

# $\chi^2$ Test for Frequencies

January 17, 2021

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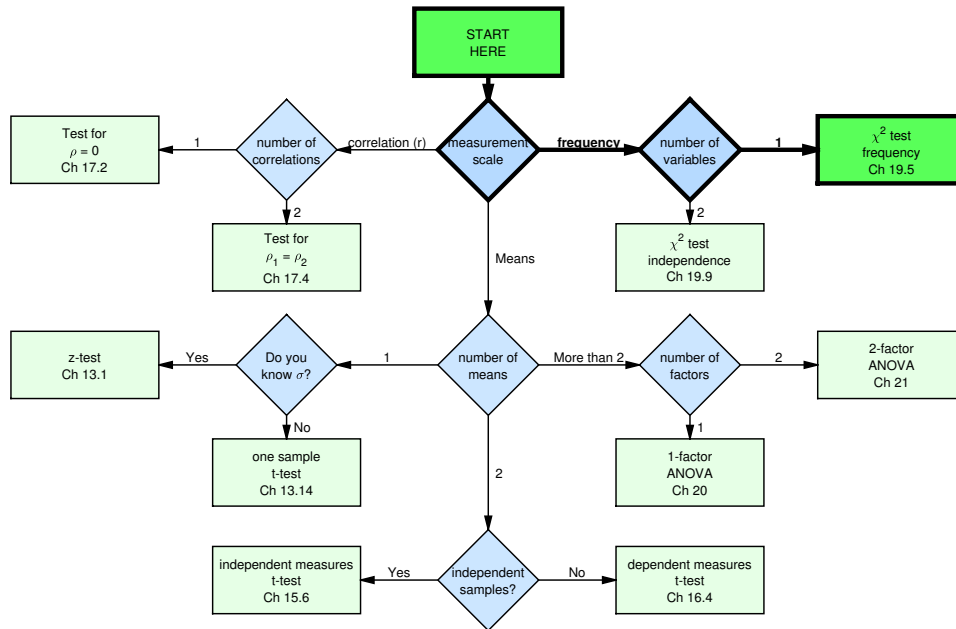
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Happy birthday to Carina Chan!

## Chi squared ( $\chi^2$ ) test for frequencies

This is a hypothesis test on the frequency of samples that fall into different discrete categories. For example, are the number of left and right-handed people in our class distributed like you'd expect from the population? Or, is the frequency distribution of birthdays by month for the students in our class distributed evenly across months? For these tests the dependent measure is a frequency, not a mean.

Here's how to get to the  $\chi^2$  test for frequencies with the flow chart:



Let's start with a simple example:

### Example 1: left vs. right handers in our class

According to Wikipedia, 10 percent of the population is left handed. For our class, 7 students reported that they are left handed, while 145 reported right handedness. A  $\chi^2$  test determines if the frequency of our sampled observations are significantly different than the frequencies that you'd expect from the population. Specifically, the null hypothesis is that our observed frequencies are drawn from a population that has some expected proportions, and our alternative hypothesis is that we're drawing from a population that does not have these expected proportions.

Like all statistical tests, the  $\chi^2$  test involves calculating a statistic that measures how far our observations are from those expected under the null hypothesis.

The first step is to calculate the frequencies expected from the null hypothesis. This is simply done by multiplying the total sample size by each of the expected proportions. Since there are 152 students in the class, then we expect  $(152)(0.1) = 15.2$  students to be left handed and  $(152)(0.9) = 136.8$  to be right handed. Expected frequencies do not have to be rounded to the nearest whole number, even though frequencies are whole numbers. This is because we should think of these expected frequencies as the **average** frequency for each category over the long run - and averages don't have to be whole numbers.

The next step is to measure how far our observed frequencies are from the expected frequencies. Here's the formula, where  $\chi^2$  is pronounced "Chi-squared".

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

Where  $f_o$  are the observed frequencies and  $f_e$  are the expected frequencies. For our example,  $f_o$  is 7 and 145 and  $f_e$  is 15.2 and 136.8:

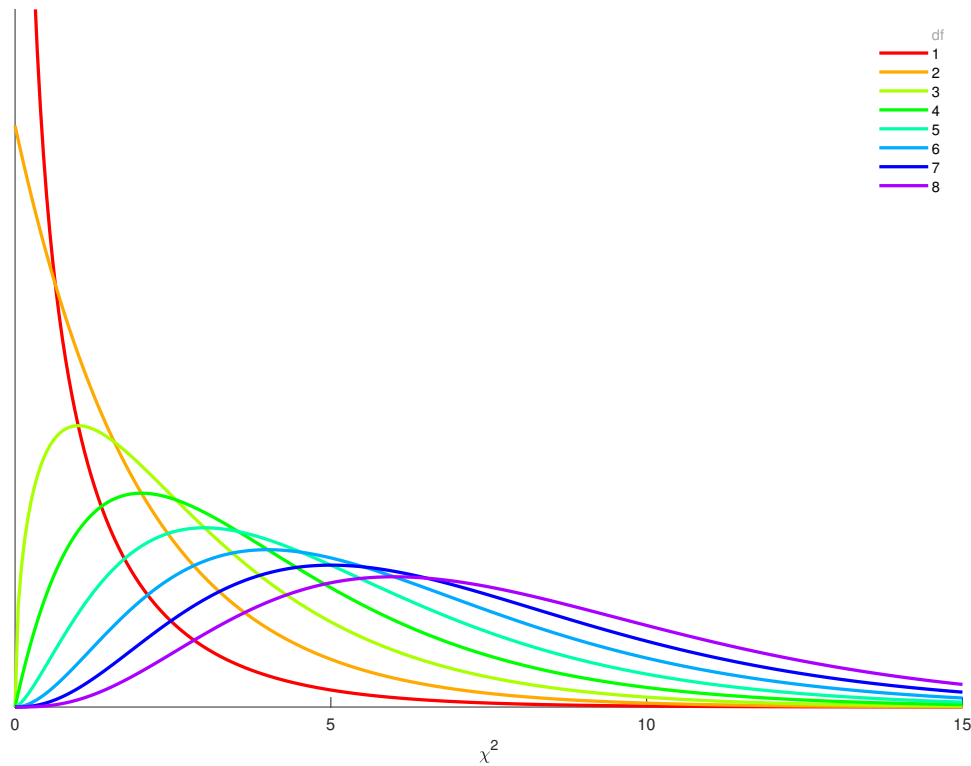
$$\chi^2 = \frac{(7-15.2)^2}{15.2} + \frac{(145-136.8)^2}{136.8} = 4.4237 + 0.4915 = 4.92$$

This measure,  $\chi^2$ , is close to zero when the observed frequencies match the expected frequencies. Therefore, large values of  $\chi^2$  can be considered evidence against the null hypothesis.

## The $\chi^2$ distribution

Just like the z and t distributions, the  $\chi^2$  distribution has a known shape and therefore has its own table in the book and page in the Excel spreadsheet (Table I). Also, like the t-distribution, the  $\chi^2$  distribution is actually a family of distributions, with a different distribution for different degrees of freedom.

The  $\chi^2$  distribution for k degrees of freedom is the distribution you'd get if you draw k values from the standard normal distribution (the z-distribution), square them, and add them up. Here's what the probability distributions look like for different degrees of freedom:



Notice how the shape of the distributions spread out and change shape with increasing

degrees of freedom. This is because as we increase df, and therefore the number of squared z-scores, the sum will on average increase too.

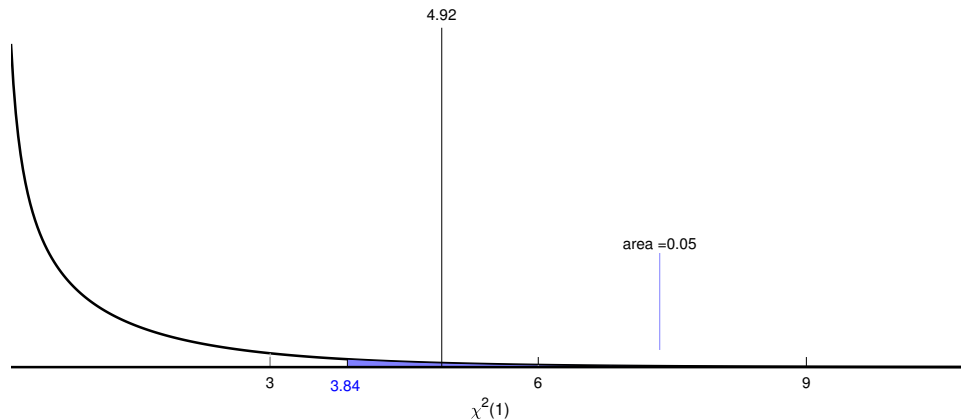
Since the  $\chi^2$  distribution is known we can calculate the probability of obtaining our observed value of  $\chi^2$  if null hypothesis is true. **The degrees of freedom is the number of categories minus one.** For this example,  $df = 2 - 1 = 1$ .

The critical value is found with Table I in our book and also in the Excel spreadsheet. All we do is look up the critical value for  $\chi^2$  for our df and value of  $\alpha$ . Let's use  $\alpha = 0.05$ :

df	0.995	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01	0.005
1	0	0.0002	0.001	0	0.02	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.1	0.21	4.61	5.99	7.38	9.21	10.6
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34	12.84
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

The table tells us that with  $df = 1$  and  $\alpha = 0.05$ , the critical value for  $\chi^2$  is 3.84.

Here's what the distribution looks like for our left-hander example, with  $df = 1$ . Shown also is the critical value of  $\chi^2$  for which 5% of the curve lies above. Also shown is the observed value of  $\chi^2$  (4.92).



You can see that our observed  $\chi^2$  value (4.92) falls above the critical value (3.84). We therefore reject the null hypothesis that our observations were drawn from the null hypothesis distribution.

Just like for the t-table, the  $\chi^2$  table is not useful for calculating p-values. Instead we can use the  $\chi^2$ -calculator on the same page in the Excel spreadsheet, which gives us a p-value of 0.0265:

$\alpha$ to $\chi^2$		
df	$\alpha$	$\chi^2$
1	0.05	3.84

$\chi^2$ to $\alpha$		
df	$\chi^2$	$\alpha$
1	4.92	0.0265

Using APA format, we'd state:

"The number of left handers in our class is significantly different from 10 percent.  $\chi^2(1, N = 152) = 4.92, p = 0.0265$ ."

