^2/c^2}} \end{array}\right\} e^{+z\sqrt{k_x^2 - \varepsilon_2 \omega^2/c^2}} e^{i(k_x x - \omega t)} . \tag{9}$$

In the above, we have used the fact that we can write $k_z = \pm i\sqrt{k_x^2 - \varepsilon \omega^2/c^2}$.

Note the differences in sign between the two solutions. Provided that certain constraints are met, these will satisfy the boundary conditions imposed by Maxwell's equations. For these solutions to be physically sensible, their amplitude must decay exponentially away from the boundary , which will be the case if $\sqrt{k_x^2 - \varepsilon_1 \omega^2/c^2} > 0$ and $\sqrt{k_x^2 - \varepsilon_2 \omega^2/c^2} > 0$. The boundary conditions require continuity of the tangential components of $\vec{E}$ and $\vec{B}/\mu$ and the continuity of the perpendicular components of $\varepsilon\vec{E}$ and $\vec{B}$ at $z = 0$. Under the assumption that we are looking at non-magnetic materials, where $\mu = 1$, for the tangential components of the fields, this is satisfied by choice of constants ($E_x = E_0$ and $E_y = 0$), and for the perpendicular components of the fields we have the constraint that

$$\varepsilon_1 \sqrt{k_x^2 - \varepsilon_2 \omega^2/c^2} = -\varepsilon_2 \sqrt{k_x^2 - \varepsilon_1 \omega^2/c^2} . \tag{10}$$

This can only be true if $\varepsilon_2 < 0$. Solving for ω we obtain the dispersion relation for surface waves:

$$\omega^2 = (ck_x)^2 \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right) . \tag{11}$$

This result tells us that we must have $-\varepsilon_2 > \varepsilon_1$ for k_x to not be purely imaginary. So for these surface waves to exist we must have $\varepsilon_2 < 0$ and $|\varepsilon_2| > |\varepsilon_1|$.

Exercise 1 *Show, by using Maxwell equation (3) that the related function for the magnetic field $\vec{B}(\vec{r}, t)$ to the solutions in Eqs. (8) and (9) is a plane wave that only has a magnetic field component lying in the y-direction. This corresponds to a "transverse magnetic" or TM mode of oscillation.*

Exercise 2 *A bulk "plasma oscillation" can be modeled by considering a slab of "jellium," in which there is a uniform, fixed positive charge of density $+\rho$ and a uniform, mobile negative charge of density $-\rho$ that extend a thickness d in the z-direction and infinitely in the x- and y-directions.*

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(a) Displace the negative charge a distance $\delta \ll d$ in the z-direction and show that it experiences a restoring force per unit area that is proportional to δ and hence will undergo simple harmonic motion. Show that the angular frequency of this motion is given by $\omega_p^2 = ne^2/m\varepsilon_0$ where n is the number of electrons per unit volume and e and m are the charge and mass, respectively, of an individual electron.

(b) Consider the slab in the presence of an external electric field in the z-direction of the form $E_z = E_0 e^{i\omega t}$ and show that this leads to the form of the dielectric constant given in equation (12) with the same resonant frequency found in (a).

We have assumed that ε_2 is real. For many metals the dielectric constant has a small imaginary part, resulting in attenuation in the direction of propagation. For a treatment of this situation see Hecht, *Optics, 3rd ed.*, pages 127–131.

For now assume that medium 1 is air ($\varepsilon_1 \approx 1$) and that medium 2 is a metal. The free electron gas model for metals gives

$$\varepsilon(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \tag{12}$$

where $\omega_p^2 = ne^2/m\varepsilon_0$ (n is the electron number density, e is the electron charge and m is the electron mass) and is called the "plasma frequency" (see Kittel, *Introduction to Solid State Physics, 6th ed.*, pages. 256-7).

