

Testing  $H_0 : \rho_1 = \rho_2$ , are two correlations significantly different?

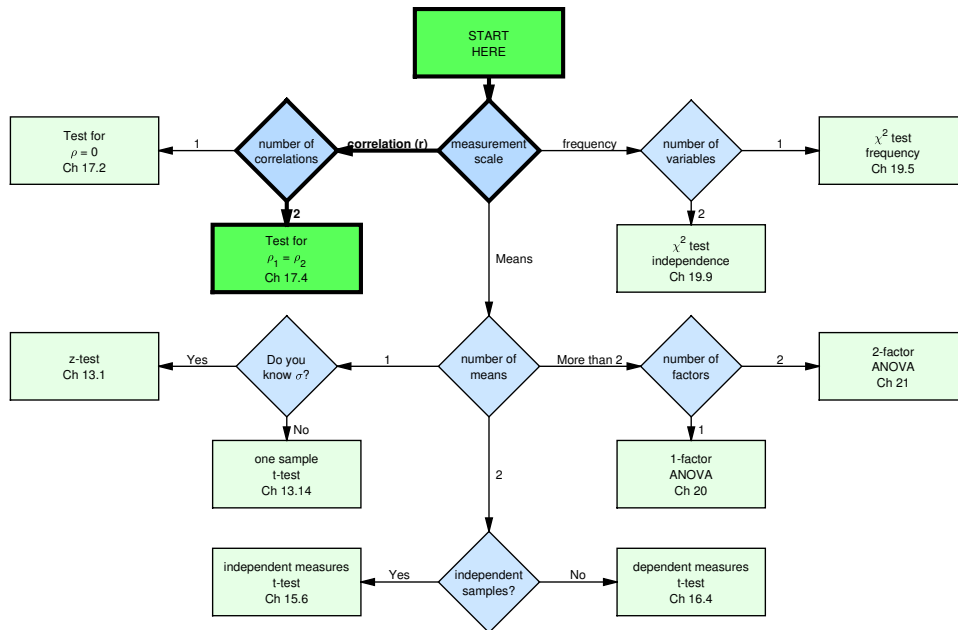
January 10, 2021

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## Testing $H_0: \rho_1 = \rho_2$

Testing to see if two correlations are significantly different from each other is one of the easier hypothesis tests. It involves converting the difference between our observed correlations into a z-score and running z-test. Here's how to get to this test in the flow chart:



The sampling distribution of correlations is very skewed for population correlations near 1 or -1. Sir Ronald Fisher figured out a way to correct for this skew by transforming the values of r into "Fisher's z":

$$z = \frac{\ln(1+r) - \ln(1-r)}{2}$$

This function is called the 'Inverse Hyperbolic Tangent' or 'atanh' on your calculator, where  $\ln$  is the 'natural logarithm', or log based e.

The z-statistic is:

$$z = \frac{z_1 - z_2}{\sigma_{z_1 - z_2}}$$

where the denominator is:

$$\sigma_{z_1 - z_2} = \sqrt{\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}}$$

Once we calculate the value of z, we can just use the z-table (Table A) to determine the statistical significance of the difference between the correlations.

To make things easy (and to never have to think about logarithms), the book and the Excel spreadsheet provide Table H, which calculates Fisher's z.

### Example: hours of sleep and caffeine consumption by gender

Let's see if the correlation between hours of sleep and caffeine consumption differs by gender. From our class, the correlation between hours of sleep and caffeine consumption for the 122 women in this class is -0.13. For the 29 men, the correlation it is -0.15. Is this difference between correlations significant? We'll use  $\alpha = 0.05$ .

First we convert our two observed correlations in to Fisher's z using table H. Note: The table for Fisher's z only has positive correlations. But since the transformation is symmetric about zero, if your correlation is negative, use the positive value in the table and then make Fisher's z negative.

For the women, z for r = -0.13 is -0.1307:

r	z
⋮	⋮
0.125	0.1257
<b>0.130</b>	<b>0.1307</b>
0.135	0.1358
⋮	⋮

For the men, z for r = -0.15 is -0.1511:

r	z
⋮	⋮
0.145	0.1460
<b>0.150</b>	<b>0.1511</b>
0.155	0.1563
⋮	⋮

For our sample sizes of 122 and 29,

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{n_1-3} + \frac{1}{n_2-3}} = \sqrt{\frac{1}{122-3} + \frac{1}{29-3}} = 0.2165$$

