

# Zeeman Effect in Mercury

The goal in this experiment is to measure the shift of atomic energy levels due to an external magnetic field. This effect is known as the Zeeman effect and was first observed by the Dutch physicist Peter Zeeman in 1896. Measurements of energy shifts are performed for the green line and both yellow lines in the mercury spectrum, and the qualitatively different results for the different lines are explained in terms of the properties of the initial and final states for each line. The results for the energy shifts are compared to theoretical predictions.

## 1 Background

Many texts offer a detailed discussion of the Zeeman effect [1, 2, 4]. Here, we offer here only a brief overview of the theory, and encourage you to consult these texts or one of your own choosing for a more comprehensive treatment.

Electrons in atoms occupy states with well-defined energies. When an electron transitions from a higher energy state to a lower energy state, a photon with energy equal to this difference is emitted by the atom. By analyzing the properties of the emitted photons, namely wavelength and type of polarization, one can learn about the states of the electron before and after the transition. For some atoms, application of a magnetic field will change the properties of the emitted light (such as its wavelength or polarization), which is in turn caused by changes in the initial and final states of the electrons in the atoms. Indeed, in the early development of quantum mechanics studying the effects of external perturbations (applied magnetic and electric fields, for example) on these transitions provided major clues leading to a detailed understanding of atomic structure.

Atomic transitions were first studied in the optical (humanly visible) portion of the electromagnetic spectrum. Transitions were observed using an instrument called a spectrograph which separates the incoming light by wavelength. Transitions at different wavelengths appear as distinct vertical lines in the spectrograph view-field, hence the term “line” to denote a transition at a particular wavelength. With the apparatus used in this experiment (a Fabry-Perot interferometer), the different “lines” are actually observed as different sets of concentric circles of different radii.

### 1.1 Magnetic moments

Magnetic moments are a key part of understanding the Zeeman effect, so let’s briefly review their basic properties. When an electrical current  $I$  circulates in a loop of wire with area  $A$  ( $= \pi a^2$ ), there is an associated magnetic moment  $\vec{\mu}$  of magnitude

$$\mu = IA , \tag{1}$$

and direction given by the right-hand rule, as shown in Fig. 1. If this loop is placed in an external magnetic field  $\vec{B}$ , the loop will experience a torque  $\vec{\tau}$

$$\vec{\tau} = \vec{\mu} \times \vec{B} . \tag{2}$$

Work is done by the magnetic field on the loop when the orientation of the loop changes relative to that of the magnetic field, and the potential energy associated with the orientation of the loop is

$$E_{\text{magnetic}} = -\vec{\mu} \cdot \vec{B} . \tag{3}$$

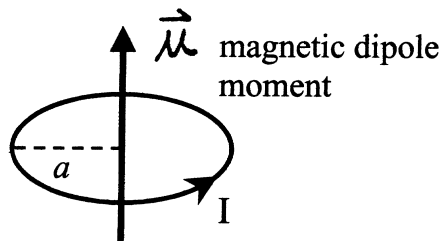


Figure 1: A current loop and its associated magnetic moment.

## 1.2 Angular momentum and magnetic moments in the atom

It is simplest to first consider an atom with only one electron outside the closed inner shells. An example of such “hydrogen-like” atoms is sodium. The optical spectrum of such an atom is due to transitions by the outer electron, and such an electron is called “optically active”. Later we will consider the case for more than one electron outside the closed shells, an example of such an atom being mercury, the subject of investigation in this experiment.

### 1.2.1 Orbital angular momentum and magnetic moment

Each state that an electron can occupy in an atom is characterized not only by a well defined energy, but also by a specific angular momentum. There are two contributions to the angular momentum, first from the orbital motion of the electron about the nucleus, and second from the intrinsic angular momentum of the electron. The first is known as *orbital* angular momentum, and the second as *spin* angular momentum. Let us consider each one separately, and then see how they combine to form the total angular momentum.

The orbital angular momentum of the electron can be thought of as due to circulation of the electron about the nucleus, and as such can be regarded as a current circulating about a loop. Just as in the macroscopic case discussed above, this circulation gives rise to a magnetic moment. The relationship between the orbital angular momentum  $\vec{l}$  and the associated magnetic moment  $\vec{\mu}_l$  is

$$\vec{\mu}_l = - \left( \frac{e}{2m_e} \right) \vec{l}, \quad (4)$$

where  $e$  is the magnitude of the electron charge, and  $m_e$  is the electron mass; the minus sign is due to the negative charge of the electron.

**Exercise (Optional)** Derive Eq. (4) from Eq. (1) under the assumption that the current  $I$  is from a single circulating electron.

This relationship is conventionally stated as

$$\vec{\mu}_l = -g_l \left[ \frac{e\hbar}{2m_e} \right] \left( \frac{\vec{l}}{\hbar} \right), = -g_l \mu_B \left( \frac{\vec{l}}{\hbar} \right), \quad (5)$$

where  $g_l = 1$  for a single optically active electron and  $e\hbar/2m_e \equiv \mu_B$  is known as the *Bohr magneton*. The dimensionless quantity  $g_l$  is known as the “ $g$  factor” and is a measure of the ratio of the magnitude of the magnetic moment in units of  $\mu_B$  to the magnitude of the orbital angular

momentum in units of  $\hbar$ ; in other words,  $g_l$  is defined by Eq. (5). Since  $g_l = 1$  for a single optically active electron, having such a factor in the equation for  $\mu_l$  may not seem to make much sense. But as we shall discover,  $g$  can take on values quite different from unity when we consider all the contributions to the angular momentum.

Now let's take a "quantum leap" over much important physical theory and simply state some results of quantum mechanics:

1. The variable  $l$  is known as the orbital angular momentum quantum number and can take on the values  $0, 1, 2, \dots$ . The magnitude of the orbital angular momentum is  $\sqrt{l(l+1)}\hbar$ .
2. The projection of the orbital angular momentum along a particular axis is quantized in units of  $\hbar$ . When the atom is placed in an external magnetic field, as when observing the Zeeman effect, the axis of quantization is parallel to the field. By convention the magnetic field is taken to point in the  $+z$  direction.
3. The quantity  $m_l$  is the quantum number specifying the projection of the orbital angular momentum along the  $z$  axis and can take on the values  $l, l-1, l-2, \dots, -l$ , for a total of  $2l+1$  values.

It follows that the possible values of the  $z$  component of the orbital angular momentum,  $l_z$ , are just  $m_l\hbar$ . Figure 2 shows the situation for  $l = 2$ , with the 5 possible values of  $m_l$  (all magnitudes are in units of  $\hbar$ ). As an aside we note that quantum mechanics allows for only one component of the angular momentum to be specified exactly—the other two components have indefinite value and average to zero over time [2, p.258]. In our scheme the  $z$  component has a definite value, so the  $x$  and  $y$  components can be anywhere on the cone denoted by the dashed line.

Since the magnetic moment and orbital angular momentum are proportional, the magnetic moment will also have quantized projections along the  $z$  axis. The question arises: what is the potential energy associated with a particular projection of the magnetic moment along the  $z$  axis? Let  $E(\vec{B})$  be the energy resulting from the interaction of the atom with the external field  $\vec{B}$ . When we turn on  $\vec{B}$  (in the  $+z$  direction) the change in energy is

$$E(\vec{B}) - E(0) = E_{\text{magnetic}} = -\vec{\mu}_l \cdot \vec{B} = -\mu_{l_z} B, \quad (6)$$

where  $\mu_{l_z}$  is the component of  $\vec{\mu}_l$  in the  $z$  direction. From Eq. (5) in terms of the  $z$  component alone, we obtain

$$\mu_{l_z} = -g_l \mu_B \left( \frac{l_z}{\hbar} \right) = -g_l \mu_B m_l, \quad (7)$$

where again the negative sign comes from the charge of the electron. Thus, the energy change from the magnetic field is

$$E_{\text{magnetic}} = -(-g_l \mu_B m_l) B = g_l m_l \mu_B B. \quad (8)$$

### 1.2.2 Spin angular momentum and magnetic moment

The treatment of the electron spin angular momentum,  $\vec{s}$ , and spin magnetic moment  $\vec{\mu}_s$  is similar to the above treatment of orbital angular momentum and its magnetic moment. The quantity  $s$  is known as the spin quantum number and can take on only the numerical value of  $1/2$  when just one electron is involved. The magnitude of the spin angular momentum is  $\sqrt{s(s+1)}\hbar$ .

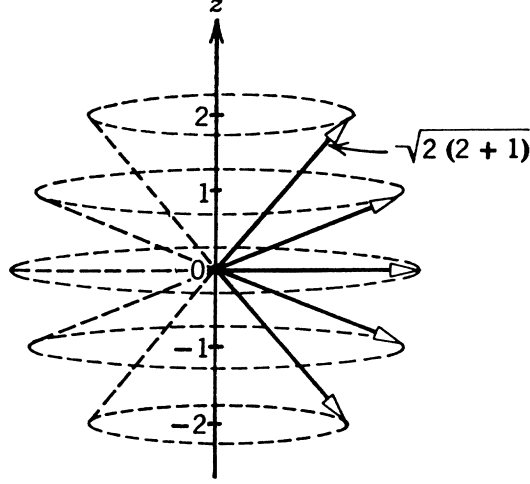


Figure 2: Angular momentum  $l = 2$  where  $|\vec{l}| = \sqrt{2(2+1)}\hbar$ . The  $z$ -projections of  $\vec{l}$  are in units of  $\hbar$ .

The projection of the spin angular momentum along a particular axis, again taken to be the  $z$  axis, is quantized in units of  $\hbar$ , with possible values  $+\hbar/2$  and  $-\hbar/2$ . Here,  $m_s$  is the quantum number specifying this projection and takes on the values  $+1/2$  and  $-1/2$ . See Fig. 3. When the atom is placed in an external magnetic field, as when observing the Zeeman effect, the axis of quantization is again parallel to the field.

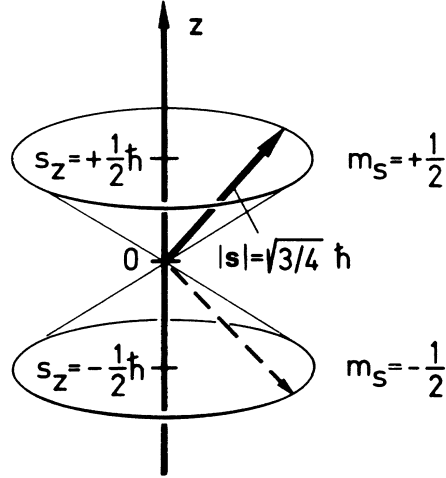


Figure 3: The spin angular momentum vector  $\vec{s}$  for a single electron.