The dispersion relation for surface waves between a metal and air now looks like

$$\omega^2 = (ck_x)^2 \left(1 + \frac{1}{1 - (\omega_p/\omega)^2}\right) , \tag{13}$$

which after some rearranging becomes

$$\omega^2 = \frac{\omega_p^2}{2} + (ck_x)^2 - \left[\frac{\omega_p^4}{4} + (ck_x)^4\right]^{1/2} . \tag{14}$$

For small k_x ($ck_x \ll \omega_p$),

$$\omega^2 \approx (ck_x)^2 \left[1 - \frac{1}{2}\left(\frac{ck_x}{\omega_p}\right)^2\right] . \tag{15}$$

And for large k_x ($ck_x \gg \omega_p$),

$$\omega^2 \approx \frac{\omega_p^2}{2}\left[1 - \frac{1}{4}\left(\frac{\omega_p}{ck_x}\right)^2\right] . \tag{16}$$

So you can see that first, as $k_x \to 0$, $\omega \to 0$; second, ω is limited by $\omega_p/\sqrt{2}$ as $k_x \to \infty$; third, for all values of $k_x > 0$ the dispersion curve for surface plasmons propagating along the metal-air boundary lies to the right of the dispersion curve for electromagnetic waves in air, $\omega = ck_x$. These features are shown in Fig. 2. Because the dispersion curve for propagation in a vacuum does not intersect the dispersion curve for surface plasmons, it is not possible to match the frequency (ω) and wave vector ($\vec{k}$) of the surface plasmons to the frequency and wave vector of incident electromagnetic radiation (light, for our purposes) in air. (This is why these surface plasmons are called non-radiative–they cannot propagate into empty space.) But if, for a given ω, the magnitude of $\vec{k}$ of the light incident on the metal-air boundary can be increased, the dispersion curves will intersect. Physically this can be accomplished by passing the incident light through a medium such as glass, on which is
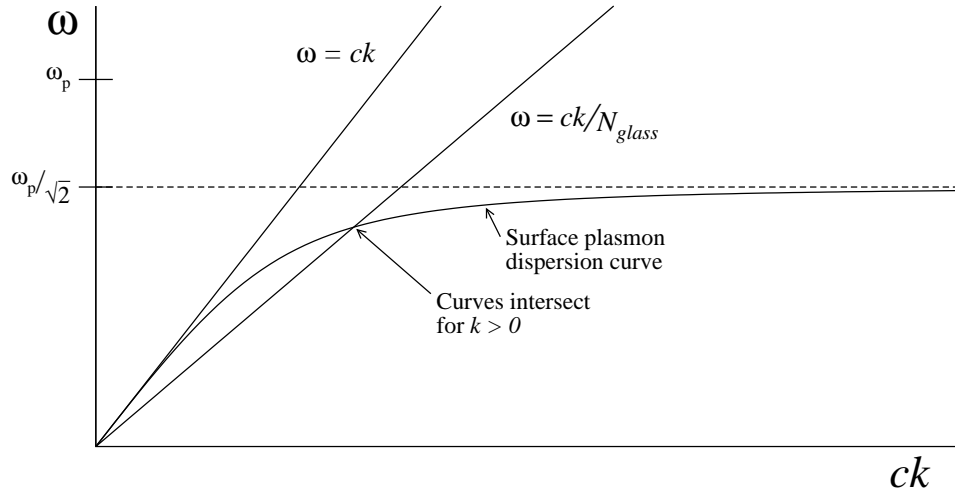
Figure 2: Dispersion curves for waves traveling in the x direction: surface plasmons, air ($\omega = ck_x$) and glass ($\omega = ck_x/N_{glass}$, assuming N_{glass} is independent of wavelength). Note that the curves do not intersect for air and plasmons, indicating that plasmons cannot be excited from light propagating from air to the metal surface.

deposited a metal film. The glass has the effect of multiplying the wave-vector by N_{glass}, the index of refraction of glass; see Fig. 2. Then, the (sufficiently) thin metal film allows some of the incident light to be transmitted through to the metal-air boundary, where the surface plasmons are excited.