which makes

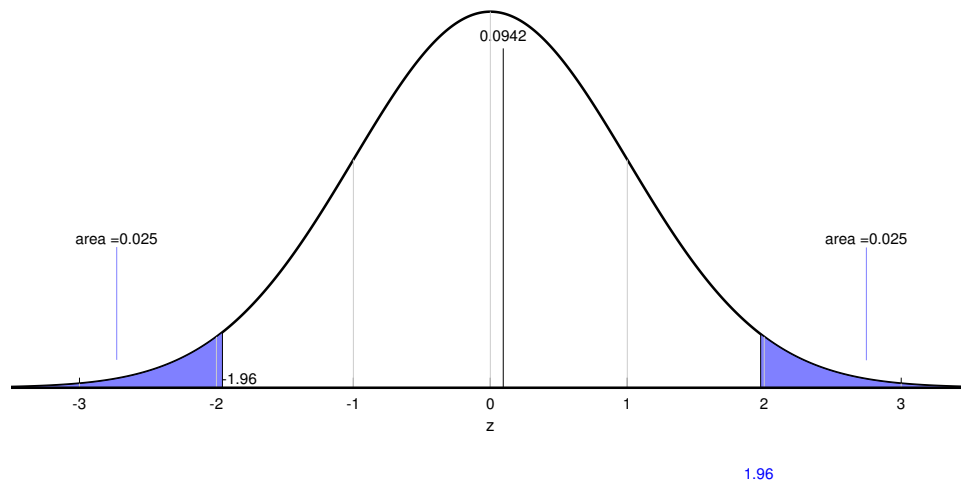
$$z = \frac{z_1-z_2}{\sigma_{z_1-z_2}} = \frac{-0.1307--0.1511}{0.2165} = 0.0942$$

This is a regular old z-value, so we can use the z-table to find the proportion of z-scores that are more extreme than our observed value of  $z = 0.09$ :

z	Area between mean and z	Area beyond z
⋮	⋮	⋮
0.07	0.0279	0.4721
0.08	0.0319	0.4681
<b>0.09</b>	<b>0.0359</b>	<b>0.4641</b>
0.10	0.0398	0.4602
0.11	0.0438	0.4562
⋮	⋮	⋮

The proportion of z-scores greater than  $z = 0.09$  is 0.4641. Since this is a two-tailed test, we **double this value**, so  $p = (2)(0.4641) = 0.9282$ .

The other way of making this decision is to find the critical value of z for a two-tailed test with  $\alpha = 0.05$ . A trick for doing this is to use the bottom row of the t-table which corresponds to an infinite degrees of freedom. You'll find that familiar number:  $z_{crit} = 1.96$ . Here's where our observed value of  $z = 0.0942$  falls in the z-distribution compared to the rejection region:



Our observed value of  $z$  does not fall in the rejection region. We therefore fail to reject  $H_0$  and conclude that there is not a significant difference between the two correlations.

I've also provided a calculator in the spreadsheet that gives you these p-values from our correlations and sample sizes (differences between the calculator and the calculations in this page are due to rounding errors. The calculator does not round each step of the way).

$r_1$ and $r_2$ to $z$							
	$r$	$n$	$z$	$\sigma_z$	$z$	$p$ (one tail)	$p$ (two tail)
1	-0.13	122	-0.1307	0.2165	0.0942	0.4625	0.9249
2	-0.15	29	-0.1511				

Using APA format, we'd conclude:

The correlation between the hours of sleep and caffeine consumption for women (-0.13) is not significantly different than the correlation for men (-0.15),  $z = 0.0942$ ,  $p = 0.925$ .

### Using R to compare two correlations

R doesn't have a function that compares two correlations, but it's not hard to do it by hand. The following implements the example in this tutorial in R.

The R commands shown below can be found here: [ComparingTwoCorrelations.R](#)

```
# Testing for the significance between two correlations
#
# Comparing two correlation requires transforming correlations
# to Fisher's z's, and the transforming the z's to a z-score.
```

```

# Load in the survey data
survey <- read.csv("http://www.courses.washington.edu/psy315/datasets/Psych315W21survey.csv")
# Example 1: Comparing the correlation between sleep and caffeine across genders
x1 <- survey$caffeine[survey$gender=="Male"]
y1 <- survey$sleep[survey$gender=="Male"]
goodvals = !is.na(x1) & !is.na(y1)
x1 <- x1[goodvals]
y1 <- y1[goodvals]
n1 <- length(x1)
r1 <- cor(x1,y1)
r1
[1] -0.1469399
x2 <- survey$caffeine[survey$gender=="Female"]
y2 <- survey$sleep[survey$gender=="Female"]
goodvals = !is.na(x2) & !is.na(y2)
x2 <- x2[goodvals]
y2 <- y2[goodvals]
n2 <- length(x2)
r2 <- cor(x2,y2)
r2
[1] -0.1334869

# Z-transform
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] -0.1480114
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] -0.1342884
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.216483
z = (z1-z2)/sigma
z
[1] -0.06339068

# use pz to get the p value for ths two-tailed test
p <- 2*(1-pnorm(abs(z)))
p
[1] 0.9494554

# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = -0.06, p = 0.9495"

```

## Questions

Get your pens pencils and paper out. It's question time! Here are 7 ridiculous practice questions followed by their answers.