The relation between the spin angular momentum  $\vec{s}$  and the associated magnetic moment  $\vec{\mu}_s$  is similar to the relations for the orbital quantities:

$$\vec{\mu}_s = -g_s \mu_B \left( \frac{\vec{s}}{\hbar} \right), \quad (9)$$

and likewise, the energy shift for a spin placed in an external magnetic field  $\vec{B}$  is

$$E_{\text{magnetic}} = g_s m_s \mu_B B . \quad (10)$$

Is  $g_s = 1$ , just like  $g_l$ ? No. It turns out that  $g_s \approx 2$ , or more precisely,  $g_s = 2.00232\dots$ . That it is nominally 2 instead of 1 derives from the relativistic treatment of the electron (i.e., the Dirac equation), and the fact that it deviates from 2 is due to the interaction of the electron with fluctuations of the electromagnetic vacuum, a concept deriving from quantum electrodynamics. The deviation of  $g_s$  from 2 has been a topic of great theoretical and experimental interest for many years. The difference  $g_s - 2$  has been measured by researchers in the University of Washington physics department to the remarkable precision of almost 12 decimal places, and increasingly refined theories result in calculations that match these very impressive experimental results.

### 1.2.3 Spin-orbit interaction

Now that we have expressions for the energy shifts due to the interaction of orbital and spin magnetic moments in an external magnetic field, are we done? No. This is because that for small external magnetic fields, typical of the magnitude that our electromagnet can generate, the orbital and spin magnetic moments do not each interact independently with the external magnetic field. Rather, the orbital and spin magnetic moments interact with each other in such a way as to form a combined magnetic moment that interacts with the external field. One way to think about this is that the orbital motion of the electron about the nucleus results in a magnetic field at the location of the electron. (From the point of view of the electron, the nucleus appears to be orbiting about it, resulting in a magnetic field at the location of the electron.) The spin magnetic moment of the electron interacts with this field so as to couple the spin and orbital magnetic moments together. The magnitude of the field seen by the electron is approximately  $10^4$  gauss [2, pp. 278-9], which is much greater than the fields we use in this experiment. Not until the applied external field greatly exceeds the internal fields in the atom do the orbital and spin magnetic moments decouple and interact independently with the external field.

How do the spin and orbital angular momenta and magnetic moments couple together? It turns out that the angular momenta combine vectorially to form a total angular momentum  $\vec{j}$ ,

$$\vec{j} = \vec{l} + \vec{s} . \quad (11)$$

Now,  $\vec{j}$  is the angular momentum with a definite projection along the  $z$  axis, as shown in Fig. 4. The magnitude of  $\vec{j}$  is  $\sqrt{j(j+1)}\hbar$ , where the values for  $j$ , called the total angular momentum quantum number, are determined by the set of numbers  $l+s, l+s-1, \dots, l-s$ , where we have assumed  $l \geq s$ . Since  $s = 1/2$ , the possible values of  $j$  are  $l+1/2, l-1/2$  only. Here,  $j_z$  is formed from the sum of  $l_z$  and  $s_z$ , which leads to a new quantum number  $m_j$  specifying the projection of total angular momentum along the  $z$  axis:

$$j_z = l_z + s_z = (m_l + m_s)\hbar = m_j\hbar . \quad (12)$$

For  $l = 2$  and  $s = 1/2$ , the two possible ranges for  $m_j$  and possible values for  $j$  ( $j = 3/2$  and  $j = 5/2$ ) are shown in Fig. 5. (Note that  $j = 2$  is not allowed as this would require that  $s_z = 0$ , which is not possible with only one electron.)

Now that we have combined spin and orbital angular momenta, can we derive a formula for the energy shift in an external magnetic field similar to the formulas above? That is, can we write

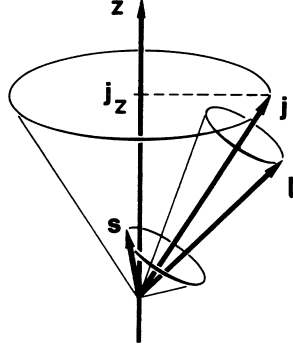


Figure 4: Angular momentum vectors  $\vec{l}$  and  $\vec{s}$  combine to form the angular momentum vector  $\vec{j}$ .

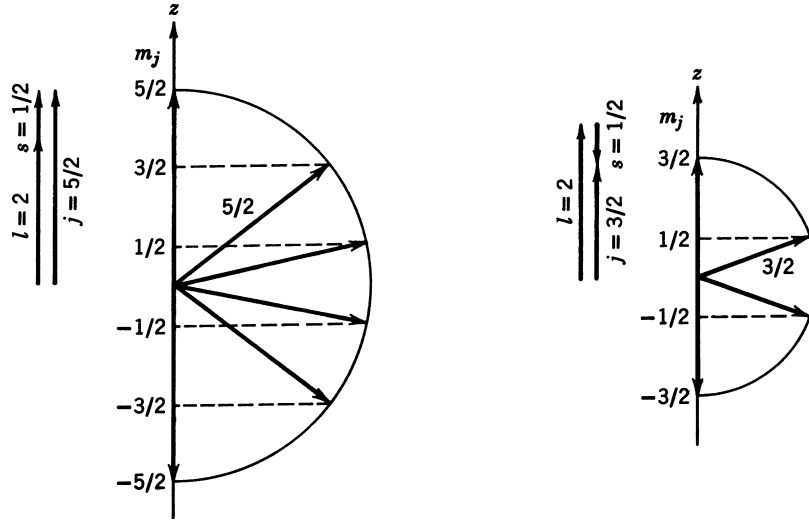


Figure 5: Example of two different possibilities for  $\vec{j}$  with  $l = 2$  and  $s = 1/2$ .

$E_{\text{magnetic}} = g_j m_j \mu_B B$ ? Yes, provided we know how to find the value of  $g_j$ . This  $g$  factor is derived from considering the projections of the spin and orbital magnetic moments along the total angular momentum  $j$ , and we state the result:

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}. \quad (13)$$

Derivations of this formula can be found in the text by Preston and Dietz [3, pp. 246–9] and in the write-up for the Optical Pumping of Rubidium experiment. The formula clearly gives the correct results for cases where either  $s$  or  $l$  vanishes,  $g_j$  then being equal to 1 or 2, respectively.

### 1.3 Multiple optically active electrons

So far we have been considering an atom with only one electron outside the closed inner shells. But mercury, the atom of interest in this experiment, has two optically active electrons. Now what? It turns out that for many species of atoms the individual orbital angular momenta combine to form a total orbital angular momentum, and the individual spin angular momenta combine to form

a total spin angular momentum. Note: upper case letters are typically used to denote combined angular momentum states. Let  $L$  be the total orbital angular momentum number and  $S$  the total spin angular momentum quantum number for the two electrons. If  $l_1$  and  $l_2$  are the values for the individual orbital quantum numbers, possible values for  $L$  are:  $l_1 + l_2, l_1 + l_2 - 1, \dots, l_1 - l_2$  (assuming  $l_1 \geq l_2$ ). For  $l_1 = 2$  and  $l_2 = 1$ , the possible values of  $L$  are 3, 2, and 1. The spins combine similarly, so for two electrons  $s_1 = s_2 = 1/2$  and the possible values of  $S$  are 0 and 1. Just as  $\vec{l}$  and  $\vec{s}$  combine to form  $\vec{j}$ ,  $\vec{L}$  and  $\vec{S}$  combine to form  $\vec{J}$ :

$$\vec{J} = \vec{L} + \vec{S}, \quad (14)$$

with the magnitude of  $\vec{J}$  being  $\sqrt{J(J+1)}\hbar$ , and the component along the  $z$  axis being specified by  $m_J$ . This scheme of coupling angular momenta for more than one electron is called  $LS$  coupling, and is also known as Russell-Saunders coupling. (The common short-hand notation for two-electron states is used in Figs. 6 which shows a particular set of transitions for the cadmium atom, and in Fig. 7 which shows a more complete picture of the energy levels of the mercury atom along with initial and final states for a number of transitions. The ground state of mercury has two  $6s$  electrons, and is designated  $^1S_0$ . A more detailed explanation of this notation system will be presented later.)

So for the multi-electron atom the expression for the energy shift in an external magnetic field is given by the now-familiar formula:

$$E_{\text{magnetic}} = g_J m_J \mu_B B. \quad (15)$$

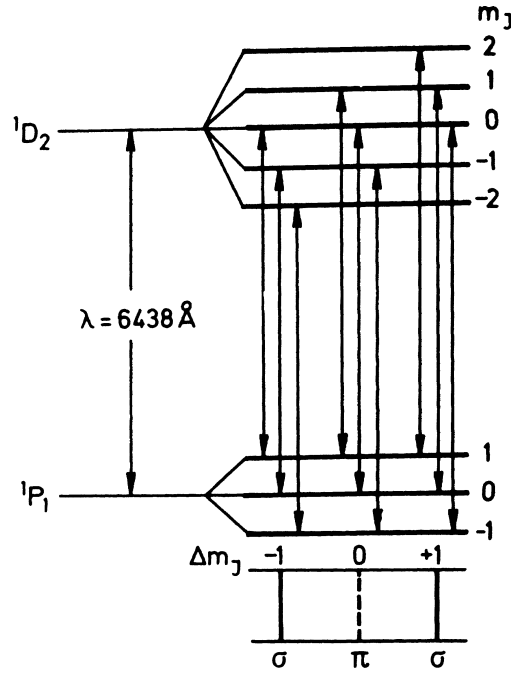


Figure 6: Energy-level diagram for the 643.8 nm spectral line of the neutral cadmium atom which shows the splitting of the  $^1D_2$  and  $^1P_1$  states caused by the application of a magnetic field.

Figure 6 is a typical diagram showing the effect of an external magnetic field on the initial and final states for a particular transition of the neutral cadmium atom which has two optically active

electrons. The multiple levels at the right are called sublevels. The arrows depict just the transitions between these sublevels that are allowed by what are called “selection rules”. The nine arrows represent nine different transitions, but note that there are only three distinct energy differences between the various initial and final states. One level is unshifted from its value for  $B = 0$ , and the other two are shifted an equal amount above and below the unshifted level. (This follows from the fact that  $g_J = 1$  for both the  $^1P_1$  and  $^1D_2$  levels.) With a total of three lines in the presence of an external magnetic field, this is an example of the “normal” Zeeman effect.

The Zeeman effect was observed long before the advent of quantum mechanics. The interpretation of this effect based on classical electrodynamics allowed for only three lines, one above and one below the unshifted line, and spectra which showed only three lines were called “normal”. See reference [1, pp. 13–16] for a derivation of the classical explanation of the Zeeman effect. The more common case, as with the mercury green line, is that there are more than three lines, and when this is the case, it is called, perhaps appropriately, the “anomalous” Zeeman effect.