### One or two tailed?

A common question for  $\chi^2$  tests is whether it is a one or a two tailed test. It seems like a one-tailed test because we reject the null hypothesis only for large values of  $\chi^2$ . However, it's really a two-tailed test since we reject the null hypothesis if our observed frequencies ( $f_e$ ) differ from the null hypothesis frequencies ( $f_o$ ) in either direction (too many or too few lefties).

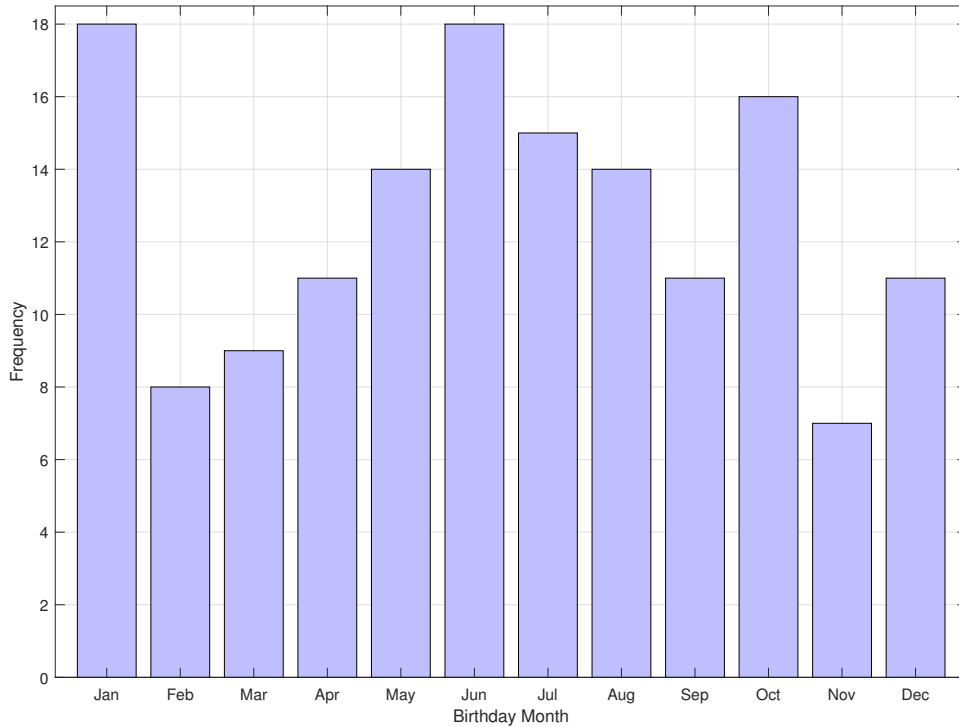
### Example 2: Birthdays by month

Let's see if the birthdays in this class are evenly distributed across months, or if there are some months for which students have significantly more birthdays than others. For simplicity, we'll assume that all months have equal probability, even though they vary in length. We'll run a  $\chi^2$  test using an alpha value of 0.05.

Here's a table showing the number of birthdays for each month for all 152 students:

observed frequencies of birthdays											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
18	8	9	11	14	18	15	14	11	16	7	11

It looks kind of uneven. A natural way to visualize this distribution is with a bar graph:



To see if this distribution is significantly uneven we calculate the expected frequencies under the null hypothesis. Here we expect equal frequencies of  $152/12 = 12.6667$  birthdays per month. Note equal frequencies assumes that each month has an equal number of days. This assumption is close enough for this example, but how would you correct the expected frequencies to account for this?

Using our  $\chi^2$  formula:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

We find that  $\chi^2$  values are:

$\chi^2$ for each cell											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2.2456	1.7193	1.0614	0.2193	0.1403	2.2456	0.4298	0.1403	0.2193	0.8772	2.5351	0.2193

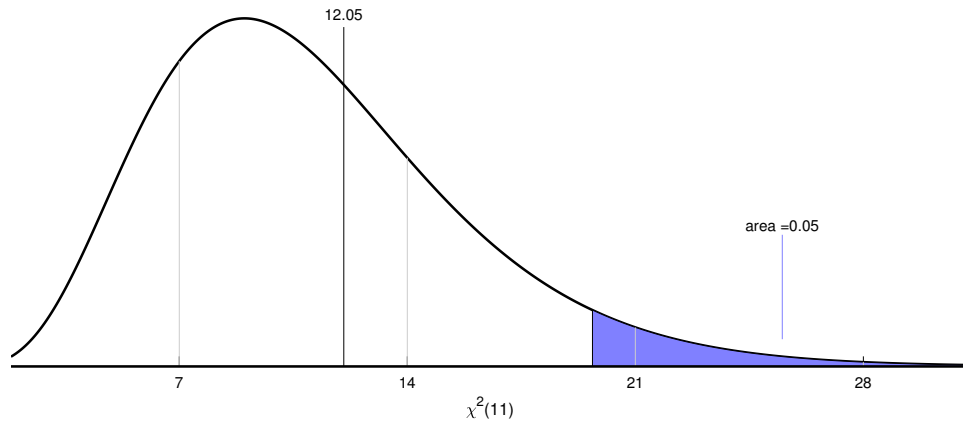
$$\text{So } \chi^2 = \frac{(18-12.6667)^2}{12.6667} + \frac{(8-12.6667)^2}{12.6667} + \dots + \frac{(11-12.6667)^2}{12.6667} =$$

$$2.2456 + 1.7193 + \dots + 0.2193 = 12.0525$$

Using the table, the critical value of  $\chi^2$  for  $df = 12 - 1 = 11$  and  $\alpha = 0.05$  is 19.68.

df	0.995	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01	0.005
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
<b>11</b>	<b>2.6</b>	<b>3.05</b>	<b>3.82</b>	<b>4.57</b>	<b>5.58</b>	<b>17.28</b>	<b>19.68</b>	<b>21.92</b>	<b>24.72</b>	<b>26.76</b>
12	3.07	3.57	4.4	5.23	6.3	18.55	21.03	23.34	26.22	28.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Here is what the  $\chi^2$  distribution looks like for 11 degrees of freedom. Note how different it is from the first example with  $df = 1$ .



