TABLE 14.1 SOME TRANSITIONS SUITABLE FOR ZEEMAN EFFECT OBSERVATIONS

Atom	*Color*	*Wavelength* (Å)	*Initial Configuration*	*Final Configuration*
Hg				
	Yellow	5791	6s 6d $^1D_2$	6s 6p $^1P_1$
	Green	5461	6s 7s $^3S_1$	6s 6p $^3P_2$
	Yellow	5770	6s 6d $^3D_2$	6s 6p $^1P_1$

The Fabry–Perot Etalon

Consider a ray of monochromatic light, of vacuum wavelength λ_0, incident on an etalon consisting of a pair of parallel glass plates separated by a gap of width d filled with a medium with index of refraction n_f, as illustrated in Figure 14.10. The inner surfaces of the plates are partially mirrored so that, if the angle of incidence θ is small, the ray is multiply reflected. The optical path for each of the rays emerging at the right is different, so that beams will exhibit interference. The phase difference between adjacent emerging rays can be calculated by considering $\delta\ell$, the difference in path lengths for the rays emerging at B and D. Relative to the ray emerging at B, the ray reflected at B has an additional path length, measured from B to the wave front at D', of $\delta\ell = BC + CD' = 2d\cos\theta$. If we neglect the relatively small additional phase changes due to the metallic film, the difference in phase, $\delta\phi$, for these two rays is

$$\delta\phi = 2\pi\frac{\delta\ell}{\lambda} = 4\pi n_f \cos\theta \frac{d}{\lambda_0} \qquad \text{(rad)} \tag{25}$$

where $\lambda = \lambda_0/n_f$ is the wavelength in the medium between the plates. The intensity I_t of the radiation transmitted through the Fabry–Perot will be a maximum if the beams interfere

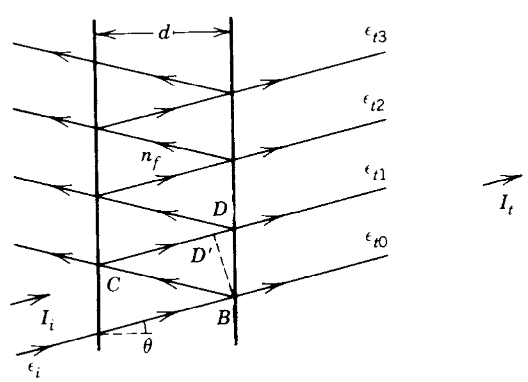

FIGURE 14.10 Multiple reflections of a beam between the plates of a Fabry–Perot etalon.

constructively, that is, if, for some integer m,

$$\delta\phi = 2\pi m \qquad \text{(rad)} \tag{26}$$

EXERCISE 11

Verify trigonometrically the relationship $\delta\ell = 2d\cos\theta$, which leads to the expression for $\delta\phi$ of equation 25.

The amplitude of the transmitted electric field ϵ_t^0 can be found in terms of the incident amplitude ϵ_i^0 by superposing, with appropriate phase factors, the time-dependent field strengths of the transmitted beams. As in Figure 14.10, the time-varying strength of the electric field ϵ_{tk} of the kth transmitted beam can be written as $\epsilon_{tk}^0 \exp(i\omega t)$, where $k = 0$ corresponds to the first transmitted beam, which has not been reflected. The field transmission and reflection coefficients for the inner surfaces of the plates, t and r, can then be used to write $\epsilon_{tk}^0 = t^2 r^{2k} \epsilon_i^0$. The phase of the kth beam lags behind that of the $k = 0$ beam by an amount $k\,\delta\phi$. The field strength ϵ_t is then expressed as

$$\begin{aligned}\epsilon_t &\simeq \sum_{k=0}^{\infty} \epsilon_{tk}^0 \exp[i(\omega t - k\,\delta\phi)] \\ &= \left[\sum_{k=0}^{\infty} (t^2 r^{2k}) \exp(-ik\,\delta\phi)\right] \epsilon_i^0 \exp(i\omega t) \qquad \text{(N/C)}\end{aligned} \tag{27}$$

where the number of reflections has been assumed so large that ϵ_t may be approximated by extending the series to an infinite number of terms. Evaluating this geometrical series yields

$$\epsilon_t = \left[\frac{T}{1 - R\exp(-i\,\delta\phi)}\right] \epsilon_i^0 \exp(i\omega t) \qquad \text{(N/C)} \tag{28}$$

where $T = t^2$ is the transmittance and $R = r^2$ the reflectance of the metal film. Since the intensity of the emerging beam is proportional to $\epsilon_t \epsilon_t^*$, equation 28 gives the ratio of transmitted and incident intensities as

$$\frac{I_t}{I_i} = \left(\frac{T}{1-R}\right)^2 \frac{1}{1 + F\sin^2(\delta\phi/2)} \tag{29}$$

where the abbreviation $F \equiv 4R/(1-R)^2$ and the identity $\cos\delta\phi = 1 - 2\sin^2(\delta\phi/2)$ have been used. This ratio is plotted in Figure 14.11*a* as a function of the phase difference $\delta\phi$ for