Even though Fig. 2 qualitatively depicts the requirement for plasmon excitation, it is actually the wave-vector component of the incident light parallel to the boundary that must be matched to the surface plasmon wave-vector, i.e.,

$$k_{x,glass} = k_{glass} \sin \theta_i = \frac{\omega}{c} N_{glass} \sin \theta_i = k_{x,plasmon} \ , \tag{17}$$

where θ_i is the incident angle, measured relative to the normal of the glass/metal-film interface. So, we can excite a surface plasmon at a given frequency (i.e., color) of incident light by first passing the light through glass and then adjusting the angle of incidence until a match of the required wave-vectors is achieved. At this point we should see a dip in the reflected light, indicating that energy is being taken out of the beam and put into the plasmon excitation. (Question: can the angle θ_i be less than the critical angle for total internal reflection, θ_c?) It is interesting to note that when the plasmon excitation occur, the amplitude of the electric field in the metal film propagating from the glass toward the air is zero! (See Simon et. al., page 632).

Some interesting points about the solutions to the wave equations are illustrated in the following exercises:

Exercise 3 *Note, from Eq. (11) we may write*

$$\frac{\omega}{k_x} = c \left(\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \right)^{-1/2} \ .$$

If medium 2 is silver, then the real part of $\varepsilon_2 = -16.3$ at $\lambda = 633$ nm. Compare the phase velocity ω/k_x of the surface plasmons at the silver-air boundary with the phase velocity of light in air. How does this result lead to the necessity that the plasmon at the metal/air interface be excited by light incident from the glass side of the metal? Can you excite a plasmon at the glass/metal interface?

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Exercise 4 *Compare how rapidly the field amplitudes fall off for $z > 0$ and for $z < 0$.*

Exercise 5 *The time-averaged Poynting vector for plane waves written in their complex form $\vec{S} = \mathrm{Re}\left[\frac{1}{2\mu_0}\vec{E} \times \vec{B}^*\right]$ represents the energy flux. Show that the x component of this vector points in opposite directions above and below the metal-air boundary, and thus there is no net energy transport along the interface.*

3 Procedure

First, some comments on the apparatus:

- The prism is kept under vacuum when not in use to help prevent degradation of the metal due to atmospheric exposure. Silver films are particularly vulnerable in this regard—they tarnish! When you are done with the experiment, ask the for assistance with replacing the glass dome and pumping it out.

- The following information is not necessary for data analysis purposes (it is already embedded in the programs), but you may want to have it for the record: The red laser wavelength is 632.8 nm in air (atmospheric pressure). The prism is a 45 x 45 x 90 degree prism of BK-7 glass. For BK-7, the index of refraction is 1.515 at 632.8 nm, and 1.519 at 543.5 nm. The index of refraction of air (atmospheric pressure) is 1.0003.

There are two LabVIEW programs to be used in collecting and analyzing data in the surface plasmon resonance (SPR) experiment. The data acquisition program is SPR Data Acquisition, and the analysis program is SPR Modeling Analysis. Shortcuts to these programs are located on the computer desktop.

3.1 Data Collection

At minimum, you should take data from at least **two different prisms** with **both the red and green lasers**. It is probably easier to take all of the red-laser data first, and then take all of the green-laser data. There is also a plain (no film) prism that you can use to see the *absence* of a plasmon resonance, and a gold film prism that shows a significantly different response than the silver films of the standard samples.

Before starting to acquire data, the configuration and alignment of the experiment needs to be checked. First turn on the laser. Check the alignment to ensure that, (1) the laser is resting on the alignment screws in the tray that holds it—these screws should press against the black aluminum rings attached near the front and back of the laser's tube; (2) the beam hits the beam-splitters close to their centers and passes through the center of the iris in front of the rotation stage; (3) the beam coming perpendicularly off of the "polarizing beam-splitter" is of minimum intensity by gently rotating the laser tube about its axis. (The laser light is mostly polarized along one axis, often marked by a label on the front of the tube. You want to maximize the intensity of the beam that hits the prism that will be mounted on the rotation stage by minimizing the part of the beam that doesn't.)