You can see that our observed  $\chi^2$  value (12.05) falls below the critical value (19.68). We therefore fail to reject the null hypothesis that our observations were drawn from the null hypothesis distribution.

Using the  $\chi^2$ -calculator gives us a p-value of 0.3599:

$\alpha$ to $\chi^2$		
df	$\alpha$	$\chi^2$
11	0.05	19.68

$\chi^2$ to $\alpha$		
df	$\chi^2$	$\alpha$
11	12.05	0.3599

Using APA format, we'd state: "The distribution of birthdays across months in our class is not significantly different from an even distribution,  $\chi^2(11, N = 152) = 12.05, p = 0.3599$ ."

## Using R to run a hypothesis test for frequencies

The following R script covers how to run a  $\chi^2$  test for frequencies for the examples in this tutorial.

The R commands shown below can be found here: [Chi2TestFrequencies.R](#)

```
# Chi-squared test for frequencies.
#
# R's 'chisq.test' provides a p-value for the chi-squared test for frequencies
# by taking in a table of frequencies and an optional list of expected frequencies.
# Here we'll run the two examples in the chi_2_test_frequencies_tutorial

# Load in the survey data
survey <- read.csv("http://www.courses.washington.edu/psy315/datasets/Psych315W21survey.csv")
# Example 1: left vs. right handers in our class, compared to 10% left handers
fo <- table(survey$hand) # observed frequencies
fe = c(.1,.9) # expected frequencies
# run the chi-squared test:
out <- chisq.test(fo,p=fe)
# The chi-squared statistic is:
out$statistic
X-squared
 4.915205

# The degrees of freedom is:
out$parameter
df
 1

# And the p-value is:
out$p.value
[1] 0.02662131

# Writing in APA format can be done like this:
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
[1] "Chi-Squared(1,N=152) = 4.92, p = 0.0266"

# Plot the results:
barplot(fo)
# Example 2: Birthdays by month
fo <- table(survey$month)
# Run the chi-squared test. If we don't specify the expected frequency the
# test assumes that expected frequencies are equal across categories.
out <- chisq.test(fo)
# result in APA format:
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
[1] "Chi-Squared(11,N=152) = 12.05, p = 0.3597"
```



```
# Plotting: first rearrange months in order
fo <- fo[c(5,4,8,1,9,7,6,2,12,11,10,3)]
# Then plot:
barplot(fo)
# If you only have the chi-squared statistic and degrees of freedom
# you can use the 'pchisq' function to get the p-value. We use
# lower.tail = FALSE to reject H0 for large values of chi-squared:
chi.squared <- out$statistic
df <- out$parameter
p.value <- pchisq(chi.squared,df,lower.tail = FALSE)
p.value
X-squared
0.3597001
```

## Questions

Your turn. Here are 10 random practice questions followed by their answers. For all questions **test the null hypothesis that there is an equal distribution of frequencies across categories.**

1) Suppose colors come in 4 varieties: plastic, useful, colossal and addicted. In the pursuit of science, you find 101 colors and count how many fall into each variety. This generates the following table:

observed frequencies of colors			
plastic	useful	colossal	addicted
29	20	21	31

Make a bar graph showing the frequencies for each variety of colors

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.01$  test the null hypothesis that the 101 colors are distributed evenly across the 4 varieties of plastic, useful, colossal and addicted.

2) Suppose proctologists come in 6 varieties: pushy, phobic, kindly, safe, unwieldy and yellow. Because you don't have anything better to do you find 155 proctologists and count how many fall into each variety. This generates the following table:

observed frequencies of proctologists					
pushy	phobic	kindly	safe	unwieldy	yellow
28	38	17	27	18	27

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.01$  test the null hypothesis that the 155 proctologists are distributed evenly across the 6 varieties of pushy, phobic, kindly, safe, unwieldy and yellow.

3) Suppose galaxies come in 3 varieties: abounding, cautious and elegant. You go out and find 72 galaxies and count how many fall into each variety. This generates the following table:

observed frequencies of galaxies		
abounding	cautious	elegant
28	19	25

Make a bar graph showing the frequencies for each variety of galaxies

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.01$  test the null hypothesis that the 72 galaxies are distributed evenly across the 3 varieties of abounding, cautious and elegant.

4) Suppose facial expressions come in 5 varieties: defeated, oceanic, abundant, handsome

and greedy. For your first year project you find 111 facial expressions and count how many fall into each variety. This generates the following table:

observed frequencies of facial expressions				
defeated	oceanic	abundant	handsome	greedy
10	34	30	11	26

Make a bar graph showing the frequencies for each variety of facial expressions

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.05$  test the null hypothesis that the 111 facial expressions are distributed evenly across the 5 varieties of defeated, oceanic, abundant, handsome and greedy.