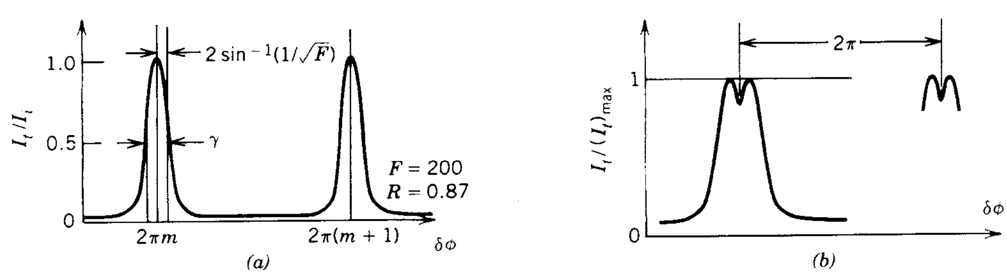

FIGURE 14.11 (*a*) The ratio of transmitted and incident intensities for a Fabry–Perot etalon with $R = .87$. (*b*) Intensity pattern from a Fabry–Perot for two barely resolved peaks.

a reflectivity of $R = 0.87$, assuming that the reflecting film absorbs no energy (i.e., that $T + R = 1$). Note, as mentioned above, that the peaks in the transmission occur for $\delta\phi = 2\pi m$.

Two ways in which the etalon can be used to measure wavelength are illustrated in Figure 14.12. If an extended source is configured with the etalon and two converging lenses, as in Figure 14.12*a*, rays enter the first plate of the Fabry–Perot with a range of incident angles θ. Each value of θ corresponds to a value of $\delta\phi$ given by equation 25 and to a radial position on a screen or photographic plate to which rays with this value of θ are focused; a series of bright rings are formed with radii corresponding to values of θ for which $\delta\phi = 2\pi m$. Measurements of these radii may then be used along with equations 25 and 26 to calculate the wavelength differences between the spectral components of the light from the source. As is discussed in reference 8, the wavelength separation for two adjacent fringes (corresponding to the same order m) with diameters D and D' is given by

$$\Delta\lambda = \lambda(D^2 - D'^2)/8f^2 \qquad \text{(m)} \tag{30}$$

where f is the focal length of the lens between the screen and the etalon.

A second method for determining spectral splittings, discussed further below in connection with the Zeeman effect measurements, is called central spot scanning, illustrated by Figure 14.12*b*. In this technique, the index of refraction n_f or the etalon spacing d is varied, thus changing $\delta\phi$. Peaks in I_t are observed for the light allowed through the pinhole, for which $\theta = 0$. From the curve of I_t versus n_f (or d), one can deduce the separations in λ for the various components of the incident beam.

