Physics 505 - Autumn 2010

HW III

Due 10/20/10

Overview: Recall that solving physics problems is not (just) about solving differential equations. Use physical reasoning to solve the following exercises and be certain to show your work. Note: the last 3 exercises in this HW set are quite similar to Classical Mechanics exercises in past Qualifying Exams!

1) Fetter & Walecka - 2.5 (6 pts) We want to exercise our understanding of motion in a rotating frame by considering how the rotation of the earth changes the apparent trajectory of cannon shells. Note that we simplify the analysis by expanding in powers of the rotation frequency ω and keeping only the first power. Strictly the dimensionless small parameter is the angular frequency ω times the characteristic time interval of the problem.

2) Fetter & Walecka – 2.7 (6 pts) In this exercise we study how the rotation of the earth perturbs the effective gravitational potential near the earth and hence its shape.

3) (4 pts) To further strengthen our understanding of rotating reference frames we should analyze the case of a pendulum in such a frame. In particular, we can use the results of Chapter 2, Section 12 to study the Foucault Pendulum in building A. Take the pendulum bob to have mass m and the massless support wire to have length l. Determine the oscillation frequency of the pendulum and the rotation rate (angular velocity) of the plane of this oscillation as observed in the rotating frame fixed to the surface of the earth at polar angle (colatitude) θ. How long does it take the plane of the pendulum in A wing to rotate through 180°? [Assume that the displacement angle of the pendulum is small enough that a linear analysis is applicable.]

4) (3 pts) As a first practice problem with minimization concepts and the calculus of variations consider a cord of indefinite length that passes freely over pulleys at heights y_1 and y_2 above the plane surface of the earth, with a horizontal distance $x_2 - x_1$ between them. If the cord has uniform linear mass density, find the differential equation that describes the cord's shape, and solve it for that shape.

How does the analysis and shape change if the cord is of fixed length, $l > \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$? Your solution will contain arbitrary constants to fit to the given endpoints. You need not evaluate these constants unless you want to. Note: We really should describe the cord as a “chain” (large mass per length) so that in Latin it is related to the word catenary. This is the label used for the classic version of this problem, which has web sites dedicated to it (try Googling it)!

5) Fetter & Walecka – 3.1 (6 pts) This is an exercise that illustrates Lagrangian methods for a system with a (trivial) constraint (confined to motion on a loop) and rotating coordinates. It also introduces us to using small (linearized) perturbations to study stability. Note that small perturbations about a stable equilibrium yield oscillatory (harmonic oscillator-like) behavior.