## 1.4 Selection rules and polarization

A natural question arises: what transitions are possible? Can an electron (or electrons) go from any arbitrary state to any other arbitrary state? The experimental evidence says no and the theory based on this evidence requires that quantities like angular momentum be conserved in transitions from one state to another. The following rules apply to an atom with two optically active electrons such as mercury (the rules are slightly different for atoms with only one optically active electron) [3, p. 252]:

$$\begin{aligned}\Delta S &= 0 \\ \Delta L &= 0, \pm 1 \\ \Delta J &= 0, \pm 1 \quad (\text{but not } J = 0 \rightarrow J = 0) \\ \Delta m_J &= 0, \pm 1 \quad (\text{but not } m_J = 0 \rightarrow m_J = 0 \text{ if } \Delta J = 0)\end{aligned}$$

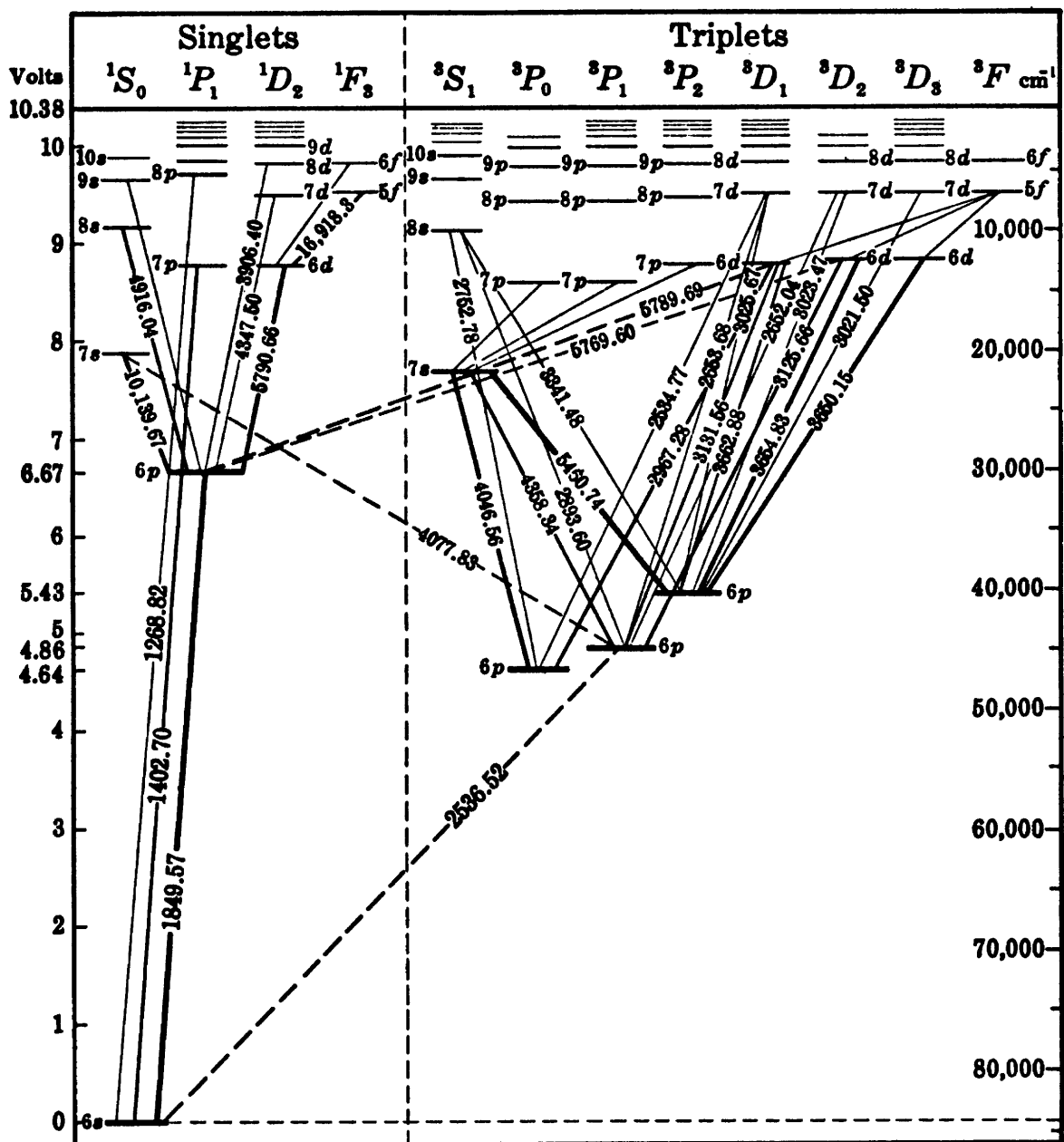
These rules apply to what are known as electric dipole transitions, which are the kind of atomic transitions that produce the most intense lines. If a transition violates these rules, occurs via some other mechanism (magnetic dipole, for example), which result in considerably weaker lines than for electric dipole transitions [2, pp. 288–295], or the associated states of the electron do not follow the  $LS$  coupling scheme assumed so far.

The polarization of the emitted photons is determined by  $\Delta m_J$ . For  $\Delta m_J = 0$ , the photons are linearly polarized parallel to the direction of  $\vec{B}$ , and the associated lines are termed  $\pi$  lines. For  $\Delta m_J = \pm 1$ , the photons are circularly polarized along the direction of  $\vec{B}$  (i.e., the electric field vector rotates clockwise or counterclockwise around the direction of  $\vec{B}$ ), and the associated lines are termed  $\sigma$  lines. When viewed at right angles to  $\vec{B}$ , as is done with our apparatus, these photons have linear polarization perpendicular to the field. (Here is a mnemonic: the two lower strokes in the Greek letter  $\pi$  are *parallel* lines, and “ $\pi$ ” transitions are parallel to  $B$ .)

### 1.4.1 Notation of states

States are specified using a long established notation,  $^{2S+1}L_J$ , where  $S$  is the total spin quantum number,  $L$  denotes the total orbital angular momentum number, and  $J$  denotes the total angular





momentum quantum number. The peculiar notation for the orbital number,  $S$  for  $L = 0$ ,  $P$  for  $L = 1$ ,  $D$  for  $L = 2$ ,  $F$  for  $L = 3$ , comes from the names originally given to the sets of spectral lines seen on photographic plates: “Sharp”, “Principal”, “Diffuse” and “Fundamental”. A state with  $S = 1$ ,  $L = 2$  and  $J = 2$  is denoted as  $^3D_2$ . The quantity  $2S + 1$  is referred to as the “multiplicity”; it represents the number of angular momentum combinations for  $J$  that can be formed from a given  $S$  and  $L$ . For example, using the above values of  $S$  and  $L$ ,  $J$  has possible values 3, 2 and 1, hence a multiplicity of 3 for this situation.

Note that there are two different uses for the letter  $S$  here: one use represents the total spin, and the second represents the orbital angular momentum state  $L = 0$ . Regarding spin, states with  $S = 0$  are called singlets, and states with  $S = 1$  are called triplets.

Initial and final states of the three mercury lines (shown in Fig. 7) to be investigated in this experiment are:

**Green line:**  $\lambda = 546.07$  nm, initial state  $(7s)^3S_1$ , final state  $(6p)^3P_2$ .

**Yellow lines:**  $\lambda = 576.96$  nm, initial state  $(6d)^3D_2$ , final state  $(6p)^1P_1$ ;  
 $\lambda = 579.07$  nm, initial state  $(6d)^1D_2$ , final state  $(6p)^1P_1$ .

Here we note that the 576.96 nm transition in mercury violates one of the selection rules, namely  $\Delta S = 0$ . This is due to the angular momentum coupling not being purely  $LS$ , but a combination of  $LS$  and what is known as  $jj$  coupling, which eases the restriction on the  $\Delta S = 0$  rule [4, Ch. 17]. One can still apply the remaining rules in constructing a diagram for this transition. Note from Fig. 7 that a large majority of the transitions do obey the selection rule  $\Delta S = 0$ , as there are few transitions that cross the singlet/triplet boundary.

## 2 Measuring the energy shift: Fabry-Perot interferometry

The energy difference between the initial and final states in an atomic transition is determined by measuring the wavelength of the light emitted in the transition. Energy is related to wavelength by the famous equation

$$E = h\nu = hc/\lambda, \quad (16)$$

where  $\nu$  and  $\lambda$  are the frequency and wavelength of the emitted light, respectively,  $h$  is Planck’s constant and  $c$  is the speed of light.

Since the goal in this experiment is to measure the shift in atomic energy levels due to an external magnetic field, then how is the shift measured? Given the magnitude of the magnetic field available with our electromagnet (around 1000 gauss) and the size of the magnetic moments involved (around a few Bohr magnetons), the shift in wavelength one would expect to see would require an absolute measurement of the wavelength of emitted light to have a precision of better than 6 decimal places. Such a measurement is very difficult and would require sophisticated and very expensive equipment beyond the means of this laboratory. Instead of absolute measurements of the wavelength, we make a relative measurement of the wavelength: we measure the shift in wavelength due to the magnetic field relative to the unshifted wavelength.

To find the energy difference in terms of a small change in wavelength differentiate Eq. (16) to obtain

$$\Delta E = - \left( \frac{hc}{\lambda^2} \right) \Delta \lambda = h\Delta\nu; . \quad (17)$$

The wavelength shift  $\Delta\lambda$  or frequency shift  $\Delta\nu$  is measured with an instrument known as a Fabry-

Perot interferometer. For a description of this device see the appendix of this write-up. Further details are given in the text by Preston and Dietz [3] and the optics text of Hecht [5]; several pages of the material from Preston and Dietz can be found on the class website. Be sure you understand the concepts of finesse and free spectral range, and how mirror separation affects the latter parameter. A very brief description of the Fabry-Perot follows.

The Fabry-Perot works by having light reflect back and forth many times in a space bounded by two very flat, highly reflecting mirrors. The set of mirrors is called an “etalon” (see Fig. 8). The intensity of the light exiting the etalon is maximum when the phase shift of the light is an integer multiple of  $2\pi$  after reflecting from both mirrors (this is the requirement for constructive interference). Light of differing wavelengths satisfies this condition at different exit angles from the etalon resulting in visible rings of differing circumference for the different wavelengths. The output of the Fabry-Perot is not just one visible ring for each unique wavelength, but rather a repeating pattern of rings, successive patterns representing an additional phase shift of  $2\pi$ .

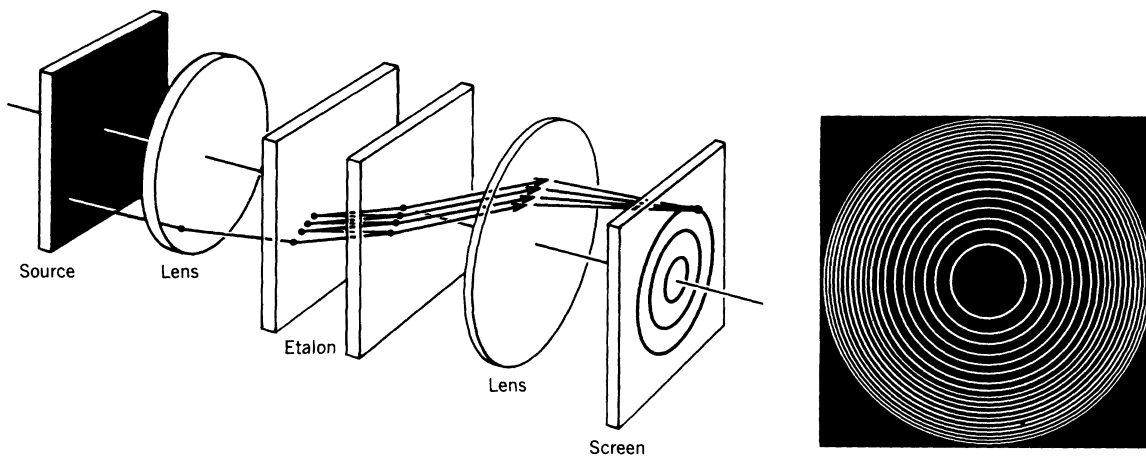


Figure 8: Schematic illustration of a Fabry-Perot interferometer and the ring pattern it produces on a screen when used with a monochromatic light source.