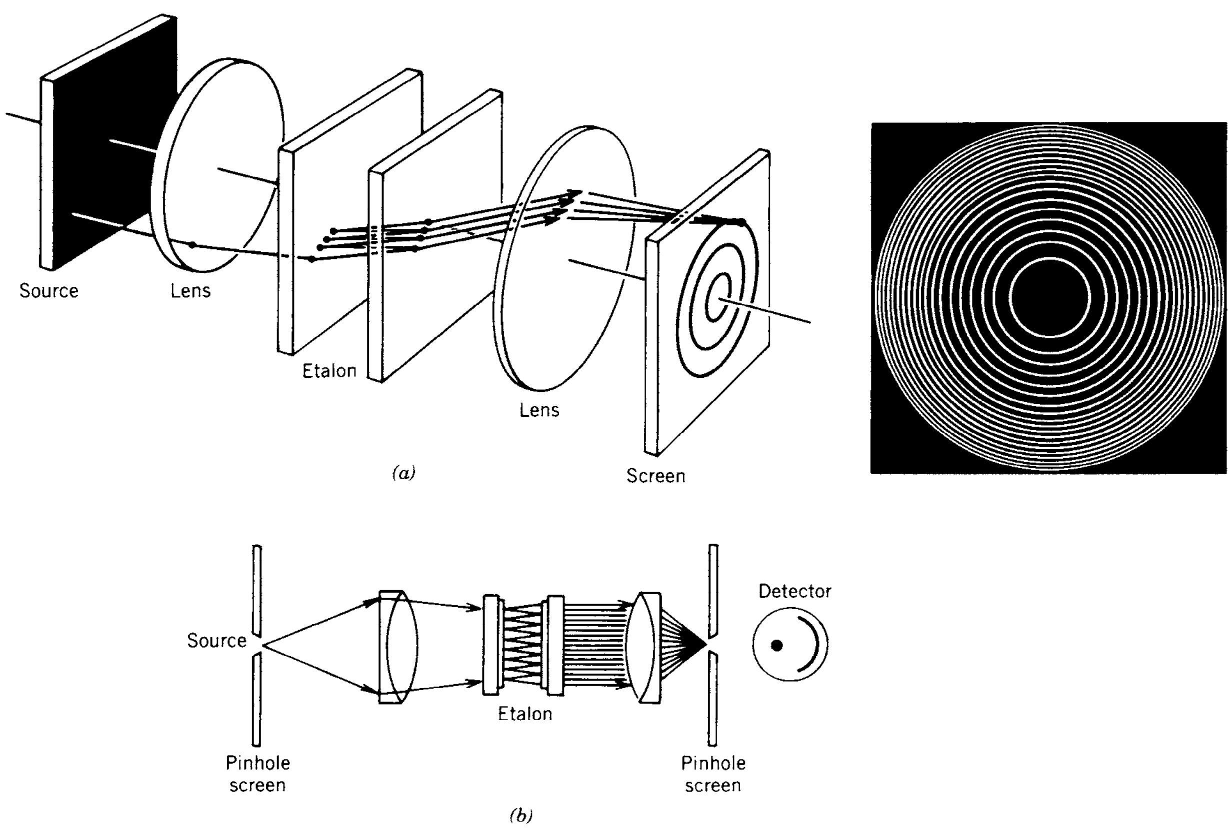

FIGURE 14.12 (*a*) Ring pattern produced by a Fabry–Perot with an extended source. (*b*) Central spot scanning with a Fabry–Perot.

Two parameters that determine the suitability of the Fabry–Perot for a particular set of measurements are the **free spectral range** $\Delta\lambda_{0f}$ and the **finesse** F_0. The free spectral range is that difference in wavelength between two spectral components of the source whose intensity maxima just overlap, that is, $\delta\phi(\lambda_0) - \delta\phi(\lambda_0 + \Delta\lambda_{0f}) = 2\pi$. Using equation 25 for $\delta\phi$ gives $\Delta\lambda_{0f}$ for $\theta = 0$:

$$\Delta\lambda_{0f} \simeq \frac{\lambda_0^2}{2n_f d} \quad \text{(m)} \tag{31}$$

EXERCISE 12

Verify that this expression for $\Delta\lambda_{0f}$ is nearly exact for visible wavelengths and an etalon having $d = 0.500$ cm and $n_f = 1$. What is $\Delta\lambda_{0f}$ in this case if $\lambda_0 \simeq 4046$ Å?

The finesse F_0 is the ratio of the separation, in $\delta\phi$, of two adjacent maxima in Figure 14.11*a* to the full width at half-maximum (FWHM) of the peaks, labeled γ. F_0 can be related to the reflectivity of the etalon surface by noting that equation 29 implies that, for large values of the parameter F (defined below, equation 29), I_t falls to one half its maximum value when $\delta\phi$ deviates by $2/F^{1/2}$ radians from $2\pi m$, its value at the mth peak. The full width γ is just double this amount, so $\gamma = 4/F^{1/2}$. Since two adjacent peaks are separated by a phase difference of 2π, $F_0 \equiv 2\pi/\gamma = \pi F^{1/2}/2$. Substituting the definition of F equation (29) gives F_0 in terms of R:

$$F_0 = \frac{\pi\sqrt{R}}{1 - R} \tag{32}$$

EXERCISE 13

Verify that I_t falls to half of its maximum value when $\delta\phi$ deviates from a peak value by $2/F^{1/2}$ radians. What approximation must be made to obtain this result?

We will say that two wavelengths λ_1 and $\lambda_2 \equiv \lambda_1 + \Delta\lambda_0$ are resolvable with the etalon if the phase differences $\delta\phi_1$ and $\delta\phi_2$ to which they correspond (for fixed values of n_f, θ, and d near a transmission peak) are different by at least the FWHM, γ, of the I_t/I_i versus $\delta\phi$ curve: $\delta\phi_1 - \delta\phi_2 \gtrsim 2\pi/F_0$ for resolution of the two wavelength components. Figure 14.11*b* illustrates the intensity pattern due to two closely spaced wavelength components whose peaks are just barely resolved. To express the above resolution criterion directly in terms of wavelength separation, an approximation for $\delta\phi_1 - \delta\phi_2$ in terms of $\Delta\lambda_0$ may be obtained by differentiating equation 25 with respect to λ_0, giving us $\delta\phi_1 - \delta\phi_2 \simeq \delta\phi(\lambda_0)(\Delta\lambda_0/\lambda_0)$, where $\lambda_0 \simeq \lambda_1 \simeq \lambda_2$. Since $\delta\phi(\lambda_0) \simeq 2\pi m$ near a peak, this means that two wavelengths may be resolved if $\Delta\lambda_0 \gtrsim \lambda_0/mF_0$. The ratio of λ_0 to the minimum resolvable wavelength difference, $(\Delta\lambda_0)_{\min}$, is the **chromatic resolution** R_0 and is given by

$$R_0 \equiv \frac{\lambda_0}{(\Delta\lambda_0)_{\min}} = mF_0 = \frac{\pi m\sqrt{R}}{1 - R} \tag{33}$$