Physics 505 - Autumn 2010

HW VI

Due 11/10/10

Overview: Recall that solving physics problems is not (just) about solving differential equations. Use physical reasoning to help solve the following exercises and be certain to show your work. It is also important that you practice completely solving these exercises, checking for errors as you go along.

1) Fetter & Walecka – 4.13 (4 pts) This is an exercise in the transition to systems with large numbers of normal modes.

2) Fetter & Walecka – 4.16 (6 pts) Here we get the opportunity to consider what happens to normal modes in 2 spatial dimensions and in the continuum limit.

3) (8 pts) Let us take one more look at our favorite problem of the rotating hoop, Fetter & Walecka – 3.1 and 4.4. Write the Lagrangian *and* the Hamiltonian in terms of the canonical variables θ, p_θ. Also construct the effective potential as in

$$H = \frac{p_\theta^2}{2ma^2} + U_{eff}(\theta),$$

taking gravity to be the only external force (other than constraint forces). In terms of Hamilton's equations, *i.e.*, think of the flow in phase space, recalling the velocity field discussed in Lecture 7, again locate all equilibria and determine their stability, permitting Ω to vary. Observe that the Hamiltonian has a certain reflection symmetry (*i.e.*, think about the symmetry with respect to the point $\theta =$ 0). Show that for $\Omega < \omega_\theta = \sqrt{g/a}$, where ω_θ is the critical rotation speed, there is only one stable equilibrium point and it exhibits the reflection symmetry of the Hamiltonian. However, when $\Omega > \omega_\theta$, there are two stable equilibrium points and they 'break' the symmetry. To illustrate the symmetry make a sketch of the effective potential ($U_{eff}(\theta)$ vs θ) in each case, where we allow plus and minus values of θ. Sketch the phase portraits, *i.e.*, the flow patterns in (θ, p_θ) phase space, for both $\Omega < \omega_\theta$ and $\Omega > \omega_\theta$. Plot the equilibrium solutions, *i.e.*, the value of θ_0, as a function of Ω. Sketch stable solutions as solid curves, unstable ones as dashed curves, and observe that the diagram has the form of a pitchfork. (This is an example of a symmetry-breaking pitchfork bifurcation and is analogous to the "spontaneous symmetry breaking" that occurs in the Higgs Phenomenon of particle physics fame.)