The strength of the Fabry-Perot is its ability to resolve lines with closely spaced wavelengths. If  $\Delta\lambda_{\min}$  is the minimum resolvable wavelength difference at wavelength  $\lambda$ , then for our instrument,  $\lambda/\Delta\lambda_{\min} \approx 10^6$ .

A limitation of the Fabry-Perot is that it can only work with wavelength or frequency differences up to a certain maximum which turns out to be

$$\Delta\lambda_{\max} = \frac{\lambda^2}{2d}, \quad (18)$$

$$\Delta\nu_{\max} = \frac{c}{2d}, \quad (19)$$

where  $\lambda$  is the nominal wavelength of the light entering the interferometer and  $d$  is the separation between the mirror surfaces. This maximum difference in frequency or wavelength is called the *free spectral range* (FSR) of the interferometer. At this wavelength difference, the ring due to light with wavelength  $\lambda \pm \Delta\lambda_{\max}$  will just overlap the adjacent ring (remember, the pattern repeats!) due to light with wavelength  $\lambda$ . This kind of overlap will result in a ring pattern that is difficult to interpret, a situation that can usually be avoided by a judicious choice of mirror spacing.

**Note:** The value for  $d$  is measured by a traveling microscope. You will need this number to determine the free spectral range, a key parameter in the analysis of your data.

To measure the light intensity of a ring we use the central spot scanning technique in which a light detector, in our case a photomultiplier tube, detects the intensity of the central spot as the ring pattern collapses in to the center as we change the mirror separation. The photomultiplier tube output current is converted to a voltage that is recorded by an oscilloscope or X-Y plotter, resulting in a plot showing the intensity and separation of the various rings. Details on how to record this pattern follow in the next section. (Note: in our instrument we change the mirror separation by changing the voltage applied to a set of piezoelectric elements that support one mirror. Another way to operate a Fabry-Perot scan is to change the optical path length by varying the pressure of air, and thus the refractive index, between two *fixed* mirrors. [1, 3])

Figure 9 shows an approximate diagram of our apparatus. The mercury discharge lamp is located between the pole pieces of the electromagnet. Light from the lamp is collimated by lens L1 and then passes into the Fabry-Perot interferometer. Light exiting the interferometer passes through the focusing lens (L2) and then through the beamsplitter. An image of the ring pattern comes to a focus on the white disk on the detector, and on the entrance to the spectrometer. The beamsplitter arrangement offers the advantage of being able to visually monitor the output of the interferometer at the same time as the light intensity is being detected and recorded.

The data acquisition procedure is described in detail in the next section.

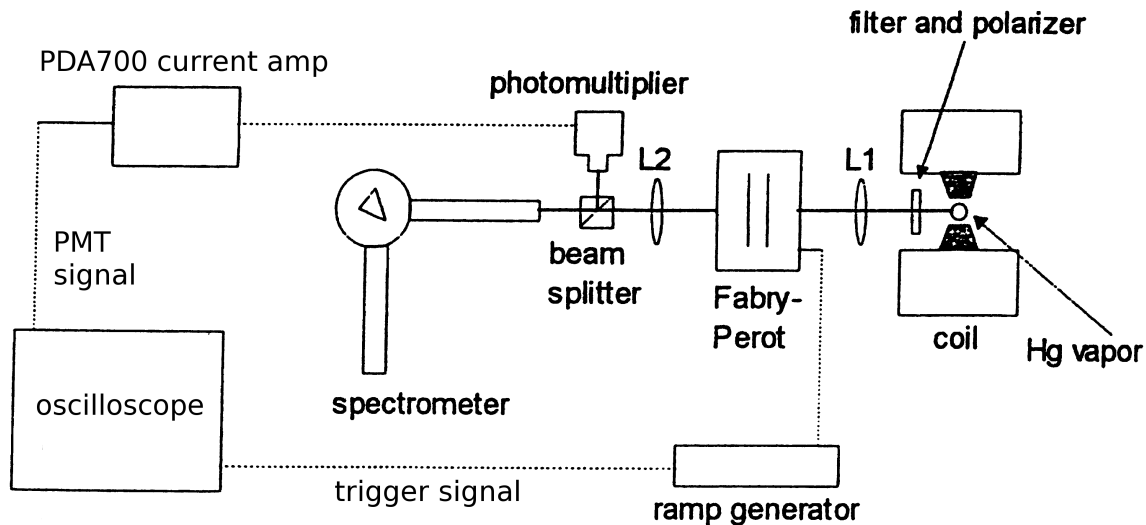


Figure 9: Schematic diagram of the Zeeman effect apparatus.

## 3 *Experiment and Procedure*

### 3.1 Apparatus alignment

The Burleigh RC-150 Fabry-Perot interferometer is a precision instrument capable of resolving wavelength shifts of 0.001 nanometers or less. To realize the full capabilities of this instrument it is necessary to optimize the alignment of the interferometer mirrors, the external optics and of the detector hardware. The challenge of performing a full alignment (from scratch) of the optical system is both time consuming and difficult; it is not reasonable for the uninitiated student to attempt such a procedure. When you come to start the experiment, the setup should be mostly aligned. You will, however, need to fine tune the electronic controls in order to get usable data.

To scan the ring pattern created by the Zeeman splitting of the green and yellow mercury lines, the spacing of the mirrors in the Fabry-Perot interferometer is varied in time. One of the interferometer mirrors is mounted on three piezoelectric elements,

120 deg apart, and the mirror spacing is changed by applying voltages to the three elements. The RC-42 Ramp Generator creates the voltages applied to these elements. A variety of knobs control how these voltages are applied:

**Bias 1, 2, 3 controls** The offset (baseline position) of each element can be adjusted separately by the “Bias 1, 2, 3” controls, and this is how the alignment of the mirrors is fine tuned, i.e., brought into the parallel condition. These will be used in the mirror alignment section below.

**Ramp Bias control** A simultaneous adjustment of the voltage to all three of the piezo elements is made with the “Ramp Bias” control. Varying this control results in a parallel translation of the mirror and movement of the ring pattern in or out from the center.

**Ramp Amplitude and Duration controls** A scan is made by applying an internally generated “ramp voltage” to the piezos. This ramp voltage changes the mirror spacing approximately uniformly in time, enabling one to record a steady change in the ring pattern under observation. Each time the ring pattern repeats as the ramp voltage increases, the mirror separation has changed  $1/2$  wavelength of the light under observation. The total change in mirror separation over one complete ramp cycle is adjusted with the “Ramp Amplitude” control. The maximum ramp amplitude setting allows for a change in mirror separation of 5 to 6 half wavelengths in the middle of the visible spectrum. For data taking purposes, scanning through 3-5 complete ring patterns is sufficient for Zeeman effect observations. The rate at which the ramp voltage changes (and repeat cycles are produced) is set by the “Ramp Duration” controls consisting of a knob and “Magnifier” switch that changes the duration by a factor of 100.

**Reset and Ramp Stop/Go switches** Because the scan speed may be very slow, it can be useful to be able to start the ramp manually. One can do this by pressing the “Reset” button which forces the ramp voltage to the beginning of the ramp. We have added another switch, the “Ramp Stop/Go” switch, which allows the user to stop the ramp at any point in order to ease the mirror alignment process.

**Trim 1, 2, 3 controls** *You should not turn these controls.* The trim controls correct for variations in the response of each piezo element to the ramp voltage so that the mirrors stay parallel as the ramp voltage is varied. These controls will have been set during the initial alignment of the instrument and should not need to be adjusted.

### *Mirror alignment first steps*

The procedure outlined below will enable you to bring the interferometer mirrors into alignment provided they are already reasonably close to being parallel. If the mirrors are too far out of alignment, a separate procedure must be followed. This more involved procedure will be performed by the Lab Manager, Prof., or TA.

1. Check that the magnet power supply (to the right and underneath the table) is off. If it is on, make sure the current is turned to zero and then turn the supply off. A flashlight is available to help see the switches and controls on this unit.
2. Check to see that the RC-42 ramp generator is on. If it is not, turn it on. Ideally, it should warm up for an hour or two to improve the stability of the instrument. Turn the Ramp Bias control fully counterclockwise (it has about 3 1/2 turns of adjustment, and the stops at the ends of the adjustment range have a very soft feel to them), and then turn it about 1 turn clockwise.
3. Set the Ramp Duration knob to “100 ms” and the magnifier switch to “ $\times 100$ ”.
4. Set the Ramp Stop/Go switch to “Stop”, and press the Reset button once.
5. If the mercury discharge lamp is not already on when you come in to the setup, before turning it on, check that it is positioned between the magnet pole pieces.

**CAUTION:** The voltage on this lamp is **5000 volts** and the supply can provide a hazardous amount of current. **Do not touch the lamp or holder while the voltage is on.** Turn on the discharge lamp and allow approximately 5 minutes for it to warm up.

6. Check that there are no filters or polarizer in the beam path. The holders for these items are right in front of the magnet pole pieces. Check that the prism spectrometer is set to about 500 nm (dial is on right side of spectrometer body).
7. Turn each of the Bias 1, 2, 3 knobs counterclockwise until you feel a slight resistance to turning (once you feel the resistance, do not turn any more—you are at the limit). Then turn each knob clockwise by exactly 1 1/2 turns. Repeat for the other two knobs. Your goal is to set them in the middle of their range.
8. After the Ramp Generator and mercury lamp have warmed up, look through the prism spectrometer. If the interferometer mirrors are not too far out of alignment, you should see a vertical slice of a pattern of concentric rings. If the rings are highly smeared out or not visible at all, the mirrors may be so far out of alignment (i.e., non-parallel) that it may be necessary to go through the complete alignment procedure. Contact Lab Manager, Prof. or TA to perform this procedure.
9. Look through the spectrometer. You should see some mainly green light, and if you are lucky, a clearly visible slice of a ring pattern. Adjust the focus of the spectrometer to bring the image of the **slits (not the rings!)** into sharp vertical lines bounding a (probably) fuzzy ring pattern. *The reason to focus the spectrometer on the slit edges is to insure that the spectrometer focus is independent of the ring pattern focus: the spectrometer focus is set for your eye, the ring pattern focus is set for the instrument, since the ring pattern is also projected on the detector, and it must be in good focus there to get good data.*

10. While you look through the spectrometer, adjust each of the bias 1, 2, 3 controls a little in either direction to see which one or two has the greatest effect on the focus of the green ring pattern. Choose the two most effective controls (typically 1 and 2), and adjust each one alternately to bring the pattern into best focus. When you have done as well as you can with just two knobs, adjust the third to improve the pattern further. Have patience: several iterations of adjustments may be necessary. Please be careful not to touch the three Trim controls underneath the Bias 1, 2, 3 controls. (The Trim controls keep the displacement of all three elements uniform, and therefore the ring pattern in focus, as the ramp voltage changes.) Note, if you try to adjust the pattern by using all three controls in a “round robin” manner, you may run out of ability to focus the pattern; it is best to make most of the adjustment with only two knobs.
11. If you are unable to bring the rings into good focus, a mechanical adjustment may be necessary. This should be performed by the instructor, as it is a very sensitive adjustment, and doing it well requires some practice. But if you attempt it yourself, first reset the bias 1, 2, 3 controls to the middle of their range. Then you should only turn the biggest two knobs (labeled “2” and “3”) *very slightly* as you look through the spectrometer. Move one knob until the pattern is clearest, then move the other, and proceed in this way, back and forth between knobs 1 and 2, until the pattern is as clear as possible. When you get close to good focus, you will find that you don’t really “turn” a knob at all, just squeeze it a little bit. *IT IS A VERY SENSITIVE ADJUSTMENT!* Then repeat the previous step with the bias 1, 2, 3 controls to optimize the focus. *DO NOT TURN THE SMALLEST KNOB “1” AT THE BOTTOM.*
12. Adjust the Ramp Bias control so that the two faint rings just inside the inner bright green ring are visible. Adjust the Bias 1, 2, 3 controls to bring these faint rings into the sharpest focus possible. Since the data are taken by observing the intensity at the center of the ring pattern, it is important to optimize the focus of the rings closest to the center of the pattern. When these two faint rings are in sharpest focus (at best they will still be a little fuzzy), the rest of the pattern should also be in good focus. Note that as you adjust each of the knobs, you may see different parts of the pattern come into focus, but you should work to get the whole pattern into as best a focus as you can.