1)

Your advisor asks you to sample 20 candy bars and 25 cats from their populations and measure both their mail and their melancholy. You calculate that for candy bars their mail correlates with melancholy with 0.18 and for cats the correlation is 0.54.

Using an alpha value of  $\alpha = 0.05$ , is the observed correlation for candy bars significantly different than for cats?

2)

You go out and sample 19 fathers and 12 baby names from their populations and measure both their chaos and their life expectancy. You calculate that for fathers their chaos correlates with life expectancy with -0.35 and for baby names the correlation is 0.51.

Using an alpha value of  $\alpha = 0.01$ , is the observed correlation for fathers significantly less than for baby names?

3)

You go out and sample 71 examples and 96 planets from their populations and measure both their mail and their baggage. You calculate that for examples their mail correlates with baggage with -0.39 and for planets the correlation is -0.02.

Using an alpha value of  $\alpha = 0.05$ , is the observed correlation for examples significantly different than for planets?

4)

Suppose you sample 21 oceans and 90 cartoon characters from their populations and measure both their frequency and their justice. You calculate that for oceans their frequency correlates with justice with 0.16 and for cartoon characters the correlation is 0.01.

Using an alpha value of  $\alpha = 0.05$ , is the observed correlation for oceans significantly greater than for cartoon characters?

5)

Your advisor asks you to sample 40 balloons and 16 dinosaurs from their populations and measure both their price and their traffic. You calculate that for balloons their price correlates with traffic with 0.72 and for dinosaurs the correlation is 0.88.

Using an alpha value of  $\alpha = 0.01$ , is the observed correlation for balloons significantly less than for dinosaurs?

6)

Because you don't have anything better to do you sample 28 fathers and 39 economists from their populations and measure both their peace and their homework. You calculate that for fathers their peace correlates with homework with -0.6 and for economists the correlation is 0.15.

Using an alpha value of  $\alpha = 0.05$ , is the observed correlation for fathers significantly

different than for economists?

7)

Let's sample 13 iPods and 12 elections from their populations and measure both their information and their anger. You calculate that for iPods their information correlates with anger with -0.07 and for elections the correlation is -0.67.

Using an alpha value of  $\alpha = 0.01$ , is the observed correlation for iPods significantly different than for elections?

## Answers

1)

$$z_1 = 0.182$$

$$z_2 = 0.6042$$

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{20-3} + \frac{1}{25-3}} = 0.3229$$

$$z = \frac{0.182-0.6042}{0.3229} = -1.3075$$

$$z_{crit} = \pm 1.96$$

We fail to reject  $H_0$ .

The correlation between the mail and melancholy for candy bars (0.18) is not significantly different than the correlation for cats (0.54),  $z = -1.3075$ ,  $p = 0.191$ .

```
r1 <- 0.18
r2 <- 0.54
n1 <- 20
n2 <- 25
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] 0.1819827
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] 0.6041556
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.3229212
z = (z1-z2)/sigma
z
[1] -1.307356
# use pz to get the p value for ths two-tailed test
p <- 2*(1-pnorm(abs(z)))
p
[1] 0.1910918
# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = -1.31, p = 0.1911"
```



2)

$$z_1 = -0.3654$$

$$z_2 = 0.5627$$

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{19-3} + \frac{1}{12-3}} = 0.4167$$

$$z = \frac{-0.3654 - 0.5627}{0.4167} = -2.2273$$

$$z_{crit} = -2.326$$

We fail to reject  $H_0$ .

The correlation between the chaos and life expectancy for fathers (-0.35) is not significantly less than the correlation for baby names (0.51),  $z = -2.2273$ ,  $p = 0.013$ .

```
r1 <- -0.35
r2 <- 0.51
n1 <- 19
n2 <- 12
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] -0.3654438
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] 0.5627298
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.4166667
z = (z1-z2)/sigma
z
[1] -2.227616
# use pz to get the p value for ths two-tailed test
p <- pnorm(z)
p
[1] 0.01295305
# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = -2.23, p = 0.0130"
```

**3)**

$$z_1 = -0.4118$$

$$z_2 = -0.02$$

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{71-3} + \frac{1}{96-3}} = 0.1596$$

$$z = \frac{-0.4118+0.02}{0.1596} = -2.4549$$

$$z_{crit} = \pm 1.96$$

We reject  $H_0$ .