5) Suppose chickens come in 7 varieties: sedate, careless, rainy, hurried, judicious, outstanding and stormy. I'd like you to find 176 chickens and count how many fall into each variety. This generates the following table:

observed frequencies of chickens						
sedate	careless	rainy	hurried	judicious	outstanding	stormy
22	27	14	26	22	29	36

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.05$  test the null hypothesis that the 176 chickens are distributed evenly across the 7 varieties of sedate, careless, rainy, hurried, judicious, outstanding and stormy.

6) Suppose teenagers come in 3 varieties: toothsome, lucky and giant. Because you don't have anything better to do you find 60 teenagers and count how many fall into each variety. This generates the following table:

observed frequencies of teenagers		
toothsome	lucky	giant
25	12	23

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.01$  test the null hypothesis that the 60 teenagers are distributed evenly across the 3 varieties of toothsome, lucky and giant.

7) Suppose statistics problems come in 5 varieties: plastic, lyrical, adhoc, horrible and cruel. One day you find 133 statistics problems and count how many fall into each variety. This generates the following table:

observed frequencies of statistics problems				
plastic	lyrical	adhoc	horrible	cruel
32	19	37	24	21

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.01$  test the null hypothesis that the 133 statistics problems are distributed evenly across the 5 varieties of plastic, lyrical, adhoc, horrible and cruel.

8) Suppose examples come in 5 varieties: lumpy, fumbling, tense, careful and agreeable. Without anything better to do, you find 65 examples and count how many fall into each variety. This generates the following table:

observed frequencies of examples				
lumpy	fumbling	tense	careful	agreeable
14	24	10	9	8

Make a bar graph showing the frequencies for each variety of examples

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.05$  test the null hypothesis that the 65 examples are distributed evenly across the 5 varieties of lumpy, fumbling, tense, careful and agreeable.

9) Suppose balloons come in 3 varieties: neat, easy and deep. Tomorrow you find 44 balloons and count how many fall into each variety. This generates the following table:

observed frequencies of balloons		
neat	easy	deep
21	13	10

Make a bar graph showing the frequencies for each variety of balloons

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.05$  test the null hypothesis that the 44 balloons are distributed evenly across the 3 varieties of neat, easy and deep.

10) Suppose statistics problems come in 4 varieties: juicy, swift, obeisant and glorious. In the pursuit of science, you find 74 statistics problems and count how many fall into each variety. This generates the following table:

observed frequencies of statistics problems			
juicy	swift	obeisant	glorious
17	26	23	8

Make a table of the expected frequencies.

Using an alpha value of  $\alpha = 0.01$  test the null hypothesis that the 74 statistics problems are distributed evenly across the 4 varieties of juicy, swift, obeisant and glorious.

## Answers

1) The frequencies of 4 kinds of colors

$$f_e = \frac{29+20+21+31}{4} = \frac{101}{4} = 25.25$$
$$\chi^2 = \frac{(29-25.25)^2}{25.25} + \frac{(20-25.25)^2}{25.25} + \frac{(21-25.25)^2}{25.25} + \frac{(31-25.25)^2}{25.25} =$$
$$0.5569 + 1.0916 + 0.7153 + 1.3094 = 3.6732$$

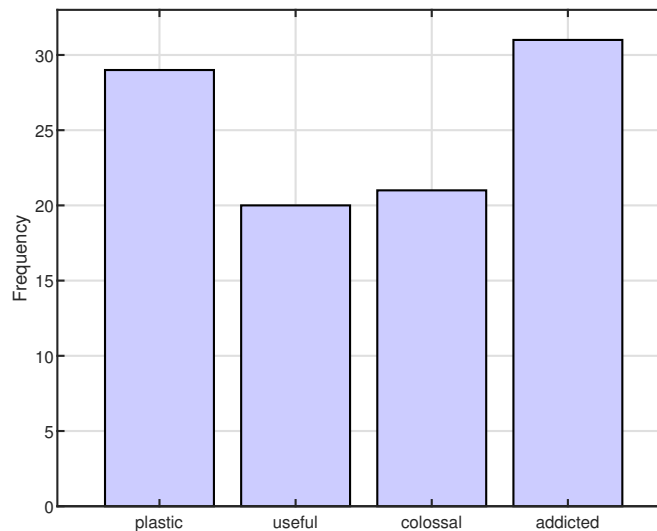
$\chi^2$ for each cell			
plastic	useful	colossal	addicted
0.5569	1.0916	0.7153	1.3094

$$df = (4-1) = 3$$

$$\chi_{crit}^2 = 11.34$$

We fail to reject  $H_0$ .

The frequency of 101 colors is distributed as expected across the 4 varieties of plastic, useful, colossal and addicted,  $\chi^2(3, N=101) = 3.67$ ,  $p = 0.2994$ .