Once you get the mirrors aligned you should see two patterns side-by-side in the spectrometer: the green pattern will be fairly bright, and the pattern from the two yellow lines will be visible to the right of it. If you look closely at the yellow pattern you should be able to see that there are in fact two overlapping sets of yellow rings whose centers are slightly displaced horizontally. The set of rings that is shifted towards the green set of rings is the slightly shorter wavelength of the yellow pair of transitions that you will study.

### ***Detector alignment check***

Look at the white disk around the entrance hole to the photomultiplier tube (PMT) housing on the far side of the beamsplitter. You should see a violet strip of light with no ring structure apparent. This strip of light is the result of allowing all the colors of the mercury spectrum to pass through the Fabry-Perot interferometer. Each line in the spectrum produces a ring pattern, and the violet strip is the result of all the ring patterns sitting on top of each other. For measurement purposes, one must isolate out the line of interest, just as the prism spectrometer does very nicely for visual observation purposes. In our apparatus, isolation of the line of interest is achieved with what is

known as an *interference filter*, which is placed close to the discharge tube. We have two such filters, one of which passes the mercury green line (546 nm filter) and the other of which passes the two closely spaced yellow lines (577 nm filter).

Your first measurements will be on the mercury green line, so now put the 546 nm filter in the filter holder (one closest to pole pieces). If you now look at the white disk (magnifying glass helps) you should see a faint vertical slice of the green ring pattern, just as in the spectrometer. To see this clearly, the room lighting must be very dim and you must be accustomed to the low light level.

1. Make sure the switch on the little box connected to the the PDA-700 amp is in the **OFF** position before proceeding. This switch helps block any voltage surges that may come when the high-voltage supply is turned on.
2. Turn on the PDA-700 amplifier and check that the range is set to .xxxx  $\mu\text{A}$  (note the position of the decimal point; select this by the small “Range” knob). If necessary, adjust the “Offset” knob so that the display reads exactly zero. The photomultiplier tube output is a current proportional to the intensity of the incident light; the PDA-700 amplifier converts this current to a voltage which is displayed on the oscilloscope. NOTE: you may need to change the range setting during the experiment, but this setting is a good place to start.
3. Now follow this procedure to turn the high voltage supply for the PMT on (located on the rack toward the back of the room). Check that the switch is off before proceeding:
  - (a) **Make sure that room light is extremely dim** (barely enough light to see at all). This is important to make sure not to damage the PMT.
  - (b) **Also make sure that a filter (start with green) is in one of the filter holders** near the magnet gap, and the switch box connected to the PDA-700 is off (as noted above). Each person in the group should have access to a flashlight to keep themselves from bumping into things and people.
  - (c) The high voltage power supply is in the rack back and to your right for the PMT. Check that the knobs are both set to the lowest (most counterclockwise) position.
  - (d) turn on the power switch to the high voltage supply.
  - (e) set the supply voltage to 800V: 500V on one knob and 300V on the other.
4. Now turn the switch on the little box connected to the PDA-700 input. You may see the display respond by showing some non-zero current. It will have a negative sign.
5. Moving back to the ramp generator, adjust the Ramp Bias control so that there is a small bright disk in the center of the ring pattern. This disk of light should fall on the center of the white disk covering the entrance hole to the PMT housing. Use the Ramp Bias control to expand the disk into a small ring and check, by visual observation, that this ring is centered on the hole in the white disk. If it is not close to being centered, consult with Lab Manager, Prof., or TA for assistance in adjusting the detector hardware.
6. The centering of the ring pattern on the entrance hole to the PMT housing can further be checked by observing the PMT output current as the Ramp Bias is adjusted to expand or contract the disk. The current observed on the PDA-700 amplifier should peak at about the same time as the brightness of the disk peaks just before it disappears (look in spectrometer to observe disk contracting and then disappearing). If this is not the case, again consult with Lab Manager, Prof., or TA for assistance in adjusting the detector hardware.



### *Fine tune the bias*

Check that the Magnifier switch is set to the “ $\times 100$ ” position and set the Ramp Duration switch to “100 ms”. Flip the Ramp Stop/Go switch to “Go”, and adjust the Ramp Amplitude knob while looking into the spectrometer so that the scan covers 4 to 5 complete cycles of the ring pattern. The rings will contract as the voltage to the piezoelectric elements is ramped up. At the end of the ramp the voltage returns very quickly to its initial value, the ring pattern snaps back to its initial configuration (so quickly you can’t see it happening), and the scan begins again. The scan can be reset to the beginning by pushing the RESET button on the RC-42 controller. The position of the rings at the beginning of the scan can be changed with the Ramp Bias control.

Note: If the ramp appears to “hold” at the beginning for a while, the Ramp Bias may be too low. Turn the Ramp Bias knob up enough so that the pattern begins to contract immediately after you press the RESET button.

As a last touch up of the mirror alignment before you switch to the oscilloscope, stop the ramp (with the Ramp Stop/Go switch) near the midpoint of the cycle, and adjust the Bias 1, 2, 3 controls to focus the pattern as sharply as possible. You should not need to make significant changes at this point. If you do, you may need to spend some more time adjusting the overall alignment as described earlier.

### *Set up the scope*

At this point, the HV supply should be on, the green filter in place, the PDA-700 current amp should be set to show a visible change in the readout when the scan runs slowly. Do not proceed until this point has been reached.

Flip the Magnifier switch from  $\times 100$  to  $\times 1$ . This will give a ramp cycle time of 100ms. When taking data with the scope, you will not be able to view the pattern with the spectrometer, since it is much too fast. We will be using the data averaging feature of the scope to eliminate much of the noise in the electrical signal from the PMT and amplifier.

Turn on the Tektronix MSO2024 oscilloscope. If it has been used recently, it may be in a state that is already usable for the Zeeman experiment, but if not, you can set it up as follows:

1. The output of the PDA-700 amplifier should be plugged into Ch 1 and the TRIGGER output from the back of the ramp generator should be plugged into another channel, e.g., Ch 4.
2. Press **Default Setup**. This puts the scope into a known configuration. (But not usable for the experiment yet.) The trace from Ch 1 should be visible as a yellow line.
3. Set the Horizontal control to 10ms/Div. You may see a signal that appears upside down at this point.
4. Turn on Ch 4 (or whichever channel is connected to the TRIGGER output on the back of the ramp generator). Set the Probe Setup to 1x, and set the channel sensitivity to 1V/Div.
5. Select the Trigger Menu on the scope, then select Source, and set the trigger source to the correct input (i.e., Ch 4). Select a positive slope, and a level of 1.4V. After you see a stable trigger on this channel, you may turn the channel display off (press the channel number again).

6. Select Ch 1 (yellow button) and set the Probe Select to 1x and set the bandwidth to limit to 20 MHz (helps reduce noise). Then set the Invert to On. This will flip the signal polarity, so peaks point up. Set the channel sensitivity to between 50 mV/Div and 500 mV/Div, and adjust the vertical position to give a clear display of the signal on Ch 1.
7. The trigger position by default is in the center of the screen. The trigger starts during the reset of the sweep. You may see a small group of peaks clustered right at that point. Use the Horizontal position adjustment to move this cluster off to the left. The peaks you want to measure are the ones that are widely spaced, and these should take up the visible part of the screen.
8. You should see 4 to 5 peaks on the screen. If you see more or fewer, adjust the Ramp Amplitude until you see the correct number.

If you have started with the Default Setup, the signal averaging will be off. If not, press **Acquire** and from the soft-keys, select **Average** and turn it Off if it is not already off.

The oscilloscope settings should now be correct to attempt the final focus adjustments with the individual Bias knobs. This step is typically the most challenging for beginners; doing it well requires practice. You may need to try the procedure multiple times before obtaining an acceptable scan.

The peaks you observe will seem to be jumping up and down chaotically. The fluctuation comes from electrical noise superimposed on the tiny currents from the PMT. To overcome this, we will use the signal averaging feature of the oscilloscope. But, because of the time needed to average out the noise, one cannot adjust the Bias with the averaging on.

To fine-tune the Bias and bring the pattern into correct focus:

1. Start with Bias 1. Turn it a little bit while watching the screen. If you turn it too far in one direction, the peaks will shrink in height and become broader, indicating poor focus. Too far in the other direction: same thing. Adjust the knob in tiny amounts until you can see the peaks reach their highest points, and when they do, they appear (briefly) very narrow.
2. Then repeat the previous step using Bias 2.
3. go back and forth between Bias 1 and Bias 2 until you can get no more improvement in the apparent sharpness of the peaks. Have patience!
4. Only when you have the best settings you are able to get with Bias 1 and 2 should you adjust Bias 3.
5. When you think you have as good a setting as you can manage, turn on the signal averaging: **Acquire** followed by **Average: On**. Sixteen (16) averages is good enough to see if you have a usable scan.
6. After averaging, you may need to adjust the Ch 1 sensitivity to make the peaks fill the display. For best data sets, make the waveform take up as much of the display area as possible without clipping it off.

Generally, you will find that you need to adjust the Bias controls very little to achieve a good result. If you find yourself turning them a lot, or in worst case, that you are near the end of their range,

you should restart your alignment procedure from the beginning: Flip the Magnifier back to  $\times 100$ , set all Bias knobs to the their midpoints, and carry out the alignment by eye, from Section 3.1. I repeat: effective use of this delicate and precise instrument requires practice; don't feel bad if you have trouble getting it to work—it's a violin, not a shovel.

In the following where you are to obtain scans of the ring pattern, make sure that the scan maintains good focus. *But, do not try to adjust the focus when the magnet current is turned up.* The Zeeman effect will cause the ring pattern to appear to be out of focus when the rings are not well separated. It is best to fine tune the bias with the magnet current off, and then turn it up shortly before you run a  $B \neq 0$  scan.

## 3.2 Required observations and data collection

### *Green lines at $B = 0$*

Using the oscilloscope, record the ring pattern with the green (546 nm) filter in the filter holder with no magnet current on. You will use this plot to determine the finesse of the interferometer and as a check of the instrument alignment quality.

Set up signal averaging for 256 or 512 averages to minimize noise. Save your data to a thumb drive so that it can be further analyzed on the computer.

