This looks like a normal distribution!

What is the mean?

\[ \mu = 20 \cdot \pi = N \cdot P = 10 \]

What is the standard deviation?

\[ \sigma = \sqrt{N \cdot P \cdot (1-P)} = \sqrt{20 \cdot 0.5 \cdot 0.5} = 2.24 \]

For example, if you flip a coin 20 times, what is the probability of obtaining 13 or more heads?

\[ Z = \frac{X - \mu}{\sigma} = \frac{12.5 - 10}{2.25} = 1.12 \]

Area above \( Z = 1.12 \) is \( 0.818 \).
Left hander example

Of the 152 students in our class that took the survey, 7 reported themselves as left-handed. If 10% of the population is left-handed, what is the probability that 7 or fewer people in will be left handed in a random sample of 152 people?

Use the normal approximation to the binomial:

\[ N = 152 \]
\[ p = 0.1 \]
\[ k = 7 \]

\[ n = N \cdot p = 152 \cdot (0.1) = 15.2 \]
\[ \sigma = \sqrt{N \cdot p \cdot (1-p)} = \sqrt{152 \cdot (0.1)(0.9)} = 3.6986 \]

\[ Z = \frac{X - \mu}{\sigma} = \frac{7.5 - 15.2}{3.6986} = -2.08 \]

\[ Pr(Z \leq -2.08) \text{ using the normal dist. table:} \]
\[ 0.0188 \]
Seahawks example

We can use our knowledge of the binomial distribution to make statistical inferences. For example, in 2019 the Seattle Seahawks football team won 11 games out of 16. Is this a better team than ‘average’? Use an \( \alpha \) value of 0.05. In other words, is the probability of winning 11 or more games out of 16 less than 0.05 under the null hypothesis that there is a 50/50 chance of winning each game?

\[
N = 16 \quad \mu = N \cdot p = 16 \cdot 0.5 = 8 \\
\sigma = \sqrt{16 \cdot 0.5 \cdot 0.5} = 2 \\
\]

\[
k = 11 \quad \]

\[
z = \frac{X - \mu}{\sigma} = \frac{10.5 - 8}{2} = 1.25 \\
\]

\[
Pr(z \geq 1.25) = 0.1056 \quad \]

\[
1 - \text{pnorm}(1.25) \\
\]

Often we compare our probability to

\[
0.05 \quad \text{or} \quad 5\% \\
\]

If we used the binomial distribution table

\[
Pr = 0.1056 \\
\]
Mariners example

How about the 2019 Seattle Mariners baseball team? They finished the 2019 with a record of 68 wins and 94 losses. What is the probability of losing 94 or more games? Assume $P = 0.5$

\[ N = 162 \]
\[ P = 0.5 \]
\[ P_r(K \geq 94) \]

\[ \mu = N \cdot P = 162 \cdot 0.5 = 81 \]
\[ \sigma = \sqrt{N \cdot P(1-P)} = \sqrt{162 \cdot 0.5 \cdot 0.5} = 6.364 \]
\[ Z = \frac{X - \mu}{\sigma} = \frac{93.5 - 81}{6.364} = 1.96 \]
\[ P_r(Z \geq 1.96) = 0.025 < 0.05 \]

There is a less than 5% chance that a 50/50 team would lose this many or more games. We conclude that the Mariners weren't unlucky, instead, we conclude that the Mariners sucked.