Lecture I: Broken symmetry, quasi-particles & collective modes

Before leaping into condensed matter phenomena, it is perhaps useful to develop some perspective on the subject in general. In this the first lecture, I would like to discuss some of the guiding principles of condensed matter physics. Most of the thoughts are not original and, therefore, perhaps they are useful!

▷ What are the guiding principles of condensed matter physics?[1]

actually, high-energy physics too!

- "Adiabatic continuity" ⇝
- Concept of quasi-particles and collective modes;
- Importance of (broken) symmetry
- and *(perhaps most importantly)* Universality

To understand why, let us contemplate a...

▷ "Theory of Everything" (on Earth) — TOE

All matter can be described Schrödinger equation

$$i\hbar\partial_t\psi = \hat{H}\psi$$

involving a many-body Hamiltonian of electrons and ions

$$\hat{H} = \overbrace{\sum_{i}^{N}\frac{\hat{\mathbf{p}}_i^2}{2m_{\mathrm{e}}} + \sum_{i<j}^{N}\frac{e^2}{|\mathbf{r}_i-\mathbf{r}_j|}}^{\hat{H}_{\mathrm{e}}} + \overbrace{\sum_{I}^{M}\frac{\hat{\mathbf{P}}_I^2}{2m_{\mathrm{i}}} + \sum_{I<J}^{N}\frac{(Ze)^2}{|\mathbf{R}_I-\mathbf{R}_J|}}^{\hat{H}_{\mathrm{i}}} - \sum_{i<J}^{N,M}\frac{Ze^2}{|\mathbf{r}_i-\mathbf{R}_J|}$$

(well, minus the electrodynamic field...)

In principle, $\hat{H}$ describes metals, insulators, semi-conductors,
and everything else besides! — it as precise as it is useless

How, then, can one explore theoretically the behaviour of a many-body system...?

▷ Adiabatic continuity and the quasi-particle concept

The development of condensed matter physics has, to a large extent,
hinged on the "unreasonable" success of non-interacting theories...

viz. free electron theory of metals, Debye theory of solids, etc.

...yet perturbation affected by interactions (and the host environment)
are rarely small (see TOE) *cf. metals...*

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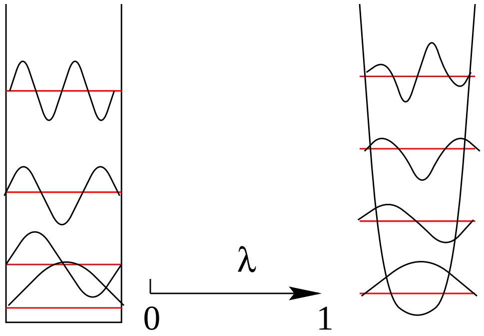
[1] terminology of "condensed matter" attributed to Anderson and Heine(?) — (a) indicates a broader remit than "solid state", (b) reference to universality implicit, and (c) resonates with funding agencies...

Apparent impotency of interactions attributed to principle of adiabatic continuity:

- The quantum numbers ($\mathbf{p}, \sigma$, etc.) that characterise a many-body system are determined by the fundamental symmetries (translation, rotation, particle exchange, etc.) — much more robust than "material parameters"

- While symmetry is maintained, the elementary excitations of an interacting system can be traced back "adiabatically" to those of the bare particle excitations in the non-interacting system, *cf. single-particle states in a quantum well*



- Elementary excitations — quasi-particles — of interacting system mirror excitations of non-interacting system

▷ "Quasi-particle correspondence is embodied in Landau's Fermi-liquid theory (see next lecture):

In the non-interacting system, a single electron is indexed by momentum $\mathbf{p}$ and spin σ. In the interacting system, the passage of electron is impaired by other particles.

Yet, while the interaction remains weak, even though the bare particles may become strongly "dressed" by their interaction with the particles in the background, the elementary excitations of the system can be classified by "quasi-particles" which share the same quantum numbers ($\mathbf{p}$, σ) as the bare particles

More formally, the non-interacting system provides a reference state from which the true ground state of the system can be inferred from perturbation theory (albeit of, perhaps, infinite order!) Indeed, radius of convergence of perturbation theory extends beyond region where perturbation is small

▷ Importance of (broken) symmetry

However, being contingent on symmetry, principle of adiabatic continuity and, with it, the quasi-particle correspondence, must be abandoned (or, at least, revised) at a phase transition — here, interactions affect a substantial rearrangement of the many-body ground state.

