Psych 315, Winter 2021, Homework 5 Answer Key

Due Friday, February 5th by 5pm.

Name ____ ID ______________________________

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Problem 1 Suppose you flip a fair coin 20 times.

a) Use the Excel spreadsheet table B (or table B cumulative) to find the probability of obtaining 13 or more heads.

Using table B with P = 0.5, n = 20, and k = 13:

Pr(13 heads) = 0.0739
Pr(14 heads) = 0.0370
Pr(15 heads) = 0.0148
Pr(16 heads) = 0.0046
Pr(17 heads) = 0.0011
Pr(18 heads) = 0.0002
Pr(19 heads) = 0.0000
Pr(20 heads) = 0.0000

So Pr(13 or more heads) = 0.0739 + 0.037 + 0.0148 + 0.0046 + 0.0011 + 0.0002 + 0 + 0 = 0.1316

b) Use R's function 'binom.test' to find the same answer. Hint: there are lots of examples in the binomial distribution tutorial. You can just write the proper command below

```
out<-binom.test(13,20,0.5,"greater")
print(out$p.value)
[1] 0.131588
```

c) Use the normal approximation to find the mean and standard deviation of the number of heads you'd expect from 20 coin flips.

μ = nP = (20)(0.5)= 10

$\sigma = \sqrt{nP(1-P)} = \sqrt{(20)(0.5)(0.5)} = 2.24$

d) What is the probability of obtaining 13 or more heads out of 20 flips based on this normal approximation? (hint: use table A to find Pr(X>12.5). Do you get a similar key as for problem 1a?

$z = \frac{X-\mu}{\sigma} = \frac{12.5-10}{2.24} = 1.12$

$Pr(z > 1.12) = 0.1314$

0.1316 from problem 1a is close to 0.1314 from problem 1c

Problem 2 Suppose the probability of funding for any grant that you send to NIH is 10%. To increase the probability of getting funded, you submit 5 separate grant proposals. What is the probability that at least 1 will get funded? You can either use the table or R.

Using table B with P = 0.1, n = 5, and k = 1:

Pr(1 getting funded) = 0.3281
Pr(2 getting funded) = 0.0729
Pr(3 getting funded) = 0.0081
Pr(4 getting funded) = 0.0005
Pr(5 getting funded) = 0.0000

So Pr(1 or more getting funded) = 0.3281 + 0.0729 + 0.0081 + 0.0005 + 0 = 0.4096

```
out<-binom.test(1,5,0.1,"greater")
print(out$p.value)
[1] 0.40951
```

Problem 3 Suppose the population body mass index (BMI) for U.S. men has a mean of $\mu_x = 24$ and a standard deviation of $\sigma_x = 2.25$. Suppose also that this population is normally distributed.

a) What is the probability that a **single sample** will be greater than 24.5?

$z = \frac{X-\mu_x}{\sigma_x} = \frac{24.5-24}{2.25} = 0.22$

$Pr(z > 0.22) = 0.4129$

b) Consider what happens when you take 100 samples from this distribution and calculate the mean. This mean is sampled from its own distribution - the 'sampling distribution of the mean'. What is is the approximate shape of this sampling distribution of the mean?

With a sample size of 100, the sampling distribution of the mean should be approximately normal according to the Central Limit Theorem

c) What is the expected mean of this sampling distribution of the mean ($\mu_{\bar{x}}$)?

The mean $\mu_{\bar{x}} = 24$, the same as the population mean, μ_x.

d) What is the expected standard deviation of this sampling distribution ($\sigma_{\bar{x}}$)?

The standard deviation is $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{2.25}{\sqrt{100}} = 0.225$

e) What is the probability that a mean of 100 samples will be greater than 24.5?

$z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{24.5 - 24}{0.23} = 2.17$

$Pr(\bar{X} > 24.5) = Pr(z > 2.17) = 0.0150$

f) Why is the answer to part (a) bigger than for part (e)?

The standard deviation of the sampling distribution of means ($\sigma_{\bar{x}} = 0.23$) is smaller than for the population ($\sigma_x = 2.25$), so it is less likely for a mean to have a large value than a single sample.

g) Answer problems a-d for a sample size of 25 (instead of 100).

The standard error of the mean is $\sigma_{\bar{x}} = \frac{\sigma_x}{n} = \frac{2.25}{\sqrt{25}} = 0.45$

$z = \frac{X - \mu_x}{\sigma_{\bar{x}}} = \frac{24.5 - 24}{0.45} = 1.11$

$Pr(z > 1.11) = 0.1335$

h) Why is it more likely for the mean to exceed 24.5 for a sample size of 25 than for a sample size of 100?

The standard deviation of the sampling distribution of means ($\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$) decreases with sample size, so it is more likely for a mean from a sample size of 25 to be be 24.5 or larger than for a mean from a sample size of 100.

Problem 4 Suppose a professor gives a multiple choice exam. The exam has 30 problems each with 5 possible choices. Suppose also that the professor accidentally gives the exam in the Hungarian language and nobody in the class speaks Hungarian. The students guess randomly on each problem.

a) What is the probability that a given student will get 8 or more problems correct? (Hint: use the normal approximation to the binomial)

The probability of getting a given question correct is 0.2
Using the normal approximation to the binomial, the mean of the distribution of correct scores will be $\mu = nP = (30)(0.2) = 6$

and the standard deviation will be $\sigma = \sqrt{n(P)(1-P)} = \sqrt{(30)(0.2)(0.8)} = 2.19$

The z-score for 8 correct answers is $\frac{X-\mu}{\sigma} = \frac{7.5-6}{2.19} = 0.68$

$Pr(z > 0.68) = 0.2483$

b) Suppose that the mean score for the 50 students in the class is 6.75 correct answers. If the students are guessing, what is the probability that the class will get a mean of 6.75 or higher?

From the Central Limit Theorem, the mean of 50 exams will be drawn from a normal distribution with mean $\mu_x = 6$ and standard deviation $\sigma\bar{x} = \frac{2.19}{\sqrt{50}} = 0.31$

The z score for a mean of 6.75 is $\frac{6.75-6}{0.31} = 2.42$

$Pr(z > 2.42) = 0.0078$

c) Do you think the students were really guessing? Why or why not?

If they were guessing, the probability of getting a mean of 6.75 or higher is low. So, either the class got lucky, or the students weren't completely guessing. Instead, maybe either some of the students could speak Hungarian, or students in general could guess better than chance by doing their best to translate the exam to English.