Physics 505 - Autumn 2010

HW VIII

Due 11/24/10

Overview: Recall that solving physics problems is not (just) about solving differential equations. Use physical reasoning to help solve the following exercises and be certain to show your work. It is also important that you practice completely solving these exercises, checking for errors as you go along.

1) Fetter & Walecka – 6.6 (7 pts) Here we apply Hamiltonian techniques to the familiar problem of the harmonic oscillator.

2) Fetter & Walecka – 6.9 (4 pts) Here we get to practice with the Hamilton-Jacobi equation.

3) (10 pts) We want to try using the numerical methods outlined in the attached notes to solve first or second order differential equations. In particular, we recall the ladder problem from exercise 3 in HW IV (F&W 3.18). While we were able to find certain features of the motion of the ladder analytically, we did not find the explicit motion. For example, in the latter part of that problem we obtained a fairly complicated expression for the time derivative of the angle of the ladder versus time after the ladder lost contact with the wall (at $\theta = \theta_{off}$), where the initial angle is $\theta = \theta_0$,

$$\frac{d\theta}{dt} = -\sqrt{\frac{\frac{12g}{L}\left[\frac{2E}{MgL} - \sin\theta\right]}{1+3\cos^2\theta}},$$

$$E = \frac{gML}{2}\sin\theta_0 - \frac{gML}{18}\sin^3\theta_0.$$

Now we want to solve this problem more fully using numerical methods.

a) (5 pts) Consider a ladder of length L = 3 meters and take the acceleration of gravity to be g = 9.8 m/s^2. Assume that the initial angle of the ladder with respect to the floor is $\theta_0 = 60°$ (before the ladder starts to fall). Using numerical methods find the orientation angle θ as a function of time. If you use NDSolve in *Mathematica*, I find it best to solve the original second order differential equation. If you use the Runge-Kutta method outlined in the notes, you should focus on the first order equation we found in our original analysis using the conservation of energy. Using your numerical solution for the angle (and our previous analytic analysis of the constraint forces) determine the time and the angle, $t = t_{off}, \theta = \theta_{off}$, when the ladder loses contact with the wall, *i.e.*, when the constraint force due to the wall vanishes.

b) (5 pts) The subsequent motion is described slightly differently (as there is no longer a force from the wall and the upper end of the ladder no longer touches the wall). Numerically determine the subsequent motion, *i.e.*, find $\theta(t)$ using $\theta = \theta_{off}$ (and possibly $d\theta/dt$ at t_{off}) as the relevant initial condition, until the time when the ladder hits the floor, $\theta = 0$. To fully specify the subsequent motion you need to also specific the motion of the CM of the ladder. Also find the time variation of the constraint force of the floor, (recall)

$$\lambda_f = \frac{Mg}{\left(4 - 3\sin^2\theta\right)^2}\left[4 + 3\sin^2\theta - 12\frac{E}{MgL}\sin\theta\right],$$

and verify that it never goes to zero.

To perform the numerical analysis you can use either *Mathematica* or the (4th order) Runge-Kutta method as outlined in the attached notes. Your results can be presented either as a printed copy of your *Mathematica* notebook or as a table of $\theta(t)$ on an evenly spaced grid of time values for $\theta_0 \geq \theta \geq 0$ (some trial & error may be required to find an appropriate time step size and number of steps). Make plots (sketches) of both $\theta(t)$ and $\lambda_f(t)$ versus *t*. You are strongly encouraged to use *Mathematica* to perform this analysis, which will also happily do the plots for you. Finally you are encouraged to make an animation of the motion of the ladder like the one that appears on our web page.