Turn on the Keithley multimeters. Set them to the DC current mode (DCI button), if they are not already reading current.

Choose a prism from the available set. Carefully rest the prism mount on the rotating stage and turn it so that the laser light hits surface A (see Fig. 3) normally and then tighten the setscrew to secure the base to the stage. Grip the base of the glass dome covering the prism firmly and open the valve at the top. You will hear a small whoosh as air enters. Then carefully tip it off the base and remove it and place it in the foam-lined box, making sure that the O-ring between the dome and the prism mount stays on the mount.

Adjust the micrometer to a reading of 8.50 mm (edge of barrel just at the line between the 8th and 9th mark on top scale). One full revolution of the barrel results in a displacement of 0.50 mm; each division on the barrel corresponds to a displacement of 0.01 mm. Loosen the prism mount clamping screw (marked with blue tape), and rotate the prism mount by the lever marked with white tape so that the laser light is reflected from surface A of the prism directly back to the laser (normal incidence). The geometry of the prism is shown in Fig. 3, and the overall setup is shown in Fig. 4. Tighten the clamping screw. The prism mount may shift slightly as the screw is tightened—a slight adjustment of the micrometer may be necessary to restore normal incidence. Note the micrometer reading at normal incidence; you will need this number in the data acquisition program.
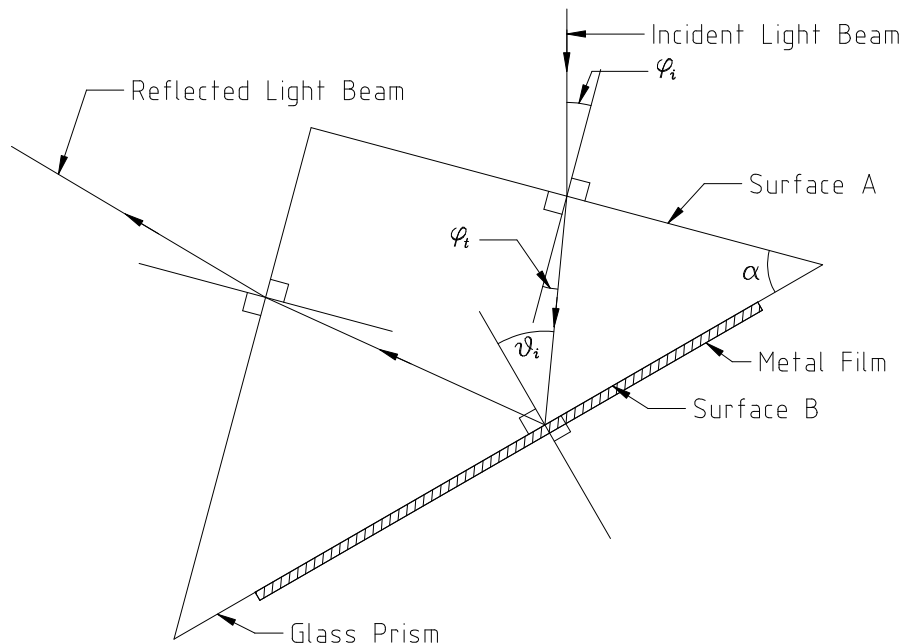

Figure 3: Geometry of SPR prism. Light beam entering at surface A at angle φ_i is refracted so that it hits the metal film at surface B at angle θ_i.

Start the data acquisition program by clicking on the icon. Check each of the initialization steps:

- Enter the micrometer reading at normal incidence. It should be close to 8.50 mm.

- Enter a selection of laser color: green or red.

- Enter a choice of folder to put your data files (or leave the default).