In the symmetry broken phase, a system may — and frequently does — exhibit elementary excitations very different from those of the non-interacting phase

These elementary excitations may be classified as new species of quasi-particles with their own characteristic quantum numbers or they may represent a new kind of excitation — a collective mode — engaging the collective motion of many bare particles

e.g. when ions or electrons condense from a liquid into a solid phase, translational symmetry is broken and the elementary excitations — phonons — involve the collective motion of many individual bare particles

e.g. in the fractional Hall liquid, the elementary quasi-particle excitations, a composite of many individual bare electron degrees of freedom, carry fractional charge and exhibit fractional statistics!

▷ Universality

Phenomenology above lends itself to a heirarchical perspective of condensed matter physics: each phase is associated with its own "non-interacting" reference ground state with its own characteristic quasi-particle excitations — a product of the fundamental symmetries that classify the phase.

Providing one stays within a given phase, one can draw on the principle of adiabatic continuity to infer the (usually benign) influence of interactions from e.g. phenomenology (as with FLT) or perturbation theory

▷ Yet, the heirarchical picture brings two further profound implications:

- Firstly, within quasi-particle picture, the underlying "bare" or elementary particles remain invisible;[2] (cf. the fractional Hall fluid)
- secondly, while the capacity to conceive of new types of interactions is almost unbound, the freedom to identify free (i.e. non-interacting) theories is strongly limited, and constrained by the space of fundamental symmetries, i.e. drawing on the principle of continuity, one can therefore anticipate a substantial degree of "universality" in properties in low-energy properties of condensed matter.

  e.g. photons, phonons, and antiferromagnetic spin waves are all classifed by the same (relativistic) low-energy theory

  For many, this unification/classification is the primary goal of condensed matter physics

All branches of condensed matter physics, quantum or classical, hard or soft, benefit from this heirarchy.

▷ Therefore, as condensed matter theorists, what is our goal? And what is the aim of these graduate lectures?

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[2] To quote from P. W. Anderson's now-famous article *More is different*, (Science **177**, 393 (1972)), "the ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe"

- Condensed matter "ab initio": although a symmetry-based classification of physics is conceptually appealing, sometimes the devil lies not in phenomenology, but in "detail" — canonical behaviours may be contingent on chemisty *(e.g. is Yb intercalcated graphite a superconductor? The phenomenology of a superconductor wont help answering this question!)*

  In these cases, the concept of adiabatic continuity often provides a means to justify density functional theory schemes to explore the TOE Hamiltonian

- Condensed matter "the Russian way": (a) isolate the "canonical" Hamiltonian of the quantum system (i.e. the simplest one that captures the essential physics); (b) apply methods of quantum field theory or diagrammatic perturbation theory to uncover the low-energy structure — foolproof but not "fool proof"!

- Condensed matter "the lazy way": Contingent on the concept of universality is the (perhaps miserable) realisation that there is (almost) "nothing new under the sun". The majority of important low-energy or "quasi-particle" theories have been identified, classified, and "canonised" in the literature.

  The ingenuity of the condensed matter physicist lies, in part, in recognising and thereby profitting from unexpected connections (or mappings) between seemingly different physics systems *(e.g. the illuminating analogy that exists between polymer dynamics and particle physics identified and harvested by Sam Edwards and others!)*

Our sights this term are set on the "lazy" — we will focus on phenomenology classifying many different themes within quantum condensed matter:

- Broken symmetry, quasi-particles & collective phenomena [1]
- Landau's Fermi-Liquid theory [1]
- Strong correlation & the Mott transition [1]
- Quantum magnetism [1]
- Bogoliubov theory of weakly interacting Bose gas [1]
- Correlated system in one-dimensional & Luttinger Liquid theory [2]
- Electron-phonon interaction & the BCS theory of superconductivity [2]
- Atomic condensation phenomena [4]
- Anderson localisation & correlated classical insulators [2] ($\rightsquigarrow$ Lent)
- Quantum Hall effects [2] ($\rightsquigarrow$ Lent)

Next term, we will attempt a more technical synthesis focusing on applications of quantum field theory in condensed matter

▷ Reading List:

P. W. Anderson, *More is Different*, Science **177**, 393 (1972)

R. B. Laughlin and D. Pines, *The Theory of Everything*, Proc. Nat. Acad. of Sci. **97**, 28 (2000). `http://large.stanford.edu/rbl/essays/p01apr99.htm`

P. W. Anderson, *Basic Notions of Condensed Matter Physics*, (Benjamin, Menlo Park, CA, 1976).