Suppose you give 3 exams over 3 years to 30 different people. Do these exams differ in difficulty over the years?

You could compare two means at a time with independent measures t-tests. 3 tests.

Avs. B, Avs. C, Bvs. C. Let \( \alpha = 0.05 \)

Problem: the probability of 1 or more type I errors is greater than \( \alpha \) if we run multiple tests.

Suppose we have \( m \) tests

\[
\text{\( p(1 \text{ or more Type I}) = (1 - \alpha)^m \)} \quad \text{if \( m=3, \alpha = 0.05 \)}
\]

"Familywise error"

\[
P = 0.14 > 0.05
\]

Means: 81, 72, 75

We really just want to test:

\[
\text{Ho: } \mu_A = \mu_B = \mu_C
\]

\[
\text{H}_a: \text{Ho is false}
\]

How different are these means from each other? Variance!

Variance of 81, 72, and 75 is 22.5

What do we compare the variance of the means to?

Suppose \( \sigma^2 \) is the population variance for all 3 years

\[
\bar{\sigma}^2 = \frac{\sigma^2}{n} \rightarrow \text{central limit theorem}
\]

\[
\bar{\sigma}^2 = \frac{\sigma^2}{n}
\]

\[
\sigma^2 = n \cdot \bar{\sigma}^2
\]

Variance of population size

\[
\text{Variance of means}
\]

\[
n \cdot \bar{\sigma}^2 = (10)(22.5) = 225
\]
Another way of estimating $\sigma^2$ from the data:

calculate the variance for each sample.

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Variances

$61, 94, 55$

$\sigma^2$ can be estimated by taking the mean of the variances.

Mean of $61, 94$ and $55$ is $70$

We have two estimates of $\sigma^2$: 1) mean variance of the means: $22.5$

2) mean of the variances: $70$

If $H_0$ is true, then these tests estimate the same thing ($\sigma^2$)

If $H_0$ is false, then the variance of the means will increase but the mean of the variances stays the same

Define $F = \frac{\text{mean variance of means}}{\text{mean of variances}}$ if $H_0$ is true, then $F = 1$

$$F = \frac{22.5}{70} = 0.325$$

$F$-table has two degrees of freedom

One for numerator: $k = \#$ of groups, $df = k-1$, $3-1 = 2$

One for denominator: $n = \text{sample size for each group}$, $k(n-1) = kn - K = 30 - 3 = 27$ total sample size

Point for $df 2, 27 = 3.35 > 3.23$

Fail to reject $H_0$. 

$22.5 < 3.35$ 

$22.5 < 3.23$