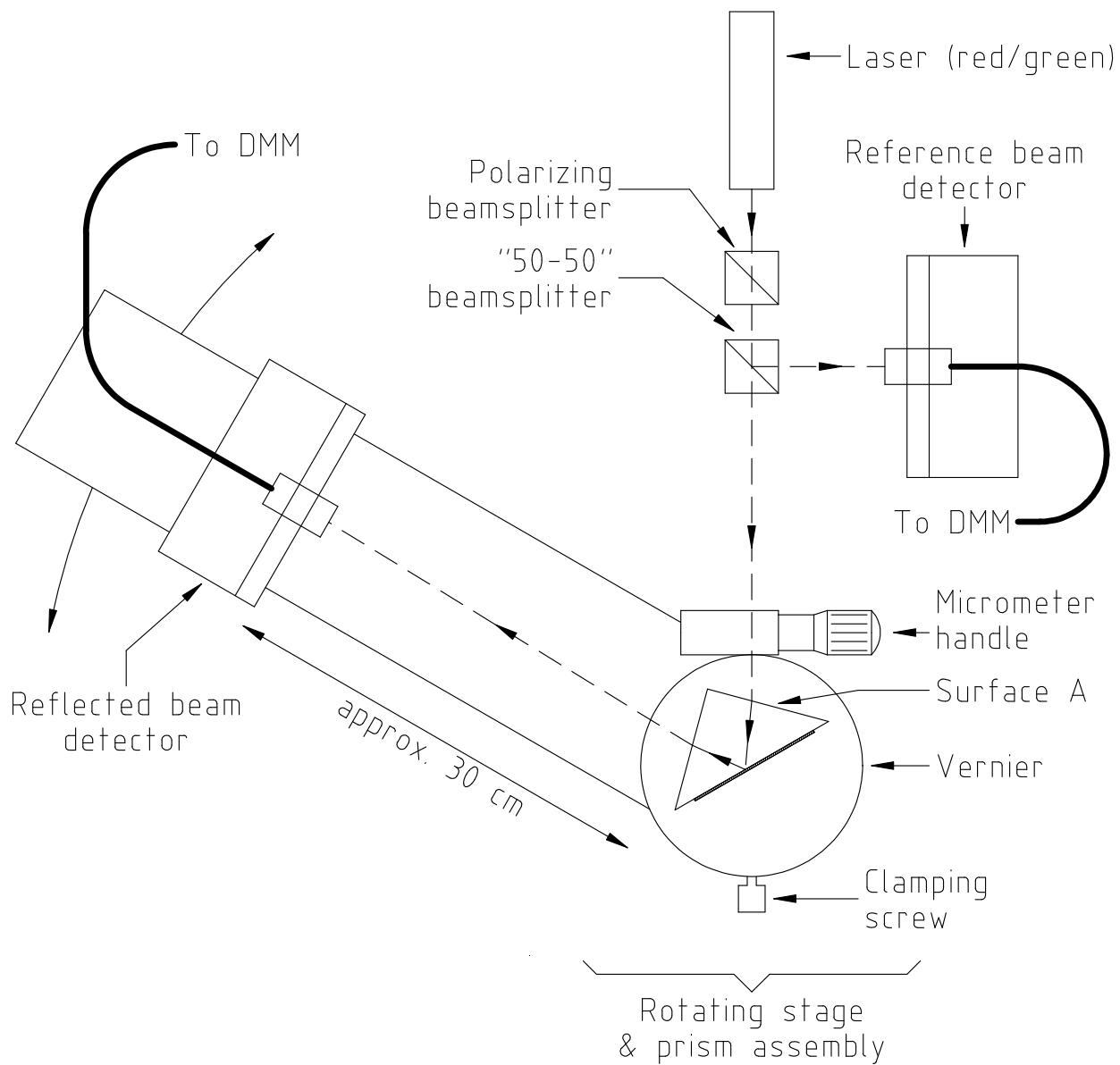- Check that the correct meters are being read for signal and reference.