To save the data to a thumb drive, you can set up the file saving under the Save/Recall buttons. Press Menu to set up waveform saving to a file. You will want to save the file as a .csv file, and under Waveform Resolution select Reduced. The default "full" resolution creates an excessively large data file which is cumbersome to analyze. You can also set the Save button to save additional data sets with one button push. For details, study the user manual for the oscilloscope.

You can also save screen-shots as .png files in in a similar manner.

Compare your scan with the peaks shown in Fig. 10. If your trace looks like the one at the bottom of the figure, you are in good shape. But if the peaks are broadened so that you cannot see the finer structure, you need to work on the mirror alignment and try again.

### *Observe and record the "anomalous" Zeeman effect for the green line.*

Stop the scan with the Ramp Stop/Go switch so that you can view the pattern through the spectrometer. Press the scan Reset button to park the piezos at their starting position. You may want to slightly adjust the overall Ramp Bias to bring the rings into a clearly visible position.

Check to be sure that the current and voltage adjust knobs on the magnet power supply are set fully counterclockwise. Turn on the supply and the digital meter, and turn the *voltage* adjust knob fully clockwise. Slowly turn the current adjust knob clockwise and observe the beautiful display as the brightest green ring begins to split. Turn up the current so that all the rings are clearly distinguishable, but not so far that any of the rings produced by the Zeeman splitting overlap.

***Make sure that everyone in your group has a chance to observe the the ring separation through the spectroscope as the current is turned up.***

Now place the linear polarizer filter in the holder, and notice that some of the rings disappear, while others remain. Then remove the polarizer, turn it 90 deg, and reinsert it into the holder and look at how the pattern has changed. Again, everyone should have a chance to see this effect in

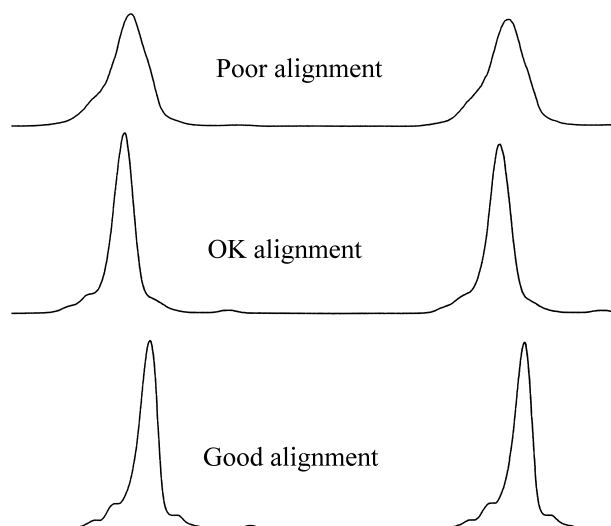


Figure 10: Scans of the 546 nm (green) line of mercury with magnet current off with different degrees of mirror alignment quality. Bottom scan shows good alignment; note easily resolved finer structure at base of main peak. Middle scan shows acceptable alignment, but not optimum. Top scan shows unacceptably poor alignment.

the spectroscope. You might want to make sketch of the ring patterns that you see with the two different orientations of the polarizer.

*These phenomena are some of the most vivid examples of the consequences of quantized angular momentum. Take a moment to appreciate how the quantum theory that you learn is evident in these observations.*

When you are satisfied with the appearance of the pattern, you can run a scan on the scope. You may very well need to readjust the Bias 1, 2, 3 controls in order to refocus the pattern. Make small adjustments and see how it affects the pattern on the scope and adjust from there. This is most easily done with a fairly large magnet current so that the splittings are well separated.

**IMPORTANT:** Watch and record the magnet current as you run the scan. You will need this number in order to analyze the data.

After recording this pattern put the linear polarizer in the beam path and separately record the pattern for both orientations of the polarizer, noting which pattern goes with which orientation. The arrow on the polarizer shows the direction of polarization. Again, keep track of the magnet current during each scan.

### *Observe and record the “normal” Zeeman effect for the yellow lines*

Now you want to repeat the entire set of observations and measurements that you made for the green line for the yellow lines.

First touch up the focus by looking at the green-line pattern—it is easier to see.

Put the 577 nm (yellow) filter in a holder, and then remove the green filter.

Record a scan with no magnet current. The yellow lines are not as intense as the green line, and it will be necessary to go to more sensitive scales on the PDA-700 amp by adjusting the gain, and adjusting the Ch 1 sensitivity on the scope.

Next, observe through the spectrometer how the yellow lines split as the magnet current is turned up under three conditions: no polarizer, polarizer vertically oriented, and polarizer horizontally oriented.

The two yellow lines are sufficiently close together that the 577 nm interference filter passes both of them. This means that a scan of the yellow lines must be done with some care; the yellow lines are shifted (slightly) in position in the spectrometer, but the ring patterns falling on the detector pinhole are not shifted at all. You will need to know which peaks correspond to which principal yellow line. When you look through the spectrometer, you will see two patterns displaced slightly along the horizontal. The yellow pattern that is displaced towards the original green pattern has the shorter wavelength of the two. With a slow scan, watch to see when a given line goes through the center first and/or last, and note this when you record the scan with the oscilloscope.

If your focus is very good and you have no filter in place (turn off the HV to the PMT first), you may also see smaller peaks. There is an evident small peak following the larger peak for the longer wavelength (579 nm) line that may be visible on the  $B = 0$  scan for yellow. This also helps identify which peaks are from 577 nm and which from 587 nm

When recording a scan that shows the Zeeman splitting of these lines, you must keep the magnetic field low enough so that it is clear which sets of rings go together. Typically, the magnet current must be kept below 2.0 amps. Fortunately, the ring pattern is simpler for the yellow lines than for the green line; the Zeeman splittings follow the model predicted by classical physics—the “normal” Zeeman effect. (Not strictly true for the 577 nm line, as you will see in the analysis.)

Remember to record the magnet current during the scans for the yellow lines, and to get scans with the polarizer in both orientations.

### *Obtain the mirror spacing*

At this point, you should have a total of eight scans: for each color you should have a scan with  $B = 0$ , a scan with  $B \neq 0$  with no polarizer, and scans with  $B \neq 0$  with the polarizer in each of the two orientations.

Turn the high voltage to the photomultiplier tube off by first flipping the little switch from the PMT to the amplifier off, then dialing down the voltage knobs on the high-voltage supply, and then flipping the HV supply power switch off.

Now you may turn the room lights back on.

In order to analyze your data, you need to know the free spectral range of the Fabry-Perot interferometer. The free spectral range can be specified in terms of either wavelength or frequency; the formulas are

$$\Delta\lambda_{\max} = \frac{\lambda^2}{2d}, \quad (20)$$

$$\Delta\nu_{\max} = \frac{c}{2d}, \quad (21)$$

where  $d$  is the mirror separation,  $c$  is the speed of light, and  $\lambda$  is the (nominal) wavelength of light. These equations are derived and discussed in the appendix.

The mirror separation is measurable with the traveling microscope mounted near the Fabry-Perot set. Peer through the microscope ocular, and you should see a light vertical band separated by two dark regions—the mirrors.

You may need to adjust the focus of the microscope tube so that the mirror edges are sharply defined, and you may also need to adjust the focus of the ocular in order to clearly see the cross-hairs.

Note that as you move the microscope position by means of the knob, the image will travel (oppositely) across your field of view.

To make a measurement, turn the knob to move the carriage toward zero, so that both mirror edges are off to one side of the cross-hair center. *IMPORTANT: You must move the knob only in the advancing direction when you make a measurement. Do NOT turn it back and forth as you approach the mirror edge. There is slop, also called “backlash” between the gears that move the carriage, and this backlash must be avoided.* Slowly advance the knob to park the exact center of the cross-hairs on one edge. Record the position. Note: each full revolution of the knob is 1 mm, so each small marking corresponds to 0.01 mm. You should be able to measure the position to  $\pm 0.005$  mm.

Advance the knob to the other edge, always turning the knob in the forward direction, and record its position. Then calculate the mirror separation. Repeat the measurements a couple of times to check for consistency. The mirror separation should be close to 1.7 mm. You will need to refer to the appendix section regarding Fabry-Perot interferometry in order to do calculations of the free spectral range and finesse, which require the mirror spacing.

## 4 Shut down

After you have completed your measurements, be sure to turn everything off. Turn the voltage and current controls on the magnet power supply to zero and the power supply off. Remove the filter and polarizer from the beam path and put them back in their storage boxes. Finally, cover up the optics with the plastic covers.

## 5 Data analysis

Your overall goal is to measure the energy spacings of the line splittings observed in your scans, and compute the corresponding differences in magnetic moments  $\Delta\mu_{J_z} = (g_{J_i}m_{J_i} - g_{J_f}m_{J_f})\mu_B$ . This requires you to understand the scans made by the Fabry-Perot interferometer in terms of the underlying energy levels and transitions evident in the mercury spectrum. To a lesser extent, you should also understand the physics of the Fabry-Perot interferometer itself.

**Exercise 1** Consult the appendix on Fabry-Perot interferometry and learn the definition of instrument finesse. From your scan of the mercury green line with no magnet current, calculate an estimate (actually, a lower bound, since the line has its own “native” width) of the finesse of this instrument.

Calculate the free spectral range of the interferometer from your measurement of the mirror spacing in frequency units. This will simplify later analysis of your scans.

Now you want to consider more carefully how the energy levels in the mercury atom relate to the Zeeman spectra.

**Exercise 2** The spectroscopic descriptions of the initial and final states for the three mercury transitions of interest are given in Section 1.4.1. Use this information to construct a diagram similar to that in Fig. 6 for each of these three transitions. The diagrams should show the sublevels for the initial and final states, and which transitions are allowed. Try to make the level separations on the diagrams proportional to the actual values of  $g_J m_J$  within each term.

**Exercise 3** From the diagrams you created in the above exercise, calculate the predicted energy shifts in the Zeeman effect spectra by first calculating the value  $g_J$  for the initial and final states in each transition (the green line and the two yellow lines). From Eq. (15) we find that the energy shift of the spectral line  $\Delta E_{\text{magnetic}}$  is given by

$$\Delta E_{\text{magnetic}} = (g_{Ji}m_{Ji} - g_{Jf}m_{Jf}) \mu_B B , \quad (22)$$

where the  $i$  and  $f$  subscripts denote the initial and final states of the transition. Then make a “scale drawing” of the predicted spectrum similar to the lower part of Fig. 6, where the position of the lines along the horizontal is proportional to the energy shift and the two polarizations are represented by solid ( $\sigma$ ) and dashed ( $\pi$ ) lines.

Now you should be ready to analyze your traces. You will need to know the values of the magnetic field strengths corresponding to the various magnet currents. You can find these by using the magnet calibration curve in Fig. 11. Assume that the dominant uncertainty is due to the uncertainty in the measurement of the current.