# Using R:

```
fo <- c( 29, 20, 21, 31)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(3,N=101) = 3.67, p = 0.2990"
```

# Plotting:

```
barplot(fo,  
        names.arg = c( "plastic", "useful", "colossal", "addicted"),  
        ylab = 'Frequencies of colors')
```

2) The frequencies of 6 kinds of proctologists

$$f_e = \frac{28+38+17+27+18+27}{6} = \frac{155}{6} = 25.8333$$

$$\chi^2 = \frac{(28-25.8333)^2}{25.8333} + \frac{(38-25.8333)^2}{25.8333} + \frac{(17-25.8333)^2}{25.8333} + \frac{(27-25.8333)^2}{25.8333} + \frac{(18-25.8333)^2}{25.8333} + \frac{(27-25.8333)^2}{25.8333} =$$

$$0.1817 + 5.7301 + 3.0204 + 0.0527 + 2.3753 + 0.0527 = 11.4129$$

$\chi^2$ for each cell					
pushy	phobic	kindly	safe	unwieldy	yellow
0.1817	5.7301	3.0204	0.0527	2.3753	0.0527

$$df = (6-1) = 5$$

$$\chi_{crit}^2 = 15.09$$

We fail to reject  $H_0$ .

The frequency of 155 proctologists is distributed as expected across the 6 varieties of pushy, phobic, kindly, safe, unwieldy and yellow,  $\chi^2(5, N=155)=11.41$ ,  $p = 0.0438$ .

# Using R:

```
fo <- c( 28, 38, 17, 27, 18, 27)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(5,N=155) = 11.41, p = 0.0438"
```

3) The frequencies of 3 kinds of galaxies

$$f_e = \frac{28+19+25}{3} = \frac{72}{3} = 24$$

$$\chi^2 = \frac{(28-24)^2}{24} + \frac{(19-24)^2}{24} + \frac{(25-24)^2}{24} =$$

$$0.6667 + 1.0417 + 0.0417 = 1.7501$$

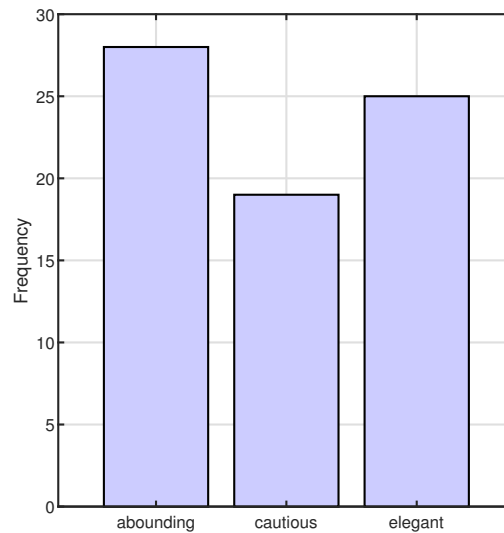
$\chi^2$ for each cell		
abounding	cautious	elegant
0.6667	1.0417	0.0417

$$df = (3-1) = 2$$

$$\chi_{crit}^2 = 9.21$$

We fail to reject  $H_0$ .

The frequency of 72 galaxies is distributed as expected across the 3 varieties of abounding, cautious and elegant,  $\chi^2(2, N=72) = 1.75$ ,  $p = 0.4169$ .



# Using R:

```
fo <- c( 28, 19, 25)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(2,N=72) = 1.75, p = 0.4169"
```

# Plotting:

```
barplot(fo,
```



```
names.arg = c( "abounding", "cautious", "elegant"),  
ylab = 'Frequencies of galaxies')
```

4) The frequencies of 5 kinds of facial expressions

$$f_e = \frac{10+34+30+11+26}{5} = \frac{111}{5} = 22.2$$

$$\chi^2 = \frac{(10-22.2)^2}{22.2} + \frac{(34-22.2)^2}{22.2} + \frac{(30-22.2)^2}{22.2} + \frac{(11-22.2)^2}{22.2} + \frac{(26-22.2)^2}{22.2} =$$

$$6.7045 + 6.2721 + 2.7405 + 5.6505 + 0.6505 = 22.0181$$

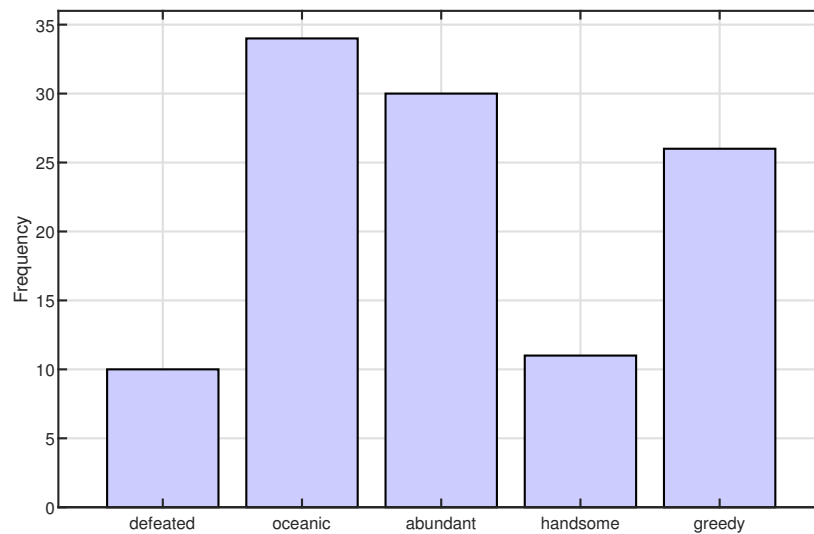
$\chi^2$ for each cell				
defeated	oceanic	abundant	handsome	greedy
6.7045	6.2721	2.7405	5.6505	0.6505

$$df = (5-1) = 4$$

$$\chi^2_{crit} = 9.49$$

We reject  $H_0$ .