Figure 4: Setup for SPR experiment. Laser light is polarized by first beam-splitter. A portion of the beam is broken off by the "50-50" beam-splitter and used as a reference intensity. The micrometer allows a fine adjustment of the prism position, while the reflected beam detector arm is set to maximize the reading on the DMM.

To enter data, click over the box you want, type the information in, and then click outside the box. Click "OK" to proceed to the data collection screen.

After the initialization screen disappears, adjust the micrometer to a reading of 14.00 mm. This will move the micrometer to just past the end of the scale. Notice that at this point, you can see two beams coming off the prism: one beam is *reflected* from the inside surface, and the other beam is *refracted* as it exits the prism.

Watch the *refracted beam* carefully as you decrease the micrometer setting. It will disappear once the refraction angle goes beyond 90 degrees (from the normal to the metal-film surface). Just past the point where the refracted beam disappears are the angles of *total internal reflection.* It is at this point that the intensity of the reflected beam is greatest. As you keep turning the micrometer, the intensity will drop as you begin to excite the surface plasmon.

You want to set the normalization just inside the critical angle for total internal reflection. Set the micrometer so that the prism is at that point. Then rotate the *signal* (reflected-beam) detector arm to maximize the reflected light signal (it should be some tenths of a milliamp, DC). The detector can be translated vertically to further maximize the signal by rotating the large fluted ring beneath the detector mount. Note how the readings on the multimeters change, and also how the information in the "Signal Current", "Reference Current" and "Normalized Current" on the LabVIEW program change. You will also see a green dot move up and down on the graph; by watching this dot you may easily maximize the reflected Signal Current. Note also the flickering of the control lights on the DMMs—they are constantly being read by the program.

After you have optimized the *signal* detector position, adjust the vertical position of the *reference* (fixed) detector to achieve a ratio

$$\text{Normalized Current} = \frac{\text{Signal Current}}{\text{Reference Current}} \approx 0.9\text{--}1.1$$

If normal incidence obtains for a micrometer reading near 8.50 mm, the resonance can be completely scanned over a range of micrometer settings from 3.00 mm to 14.00 mm. The data acquisition program converts micrometer readings to degrees and calculates the angle of incidence at the glass/metal-film interface; this is the number given on the x axis. Coarse changes in micrometer setting (say, 0.25 mm—one-half turn of the micrometer between points) work fine away from the resonance, but close in to the resonance smaller changes (≈ 0.10 mm) are necessary to accurately map out the resonance.

With the micrometer set near 13.00 mm you are ready for data acquisition.

The data acquisition screen displays the current data point and all recorded data points. To record a data point:

1. Set the micrometer to the desired position.

2. Enter the current micrometer reading in the "Micrometer reading" box under "Data Acquisition". *Do not change the value of the "Micrometer reading at normal incidence" in the "Initialization" box.* This value (and the laser wavelength) should stay the same for all of the data recorded.

3. Optimize the intensity reading on the *signal* detector. **Important: always adjust both angular and vertical positioning at each point to make the reading as high as possible.** The vertical angle of the reflected beam changes systematically as you rotate the

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prism; not adjusting the detector height will cause a skew in the intensity on top of the effect of the plasmon resonance.

4. Click on the "Record data point" button.

5. **Then, do not touch anything for three seconds.** During data acquisition, the software takes 16 readings from the meters and averages the result. If you bump the apparatus, or move too quickly to take the next reading, you will disturb this process.

If you want to remove the last recorded data point, click on the "Delete last point" button. You may also clear out the whole set with "Delete all data".

Note: if any point appears to deviate significantly from a smooth curve, it may be wise to delete that point and try taking it again.

When you have taken enough data, Click "Save Data" to open a dialog box that allows you to write the data to a file. You will find it easier to keep track of your data sets if you put them in an appropriately named subdirectory.

When you are done taking data from a particular prism, you must reseal it in the vacuum cover. Here's how (please read all the way through before attempting the procedure):

1. Check that the valve on the glass cover is open.

2. Check that the valve on the pump inlet (under the table) is *closed*: fully clockwise. Then turn on the pump.

3. Place the cover carefully over the base. There should be no bits of dirt between the cover an the O-ring, to insure a good seal.