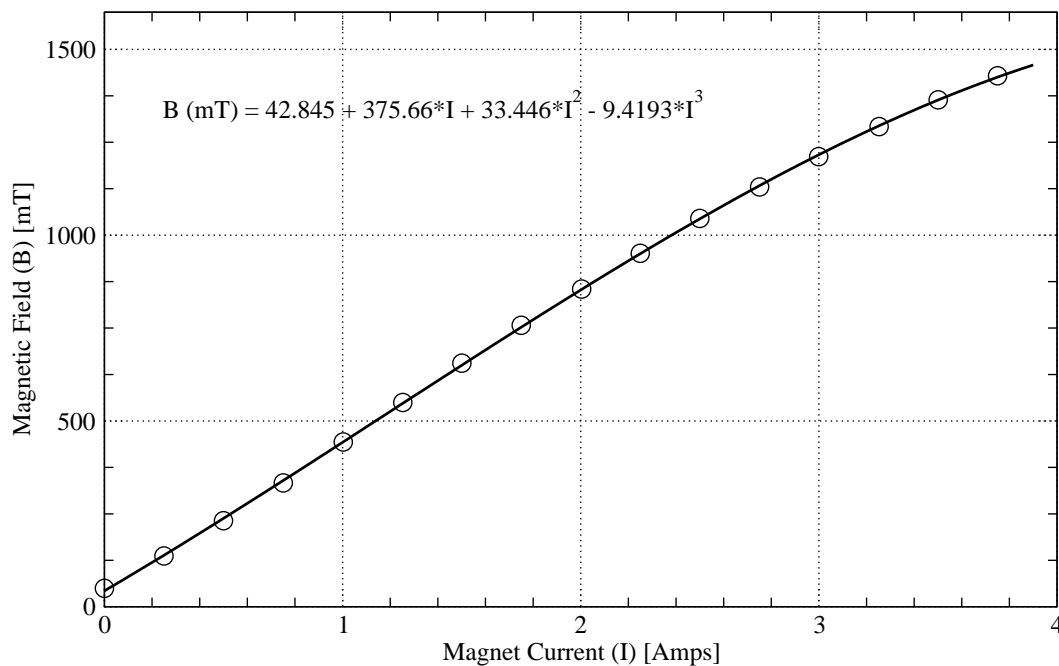


Figure 11: Calibration curve for the Zeeman effect electromagnet. Note the polynomial fit equation which allows one to calculate the field strength for any value of the magnet current.

In principle, it is a simple matter to find the shift of a Zeeman peak in an interferometer scan. You know that the shift corresponding to one free spectral range would correspond to the distance between two similar features. Thus, any peak that appears to be shifted less than a full free spectral range will have an energy shift that is proportionately less. So to analyze a trace, you first measure the distance between repeating features (typically the main peaks), and then calculate a conversion factor from the free spectral range.

The challenge is that the x-axis on your sweeps corresponds to time, and although the voltage ramp sent to the piezo-electrics that adjust the mirror spacing is linear in time to a very good approximation, the response of the piezo electrics is not. Upon close inspection, you will notice that the spacing of repeated peaks changes. You will need to account for this variation in the peak spacing.

Here is one way to do it:

Locate the positions of the 9 peaks within each cluster over as many complete cycles as you have. If you take these directly off the scope scan, these will be in units of time. For a data file from the scope, you may work with the array indices. If using a computer to carry this out, plot symbols at the peak locations over the data set to confirm you have the correct ones.

Obtain the positions of the 4 central peaks. Because of hysteresis in the piezos, the cycles are not evenly spaced. But the change in spacing follows a smooth function. Fit the peak number (e.g., 1, 2, 3, 4) versus peak positions with a 3rd order polynomial. Because a change in energy by one free-spectral-range corresponds to a change by one peak position, this fit will allow a calibration of the data into FSR units.

Apply the calibration to obtain the positions of all peaks in FSR units. You should see that the central peaks have FSR locations of exactly (or very close to) 1, 2, 3,... Make a table to check the calculation.

Each cluster is a cycle showing the same pattern repeated. The peaks surrounding each central peak are shifted from that peak either up or down in energy by the Zeeman effect. Calculate the difference between the position of each peak in a cluster and the position of the central peak in that cluster. (This would be equivalent to subtracting 1 from the peaks in the first cluster, 2 from the peaks in the second cluster, etc.) Then average the results for each satellite peak to find the average separation from the central peak across the scan. Make a table of the average separation of each peak from the center in units of free spectral range.

To compare to expected values (from Exercises 2 and 3), you will want to calculate these shifts in units of  $B\mu_B$  where  $B$  is the magnetic field applied during the scan. It is most convenient to work in frequency units. For the measured shifts from the spectrum, you can obtain the shifts in terms of frequency by multiplying the FSR units by the free spectral range in Hertz. For  $B\mu_B$  you can use the fact that

$$\mu_B = 1.39962 \times 10^{10} \text{ Hz/T.} \quad (23)$$

This will give both sides of Eq. (22) in units of frequency. By dividing through with  $B\mu_B$  you can compare to the simple numbers predicted for the  $g$  and  $m$  values for the transition. Make a table containing the predicted ( $g_i m_i - g_f m_f$ ) versus the measured ones from this analysis.

The same method will work for the yellow lines. However be advised that you will need to create two separate calibrations for the two different sets of peaks. Even though the scan includes both lines, the central peaks are in different locations.



In this way (or another of your own devising) determine your measured values of  $\Delta E/(B\mu_B)$  for each line. Compare to the values you worked out in Exercises 2 and 3.

If you followed the prescription for calculating the predicted spectra, you should have found that the  $^3D_2 \rightarrow ^1P_1$  (576.96 nm) yellow transition should give a more complicated spectrum than what is observed. Can you explain this discrepancy? What would you do, assuming you could go back to the experiment, to try to resolve it?

## References

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## Appendix: Fabry-Perot interferometry

### REFERENCE

Hecht, *Optics (4th ed.)*, Addison Wesley, San Francisco, 2002, pp. 414–425.

To effectively use a Fabry-Perot (FP) interferometer as a spectroscopic tool, you need to understand the concepts of **fineness** and **free spectral range**. This section presents these concepts in a descriptive form with emphasis on the underlying physics; see Hecht, pages 416–425, for the mathematical details.

The Fabry-Perot interferometer uses the phenomenon of multiple beam interference that arises when light shines through a cavity bounded by two reflective parallel surfaces. Each time the light encounters one of the surfaces, a portion of it passes through to the other side, and the remainder is reflected back; the net effect is to break a single beam into multiple beams which interfere with each other. If the additional optical path length traveled by a (multiply) reflected beam is an integer multiple of the light's wavelength, then the reflected beams will interfere constructively. Conceptually, the multiple reflections in the cavity interfere with each other in the same way that beams from a multiple-slit grating do: as the number of beams (or reflections) becomes higher, the interference maxima become sharper.

With this in mind, consider the arrangement shown in Fig. 12. The He-Ne laser produces a (nearly) monochromatic beam of light. The beam passes through the microscope objective which causes the beam to diverge. The diverging rays encounter the FP cavity where they undergo multiple reflections, and the light leaving the cavity creates an interference pattern on the screen. The additional phase  $\delta$  acquired by a beam reflecting once from each mirror surface before leaving the cavity depends on the mirror spacing  $d$ , the wavelength  $\lambda$  and the angle from the optical axis  $\theta$

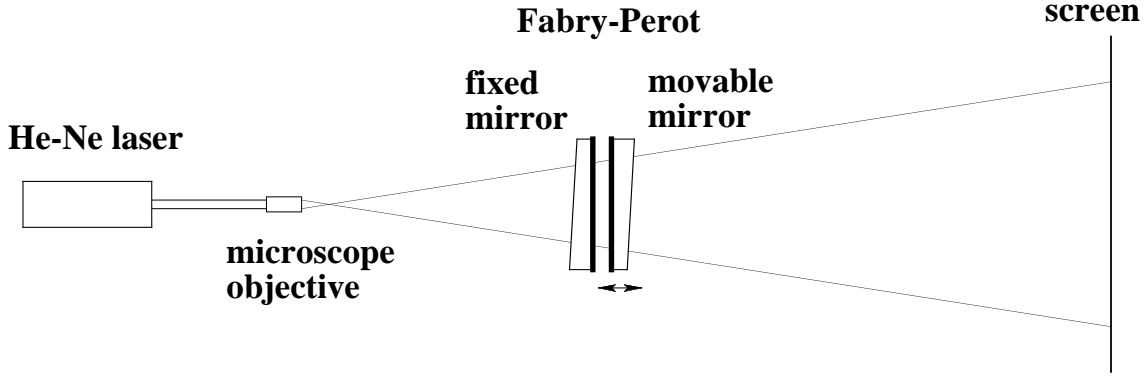


Figure 12: Basic setup for Fabry-Perot interferometry.

(Hecht, p. 422.):

$$\delta = \frac{4\pi d \cos \theta}{\lambda} . \quad (24)$$

Under the ideal conditions of no absorption (all of the light either transmitted or reflected), perfect mirror flatness, and no diffraction limiting, the intensity of the light transmitted by the cavity  $I_t$  will follow the mathematical form of an Airy function:

$$I_t \propto \mathcal{A}(\theta) \equiv \frac{1}{1 + F \sin^2(\delta/2)} . \quad (25)$$

(Hecht, pp. 416–420.) The quantity  $F$  is a parameter given the somewhat confusing name *coefficient of finesse* (it is not quite the same as “the *finesse*”, but they are related, see below). If you examine Eq. (25) a moment, you can see that the Airy function equals 1 whenever  $\delta/2$  is a multiple of  $\pi$ ; otherwise the function is less than one. When the coefficient of finesse  $F$  is large, the function is sharply peaked at those multiples of  $\pi$ . The size of  $F$  increases with the number of reflections between the two mirrors, that is, if the mirrors have reflectance  $R$ , then  $F = 4R/(1 - R)^2$ .

The dependence of  $\delta$  on  $\theta$ , along with the axial symmetry of the setup, produces a ring pattern on the screen. Each ring in the pattern corresponds to a change in the optical path length by one wavelength of light, or one *order*. If the mirror spacing  $d$  were such that exactly  $m$  half wavelengths of light fit between them, then along the axis ( $\theta = 0$ ) the phase parameter  $\delta$  would equal  $2m\pi$ , and one would see a bright spot at the center of the screen. This spot would correspond to order  $m$ . As  $\theta$  increases from 0,  $\delta$  decreases (since  $\cos \theta$  decreases), and at a certain angle, say  $\theta_1$ , the phase would have decreased by  $2\pi$  to become  $2(m - 1)\pi$ , and one would see a bright ring, corresponding to order  $m - 1$ . More generally, the order of the  $p$ th ring out from the center has order  $m - p$ . A small-angle approximation for  $\cos \theta$  gives the angular location  $\theta_p$  as

$$\theta_p^2 = \left( \frac{r}{L} \right)^2 = 2 - (m - p) \frac{\lambda}{d} , \quad (26)$$

where  $r$  is the radius of the ring on the screen and  $L$  is the distance between the screen and the focal point of the microscope objective.

The usual way to use an FP to make measurements is to vary the spacing between the mirrors in a controlled manner while measuring the light intensity at the very center of the ring pattern. This technique is called *central spot scanning*. For monochromatic light, the transmission will peak once

every time the distance between the mirrors moves by  $\lambda/2$ : in central spot scanning,  $\theta = 0$  and  $\delta/2 = 2\pi d/\lambda$ , so a change in  $d$  by a half wavelength changes  $\delta$  by  $2\pi$ . Since  $d$  is proportional to  $\delta$ , a steady change in  $d$  will produce a graph of evenly spaced peaks.

