The Normal Distribution

January 9, 2021

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Normal distributions can only vary by their means and standard distributions. If you know a distribution is normal, and you know the mean and standard deviation, then you have everything you need to know to calculate areas and probabilities.

To find areas under any normal distribution we convert our scores into z-scores and look up the answer in the z-table. Given a normal distribution of scores, X, that has a mean μ and standard deviation σ, we convert X to z with:

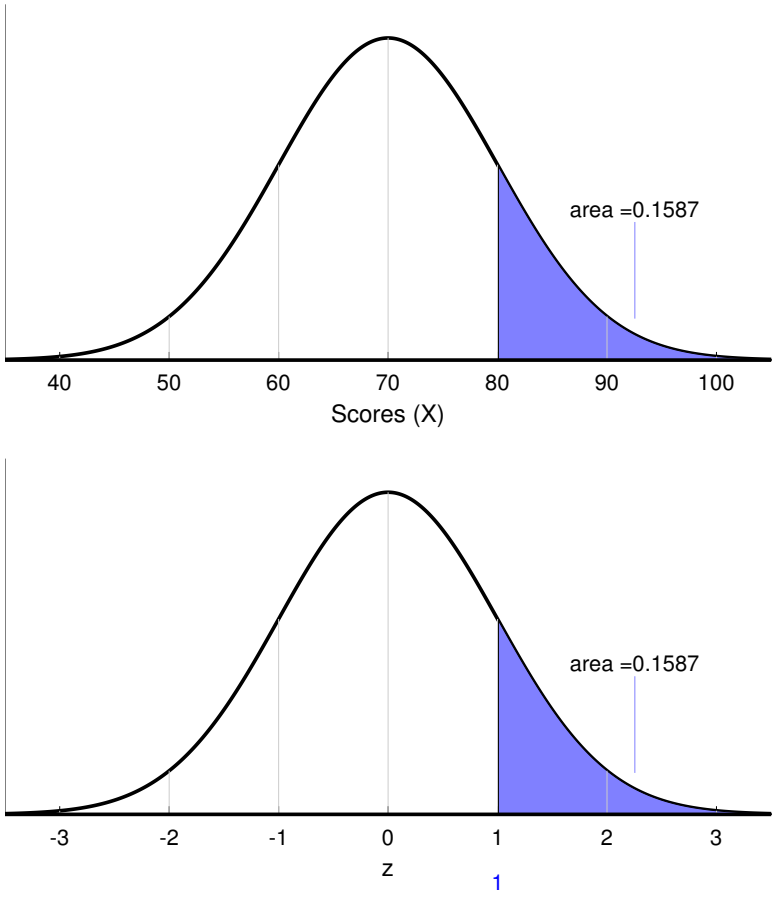
$$z = \frac{X - \mu}{\sigma}$$

Example: Suppose you know that a population of test scores has a mean of 70 and a standard deviation of 10. What proportion of scores fall above 80?

These problems can be solved by converting scores into standard deviation units and then looking up values in the z-table. To convert scores to standard deviation units, we subtract the mean and then divide by the standard deviation:

$z = \frac{X-\mu}{\sigma} = \frac{80-70}{10} = 1$

The standard normal distribution tells you the proportion of areas under the normal distribution in standard deviation units. So the proportion of scores above X = 80 is the same as the area under the standard normal distribution above z = 1:



From the z-table, this area is 0.1587.