4. Loosen the collar a little on the open end of vacuum connector attached to the rubber hose. Then, while holding the glass cover firmly on the base, slide the connector on to the glass inlet tube. ***IMPORTANT: don't let go of the glass vacuum cover, or else the rubber hose will tip it off the base, possibly causing it to break. The vacuum connector should slide easily onto the glass tube.***

5. Tighten the connector collar so that the O-ring grips the tube snugly. *Don't over-tighten it.*

6. While holding the cover firmly to the base, open the valve on the pump. You should hear the air exit the cover, and then the sound will return to what it was before you opened the valve. **If this does not happen, there is a leak.**

7. Let the pump run for thirty (30) seconds to insure a good vacuum. You can relax your hold on the base at this point, since it should stay attached because of the outside air pressure.

8. Close the valve on the vacuum cover. (Just enough so that the O-ring seals: **Don't over-tighten this valve.**)

9. Finally, close the valve on the pump, shut the pump off, and loosen the collar on the vacuum connector. You should be able to slide the connector off of the tube with a light tug.

Please ask for help if you do not feel confident about properly carrying out this procedure to reseal the prism vacuum covers.

3.2 Data Analysis

After you have acquired data and saved it to a file, exit the SPR Data Acquisition program and open the SPR Modeling Analysis program. Three buttons at the lower right allow you to load a data file, print the results of your fit, or quit the program. Click the button to open a data file, and in the dialog, select your data file and open it. The program will plot your data (red points), and also a model for the data (blue line) using default values for the parameters in the model. These parameters are: index of refraction of the prism, laser wavelength, film thickness, real and imaginary parts of the dielectric constant, and normalization factor.

Enter the appropriate values of the index of refraction of the prism and the wavelength of the light. These will stay constant, but the others will need to be adjusted during the fit. For the red laser, $\lambda = 632.8$ nm and $N_{prism} = 1.515$; for the green laser, $\lambda = 543.5$ nm and $N_{prism} = 1.519$.

Beneath the graph of your data and the model are slider controls that allow you to vary the values of the unknown parameters. Parameter values can be coarsely varied by moving the sliders up and down, and more finely varied by clicking the arrows to the left of the number boxes. As the values of the parameters are varied, the model output is recalculated and will update on the screen. Vary the parameters (except N_{prism} and λ, as they are known) until you achieve a good match between the model and the data.

As you get close to a decent match, pay attention to the graph of fitting residuals (the point-by-point difference between the model and the data) and the overall reduced chi-square, which is defined as

$$\chi_\nu^2 = \frac{1}{N - P} \sum_{i=1}^{N} \frac{[M(\theta_i) - I(\theta_i)]^2}{\sigma_i^2} \ , \tag{18}$$

where N is the number of data points, P is the number of fitting parameters (here taken to be 3, since the others are known and fixed), $M(\theta)$ is the theoretical model and $I(\theta)$ is the measured normalized current. The uncertainty-per-point σ is taken to be the same for all points; the default is 0.01, or 1%. A good fit would give a χ_ν^2 equal to around 1. If this number is much greater than 1, then the model fit is often taken to be poor; if the number were much less than 1, then the uncertainty per point is probably overestimated. If you get a reduced chi-square in the neighborhood of 1 and your residuals graph looks fairly flat and somewhat randomly distributed about zero, you have a very good fit—congratulations!

Print your results using the "Print results" button. You can add notes in the text box at the top. Notice that the text box already contains the values of the various model parameters. The printout should appear on the default printer in room B248.

To understand your data, compare your results for the complex dielectric constant with the data given by Christy and Johnson (PRB, 1972; see references). Also compare the results you get among the various films you measured. Think about (and comment on in your report) how the measurements may be used to estimate film thickness. Also, discuss any systematic errors and their sources that you find.

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