The frequency of 111 facial expressions is not distributed as expected across the 5 varieties of defeated, oceanic, abundant, handsome and greedy,  $\chi^2(4, N=111)=22.02$ ,  $p = 0.0002$ .



# Using R:

```
fo <- c( 10, 34, 30, 11, 26)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(4,N=111) = 22.02, p = 0.0002"
```

# Plotting:

```
barplot(fo,  
        names.arg = c( "defeated", "oceanic", "abundant", "handsome", "greedy"),  
        ylab = 'Frequencies of facial expressions')
```

5) The frequencies of 7 kinds of chickens

$$f_e = \frac{22+27+14+26+22+29+36}{7} = \frac{176}{7} = 25.1429$$

$$\chi^2 = \frac{(22-25.1429)^2}{25.1429} + \frac{(27-25.1429)^2}{25.1429} + \frac{(14-25.1429)^2}{25.1429} + \frac{(26-25.1429)^2}{25.1429} + \frac{(22-25.1429)^2}{25.1429} + \frac{(29-25.1429)^2}{25.1429} + \frac{(36-25.1429)^2}{25.1429} =$$

$$0.3929 + 0.1372 + 4.9383 + 0.0292 + 0.3929 + 0.5917 + 4.6883 = 11.1705$$

$\chi^2$ for each cell						
sedate	careless	rainy	hurried	judicious	outstanding	stormy
0.3929	0.1372	4.9383	0.0292	0.3929	0.5917	4.6883

$$df = (7-1) = 6$$

$$\chi_{crit}^2 = 12.59$$

We fail to reject  $H_0$ .

The frequency of 176 chickens is distributed as expected across the 7 varieties of sedate, careless, rainy, hurried, judicious, outstanding and stormy,  $\chi^2(6, N=176)=11.17$ ,  $p = 0.0833$ .

# Using R:

```
fo <- c( 22, 27, 14, 26, 22, 29, 36)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f', out$parameter, sum(fo), out$statistic, out$p.value)
```

```
[1] "Chi-Squared(6,N=176) = 11.17, p = 0.0832"
```

6) The frequencies of 3 kinds of teenagers

$$f_e = \frac{25+12+23}{3} = \frac{60}{3} = 20$$
$$\chi^2 = \frac{(25-20)^2}{20} + \frac{(12-20)^2}{20} + \frac{(23-20)^2}{20} =$$

$$1.25 + 3.2 + 0.45 = 4.9$$

$\chi^2$ for each cell		
toothsome	lucky	giant
1.25	3.2	0.45

$$df = (3-1) = 2$$

$$\chi_{crit}^2 = 9.21$$

We fail to reject  $H_0$ .

The frequency of 60 teenagers is distributed as expected across the 3 varieties of toothsome, lucky and giant,  $\chi^2(2, N=60) = 4.90$ ,  $p = 0.0863$ .

# Using R:

```
fo <- c( 25, 12, 23)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(2,N=60) = 4.90, p = 0.0863"
```

7) The frequencies of 5 kinds of statistics problems

$$f_e = \frac{32+19+37+24+21}{5} = \frac{133}{5} = 26.6$$

$$\chi^2 = \frac{(32-26.6)^2}{26.6} + \frac{(19-26.6)^2}{26.6} + \frac{(37-26.6)^2}{26.6} + \frac{(24-26.6)^2}{26.6} + \frac{(21-26.6)^2}{26.6} =$$

$$1.0962 + 2.1714 + 4.0662 + 0.2541 + 1.1789 = 8.7668$$

$\chi^2$ for each cell				
plastic	lyrical	adhoc	horrible	cruel
1.0962	2.1714	4.0662	0.2541	1.1789

$$df = (5-1) = 4$$

$$\chi_{crit}^2 = 13.28$$

We fail to reject  $H_0$ .

The frequency of 133 statistics problems is distributed as expected across the 5 varieties of plastic, lyrical, adhoc, horrible and cruel,  $\chi^2(4, N=133) = 8.77$ ,  $p = 0.0671$ .

# Using R:

```
fo <- c( 32, 19, 37, 24, 21)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(4,N=133) = 8.77, p = 0.0672"
```

8) The frequencies of 5 kinds of examples

$$f_e = \frac{14+24+10+9+8}{5} = \frac{65}{5} = 13$$

$$\chi^2 = \frac{(14-13)^2}{13} + \frac{(24-13)^2}{13} + \frac{(10-13)^2}{13} + \frac{(9-13)^2}{13} + \frac{(8-13)^2}{13} =$$