The correlation between the mail and baggage for examples (-0.39) is significantly different than the correlation for planets (-0.02),  $z = -2.4549$ ,  $p = 0.0141$ .

```
r1 <- -0.39
r2 <- -0.02
n1 <- 71
n2 <- 96
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] -0.4118
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] -0.02000267
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.1595574
z = (z1-z2)/sigma
z
[1] -2.455526
# use pz to get the p value for ths two-tailed test
p <- 2*(1-pnorm(abs(z)))
p
[1] 0.01406786
# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = -2.46, p = 0.0141"
```

4)

$$z_1 = 0.1614$$

$$z_2 = 0.01$$

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{21-3} + \frac{1}{90-3}} = 0.2589$$

$$z = \frac{0.1614-0.01}{0.2589} = 0.5848$$

$$z_{crit} = 1.645$$

We fail to reject  $H_0$ .

The correlation between the frequency and justice for oceans (0.16) is not significantly greater than the correlation for cartoon characters (0.01),  $z = 0.5848$ ,  $p = 0.2793$ .

```
r1 <- 0.16
r2 <- 0.01
n1 <- 21
n2 <- 90
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] 0.1613867
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] 0.01000033
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.2589398
z = (z1-z2)/sigma
z
[1] 0.5846393
# use pz to get the p value for ths two-tailed test
p <- 1-pnorm(z)
p
[1] 0.2793951
# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = 0.58, p = 0.2794"
```

5)

$$z_1 = 0.9076$$

$$z_2 = 1.3758$$

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{40-3} + \frac{1}{16-3}} = 0.3224$$

$$z = \frac{0.9076 - 1.3758}{0.3224} = -1.4522$$

$$z_{crit} = -2.326$$

We fail to reject  $H_0$ .

The correlation between the price and traffic for balloons (0.72) is not significantly less than the correlation for dinosaurs (0.88),  $z = -1.4522$ ,  $p = 0.0732$ .

```
r1 <- 0.72
r2 <- 0.88
n1 <- 40
n2 <- 16
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] 0.907645
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] 1.375768
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.3224129
z = (z1-z2)/sigma
z
[1] -1.451935
# use pz to get the p value for ths two-tailed test
p <- pnorm(z)
p
[1] 0.07325982
# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = -1.45, p = 0.0733"
```

6)

$$z_1 = -0.6931$$

$$z_2 = 0.1511$$

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{28-3} + \frac{1}{39-3}} = 0.2603$$

$$z = \frac{-0.6931-0.1511}{0.2603} = -3.2432$$

$$z_{crit} = \pm 1.96$$

We reject  $H_0$ .

The correlation between the peace and homework for fathers (-0.6) is significantly different than the correlation for economists (0.15),  $z = -3.2432$ ,  $p = 0.0012$ .

```
r1 <- -0.6
r2 <- 0.15
n1 <- 28
n2 <- 39
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] -0.6931472
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] 0.1511404
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.2603417
z = (z1-z2)/sigma
z
[1] -3.242999
# use pz to get the p value for ths two-tailed test
p <- 2*(1-pnorm(abs(z)))
p
[1] 0.001182788
# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = -3.24, p = 0.0012"
```

7)

$$z_1 = -0.0701$$

$$z_2 = -0.8107$$

$$\sigma_{z_1-z_2} = \sqrt{\frac{1}{13-3} + \frac{1}{12-3}} = 0.4595$$

$$z = \frac{-0.0701+0.8107}{0.4595} = 1.6118$$

$$z_{crit} = \pm 2.576$$

We fail to reject  $H_0$ .

The correlation between the information and anger for iPods (-0.07) is not significantly different than the correlation for elections (-0.67),  $z = 1.6118$ ,  $p = 0.107$ .

```
r1 <- -0.07
r2 <- -0.67
n1 <- 13
n2 <- 12
z1 <- (log(1+r1) - log(1-r1))/2
z1
[1] -0.07011467
z2 <- (log(1+r2) - log(1-r2))/2
z2
[1] -0.8107431
sigma <- sqrt(1/(n1-3) + 1/(n2-3))
sigma
[1] 0.4594683
z = (z1-z2)/sigma
z
[1] 1.611925
# use pz to get the p value for ths two-tailed test
p <- 2*(1-pnorm(abs(z)))
p
[1] 0.1069782
# APA format:
sprintf('z = %4.2f, p = %5.4f',z,p)
[1] "z = 1.61, p = 0.1070"
```