### *Instrument Finesse*

The sharpness of the peaks in a central spot scan depends on a number of physical conditions: the reflectivity of the mirrors, the flatness of the mirrors, diffraction from apertures in the optical path, and the angular acceptance of the central-spot detector. A practical measure of the peak sharpness from a real instrument is called the *instrument finesse* or simply *finesse*. It is defined as follows:

$$\text{Finesse} \equiv \mathcal{F} = \frac{\text{Peak separation}}{\text{Peak width}} . \quad (27)$$

In the ideal conditions that would allow a real instrument to obey Eq. (25), one would obtain a finesse dependent only on the mirror reflectivity. This is called *mirror finesse*  $\mathcal{F}_m$ , and a simple calculation shows that it depends on the parameter  $F$ , and thus, the mirror reflectance  $R$ :

$$\mathcal{F}_m = \frac{\pi}{2} \sqrt{F} = \frac{\pi \sqrt{R}}{1 - R} . \quad (28)$$

To show this, simply find the change in  $\delta$ ,  $\Delta\delta_m$  that allows  $\mathcal{A}(\theta)$  to drop by  $1/2$  compared to the value of  $\delta$  that separates two peaks, i.e.,  $\delta = 2\pi$ .

A real instrument has a finesse typically much lower than this limit. To estimate the contribution of a couple of other imperfections, assume for a moment that the mirror finesse is infinite, that is, for a monochromatic source, a central-spot scan would produce delta-function peaks at every point that  $d$  equals a half-wavelength multiple.

If the mirror surfaces are not exactly flat over the aperture that contains the light beam, then the phase of the light reflected between the mirrors will also vary—our hypothetical delta function at  $d = m\lambda/2$  would be smeared out over a variation in mirror separation  $\Delta d$ , which would produce a variation in phase  $\Delta\delta_f = 4\pi\Delta d/\lambda$ . The quality of mirror flatness is usually given in terms of a “mirror figure”  $M$  defined according to the fraction of a wavelength that the surface departs from flatness, that is,  $\Delta d = \lambda/M$ . This would make the phase variation  $\Delta\delta_f = 4\pi/M$ , and since the phase change between peaks is  $2\pi$ , the so-called *figure finesse*  $\mathcal{F}_f$  would be

$$\mathcal{F}_f = \frac{M}{2} . \quad (29)$$

The mirrors in our student-grade FP setup have a flatness of  $\lambda/20$  over their entire surface. However, this number should be taken as a worst-case value since the area of the mirrors which produce the central spot is considerably smaller than the size of the mirror itself.

Now return, mentally, to our ideal FP set with an infinite finesse, but imagine that our detector has an pinhole aperture of diameter  $D$ . If we start with the condition that a spot is present at the center of the detector, the phase parameter  $\delta = 4\pi d/\lambda$ , and we will record a peak in transmission from our detector. Now let the mirror separation increase. The spot will grow into a sharp ring, but the detector will continue to record a peak until this ring reaches the edge of the detector pinhole. At this point, the phase parameter has changed by an amount  $\Delta\delta_p$  so that  $\delta + \Delta\delta_p = (4\pi d/\lambda)[1 - \theta^2/2]$  (using the small-angle approximation for  $\cos \theta$ ), consequently the variation in phase that still allows

a peak to be seen is  $\Delta\delta_p = 2\pi d\theta^2$  where, in this case,  $\theta = D/(2L)$ , the angular acceptance of the pinhole. Thus, the *pinhole finesse*  $\mathcal{F}_p$  is given by

$$\mathcal{F}_p = \frac{4\lambda L^2}{D^2 d}. \quad (30)$$

In order to combine the contributions of each effect into a single estimate for the instrument finesse, note that the consequence of a finite finesse is to introduce an error  $\Delta\delta$  into the phase parameter  $\delta$ . Since each effect (pinhole size, mirror flatness, and mirror reflectivity) is independent of the other, it is reasonable as a first approximation to add the errors due to each effect in *quadrature*, that is, the total fractional error in  $\delta$ ,  $\Delta\delta/\delta$  is found by assuming  $(\Delta\delta/\delta)^2 = (\Delta\delta_m/\delta)^2 + (\Delta\delta_f/\delta)^2 + (\Delta\delta_p/\delta)^2$ , or in terms of finesse,

$$\frac{1}{\mathcal{F}^2} = \frac{1}{\mathcal{F}_m^2} + \frac{1}{\mathcal{F}_f^2} + \frac{1}{\mathcal{F}_p^2}. \quad (31)$$

### ***Free Spectral Range***

Up to this point the discussion has assumed that the light entering the FP cavity is perfectly monochromatic. Now consider what you would see if light of two different colors were used. For each wavelength, you would see a different spacing of the rings in the ring pattern, corresponding to the different values of  $\lambda$  in Eqs. (24) and (26), for a given mirror separation  $d$ . For example, the spacing between rings of red light would be larger than the spacing between rings of green light since the longer wavelength of red light would require  $\cos\theta$  to change more in order to have the same change of  $\delta$  in Eq. (24).

Now imagine that the two colors are very close to each other, for example, two slightly different shades of green. The ring pattern would be composed of “doublets”: pairs of closely-spaced rings. Which ring in a given pair corresponds to which wavelength? The answer is a bit counter-intuitive, given that the longer-wavelength light must produce more widely-spaced rings. The *larger* diameter ring in a doublet corresponds to the *shorter* wavelength light. To see why this is true, consider how  $\delta$  changes as  $\lambda$  changes, with  $d$  and  $\theta$  held constant. As  $\lambda$  shrinks,  $\delta$  must grow, and this pushes all of the rings *outward*. Moreover, as the rings themselves become larger, the spacing between successive rings shrinks; one can see this by taking differentials of Eq. (26). Let  $\Delta\theta_p$  be the difference in angles between two rings. From Eq. (26), we get

$$\Delta\theta_p = \frac{\lambda}{2d\theta_p} \Delta p. \quad (32)$$

So for larger rings (larger  $\theta_p$ ), the ring spacing  $\Delta\theta_p$  is smaller.

In a central-spot scan, with  $d$  increasing, the rings would expand from the center, and the shorter-wavelength light would make a peak a bit before the longer-wavelength light would, as the next order of the shorter-wavelength light would come into the pattern before the corresponding order of the longer-wavelength light. In a very long scan, however, the peaks would separate (since the spacing between the longer-wavelength rings must be greater), until one would see the the shorter-wavelength peak coming *before* the longer-wavelength peak; indeed, the two peaks would eventually overlap! What is happening in this case is that the *orders* of the two peaks at a particular separation  $d$  are different: the  $m$ th order of the longer-wavelength light is making a peak at the same time as the  $(m+1)$ th order of the shorter-wavelength light.

If the scan were run in the opposite direction, with  $d$  decreasing, the rings would collapse toward the center. In this case, the longer-wavelength peak would be recorded before the shorter-wavelength peak. In a long scan the peaks would get closer together, but they would overlap completely only at  $d = 0$ . However, because the finesse is not infinite in a real instrument, one would record a single peak well before zero separation.

One may ask the question: what wavelength difference  $\Delta\lambda$  would cause the “order mixing” to occur at a particular separation  $d$ ? The answer to this question is called the *free spectral range* of the FP cavity.

Mathematical derivations of the free spectral range are given in Hecht and other texts. Here is a derivation based on a mental picture. What we are after is the change in wavelength  $\Delta\lambda$  necessary to make the  $m$ th order of the longer-wavelength light happen at the same separation  $d$  as the  $(m + 1)$ th order of the shorter-wavelength light. Assume for a moment that  $d$  is such that only one-half wavelength of the longer- $\lambda$  light fits between the mirrors; in other words  $m = 1$ . The wavelength would have to shrink by a factor of 2 to fit the next order of the shorter wavelength in the same space:  $\Delta\lambda$  would equal  $\lambda/2$ . See the upper picture in Fig. 13.

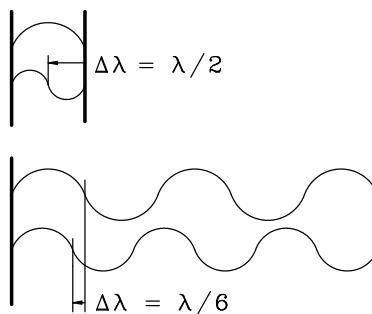


Figure 13: Illustration showing that the free spectral range  $\Delta\lambda_{FSR}$  decreases with increasing mirror separation.

As the separation  $d$  is increased, the wavelength change needed to fit the next order between the mirrors decreases; note that the change needed to go from  $m = 5$  to  $m = 6$  in Fig. 13 is about  $\lambda/6$ . Typically the mirror separation  $d$  is much larger than  $\lambda$ —many orders fit between the mirrors. One can see that for large  $m$ ,  $\Delta\lambda = \lambda/m$  (since there’s not that much difference between  $m$  and  $m + 1$ ). Because there are  $m$  half-wavelengths which fit between the mirrors,  $m = d/(\lambda/2)$ , so in terms of  $d$ , the free spectral range is

$$\Delta\lambda_{FSR} = \frac{\lambda^2}{2d} . \quad (33)$$

In many instances, the free spectral range is given in terms of the change in frequency of the light, rather than the change in wavelength. It is easy to show by treating  $\Delta\lambda$  as a differential and using the well-worn relation  $\nu = c/\lambda$  that

$$\Delta\nu_{FSR} = \frac{c}{2d} , \quad (34)$$

which has the advantage that the free spectral range can be given without specifying a particular wavelength  $\lambda$ .

Knowledge of the free spectral range allows one to use a central spot scan to measure wavelength shifts in a spectroscopic measurement. For example, in the Zeeman effect, the application of a magnetic field to a gas of atoms emitting light causes the emission lines to separate into components

of slightly different wavelengths. The amount of wavelength separation is proportional to the applied field. If the light is passed through an FP cavity, these separations can be resolved into different peaks. The distance between *different-wavelength* peaks in a scan is directly proportional to the free spectral range. Since, by definition, the wavelength change of an amount *equal* to the free spectral range is given by the distance between the peaks of two successive orders of the same wavelength, one can use this distance, along with the free spectral range calculated from the known value of  $d$  to make a conversion factor.

For example, suppose the mirror separation is 4 cm. By Eq. (34),  $\Delta\nu_{FSR} = 3.74$  GHz for this separation. A scan is made of some light source, and it is found that the distance between peaks of the *same wavelength* is 6 cm, and that two *different-wavelength* peaks are separated by 2 cm. The conversion factor would be  $(3.74 \text{ GHz})/(6 \text{ cm}) = 0.623 \text{ GHz/cm}$ . So the frequency separation between the two different-wavelength peaks would be 1.25 GHz.

Two final notes on the use of FPs for spectroscopy: First, the minimum resolvable difference in wavelengths depends, of course, on the instrument finesse. You can estimate this by using the free spectral range and the width of the (monochromatic) peaks in your scan. Second, watch out for peaks which appear to move a lot with respect to the others as the scan runs from order to order; such peaks may have significantly different wavelengths than the one used for calibration of your scan. The estimate of wavelength differences from fractions of the free spectral range is only valid when those differences are indeed less than the free spectral range.

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zeeman\_effect\_2018.tex -- Updated 7 May 2021