$$0.0769 + 9.3077 + 0.6923 + 1.2308 + 1.9231 = 13.2308$$

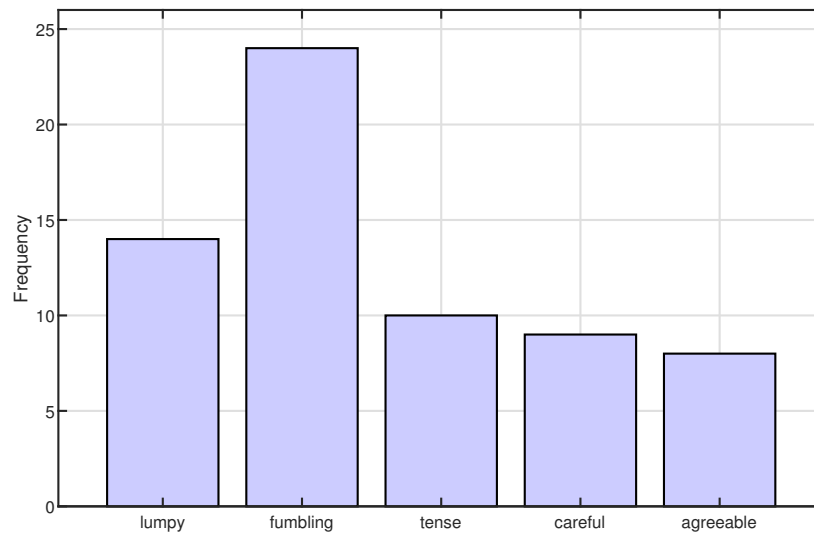
$\chi^2$ for each cell				
lumpy	fumbling	tense	careful	agreeable
0.0769	9.3077	0.6923	1.2308	1.9231

$$df = (5-1) = 4$$

$$\chi_{crit}^2 = 9.49$$

We reject  $H_0$ .

The frequency of 65 examples is not distributed as expected across the 5 varieties of lumpy, fumbling, tense, careful and agreeable,  $\chi^2(4, N=65)=13.23$ ,  $p = 0.0102$ .



# Using R:

```
fo <- c( 14, 24, 10, 9, 8)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(4,N=65) = 13.23, p = 0.0102"
```

# Plotting:

```
barplot(fo,
```

```
names.arg = c( "lumpy", "fumbling", "tense", "careful", "agreeable"),  
ylab = 'Frequencies of examples')
```



9) The frequencies of 3 kinds of balloons

$$f_e = \frac{21+13+10}{3} = \frac{44}{3} = 14.6667$$

$$\chi^2 = \frac{(21-14.6667)^2}{14.6667} + \frac{(13-14.6667)^2}{14.6667} + \frac{(10-14.6667)^2}{14.6667} =$$

$$2.7348 + 0.1894 + 1.4849 = 4.4091$$

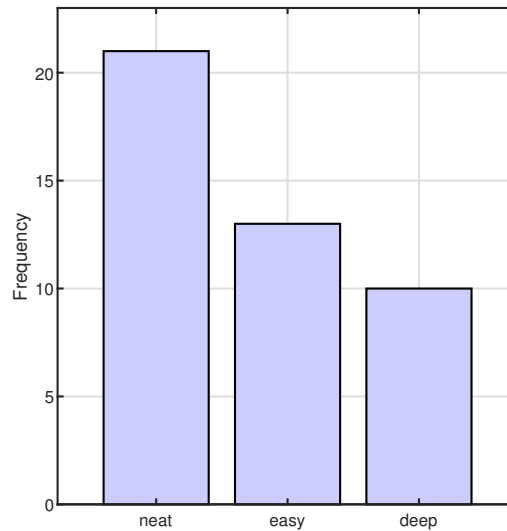
$\chi^2$ for each cell		
neat	easy	deep
2.7348	0.1894	1.4849

$$df = (3-1) = 2$$

$$\chi^2_{crit} = 5.99$$

We fail to reject  $H_0$ .

The frequency of 44 balloons is distributed as expected across the 3 varieties of neat, easy and deep,  $\chi^2(2, N=44) = 4.41, p = 0.1103$ .



# Using R:

```
fo <- c( 21, 13, 10)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f',out$parameter,sum(fo),out$statistic,out$p.value)
```

```
[1] "Chi-Squared(2,N=44) = 4.41, p = 0.1103"
```

# Plotting:

```
barplot(fo,
```

```
names.arg = c( "neat", "easy", "deep"),  
ylab = 'Frequencies of balloons')
```

10) The frequencies of 4 kinds of statistics problems

$$f_e = \frac{17+26+23+8}{4} = \frac{74}{4} = 18.5$$

$$\chi^2 = \frac{(17-18.5)^2}{18.5} + \frac{(26-18.5)^2}{18.5} + \frac{(23-18.5)^2}{18.5} + \frac{(8-18.5)^2}{18.5} =$$

$$0.1216 + 3.0405 + 1.0946 + 5.9595 = 10.2162$$

$\chi^2$ for each cell			
juicy	swift	obeisant	glorious
0.1216	3.0405	1.0946	5.9595

$$df = (4-1) = 3$$

$$\chi_{crit}^2 = 11.34$$

We fail to reject  $H_0$ .

The frequency of 74 statistics problems is distributed as expected across the 4 varieties of juicy, swift, obeisant and glorious,  $\chi^2(3, N=74)=10.22$ ,  $p = 0.0168$ .

# Using R:

```
fo <- c( 17, 26, 23, 8)
```

```
out <- chisq.test(fo)
```

```
sprintf('Chi-Squared(%d,N=%d) = %5.2f, p = %5.4f', out$parameter, sum(fo), out$statistic, out$p.value)
```

```
[1] "Chi-Squared(3,N=74) = 10.22, p = 0.0168"
```