Physics 505 - Autumn 2005

HW VII

Due 11/16/05

Overview: Recall that solving physics problems is not (just) about solving differential equations. Use physical reasoning to help solve the following exercises and be certain to show your work. It is also important that you practice completely solving these exercises, checking for errors as you go along.

1) Let us take one more look at our favorite problem of the rotating hoop, Fetter & Walecka – 3.1 and 4.4. Write the Lagrangian *and* the Hamiltonian in terms of the canonical variables θ, p_θ. Also construct the effective potential as in

$$H = \frac{p_\theta^2}{2ma^2} + U_{eff}\left(\theta\right),$$

taking gravity to be the only external force (other than constraint forces). In terms of Hamilton's equations, *i.e.*, think of the flow in phase space, recalling the velocity field discussed in Lecture 7, again locate all equilibria and determine their stability, permitting Ω to vary. Observe that the Hamiltonian has a certain reflection symmetry (*i.e.*, think about the symmetry with respect to the point $\theta =$ 0). Show that for $\Omega < \omega_\theta = \sqrt{g/a}$, where ω_θ is the critical rotation speed, there is only one stable equilibrium point and it exhibits the reflection symmetry of the Hamiltonian. However, when $\Omega > \omega_\theta$, there are two stable equilibrium points and they 'break' the symmetry. To illustrate the symmetry make a sketch of the effective potential ($U_{eff}\left(\theta\right)\text{vs}\,\theta$) in each case, where we allow plus and minus values of θ. Sketch the phase portraits, *i.e.*, the flow patterns in (θ, p_θ) phase space, for both $\Omega < \omega_\theta$ and $\Omega > \omega_\theta$. Plot the equilibrium solutions, *i.e.*, the value of θ_0, as a function of Ω. Sketch stable solutions as solid curves, unstable ones as dashed curves, and observe that the diagram has the form of a pitchfork. (This is an example of a symmetry-breaking pitchfork bifurcation and is analogous to the "spontaneous symmetry breaking" that occurs in the Higgs Phenomenon of particle physics fame.)

2) Fetter & Walecka – 5.3 We want to practice analyzing rigid body motion.

3) As another rigid body analysis (with clear application to your spare time pursuits) we want to work through the problem illustrated in Fig. 28.3 in Fetter & Walecka. A homogeneous billiard ball of mass M and radius a is struck impulsively with a horizontal force (*i.e.*, a cue stick), with the force applied a distance h above the center of the ball. The subsequent motion is determined by the force of friction that retards motion, with the frictional acceleration given in units of g (the gravitational acceleration),

$$\ddot{x} = -\mu g .$$

Here μ is assumed to be constant (independent of the ball's velocity). Let v_0 be the ball's initial velocity (after been struck). For h positive (as in Fig. 28.3), but otherwise arbitrary, determine the time and velocity at which the ball begins to roll without sliding. As noted in F&W, there are 3 distinct regimes in h to consider.

4) As practice with Euler consider the following system. A uniform right circular cone of height h, half angle α, and density ρ rolls on its side without slipping on a uniform horizontal plane in such a way that it returns to its original position in a time τ. Find expressions for the kinetic energy and the components of the angular momentum of the cone.