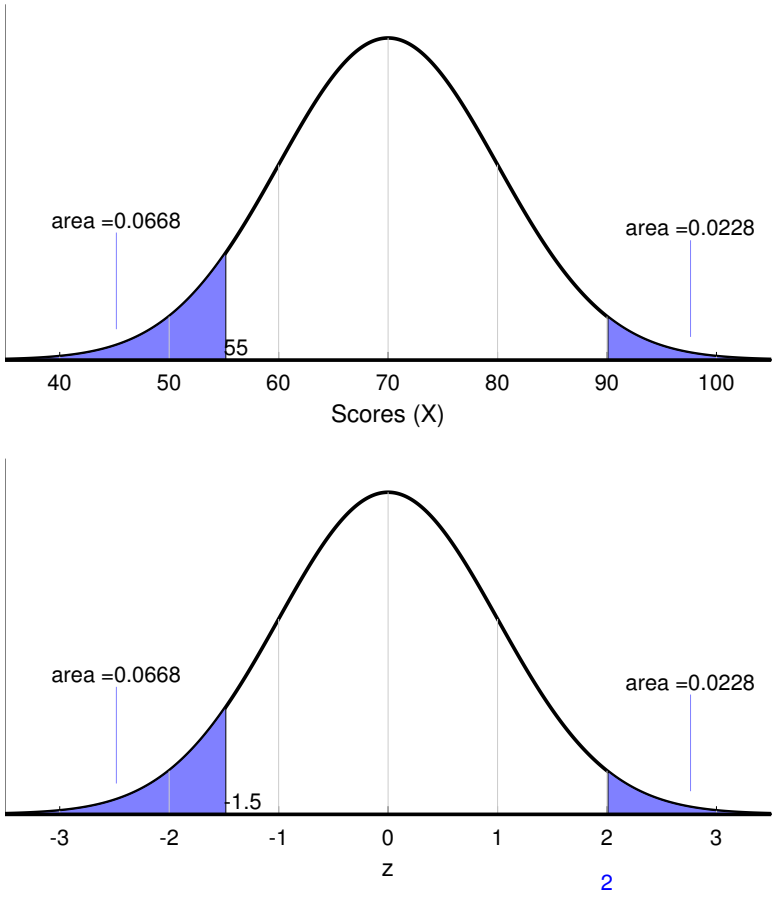
Converting scores to areas

Example: What porportion of scores fall either below 55 or above 90?

This time we need to convert two test scores into z-scores:

$z_1 = \frac{X_1 - \mu}{\sigma} = \frac{55-70}{10} = -1.5$

$z_2 = \frac{X_2 - \mu}{\sigma} = \frac{90-70}{10} = 2$



We then use the z-table to find the two areas and add them up: $0.0668 + 0.0228 = 0.0896$

Converting areas to scores

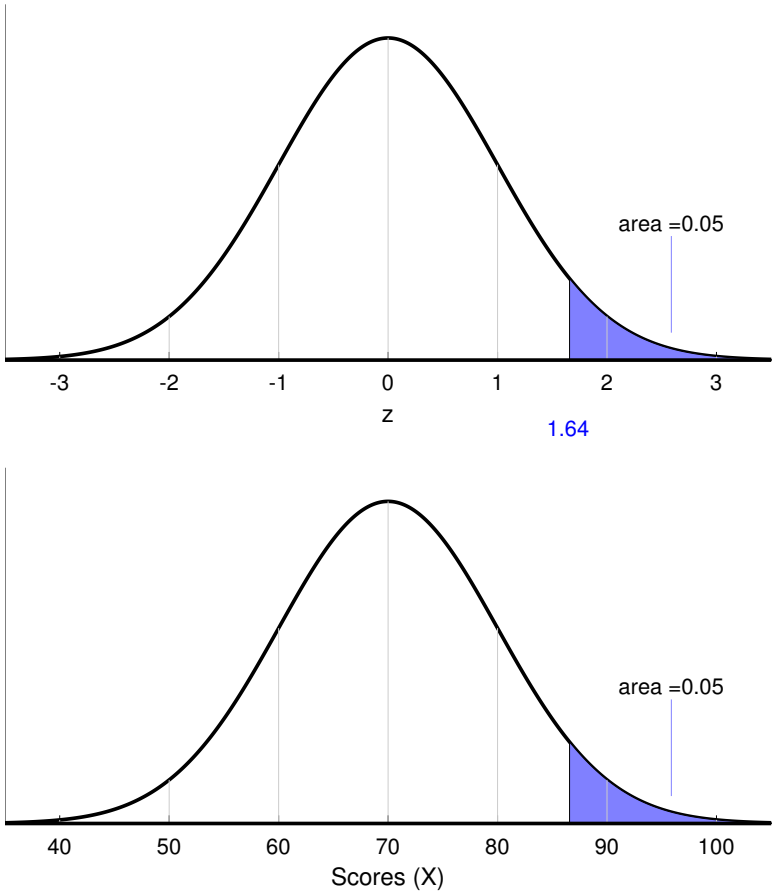
If we're given an area under the standard normal distribution and we need to convert it to a score, X, we first find the z-score for that area in the z-table and use:

$$X = z\sigma + \mu$$

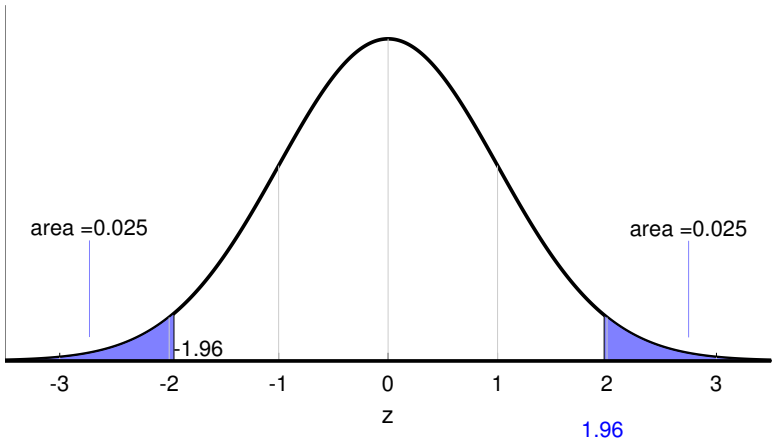
Example: For what test score does 5% of the scores lie above?

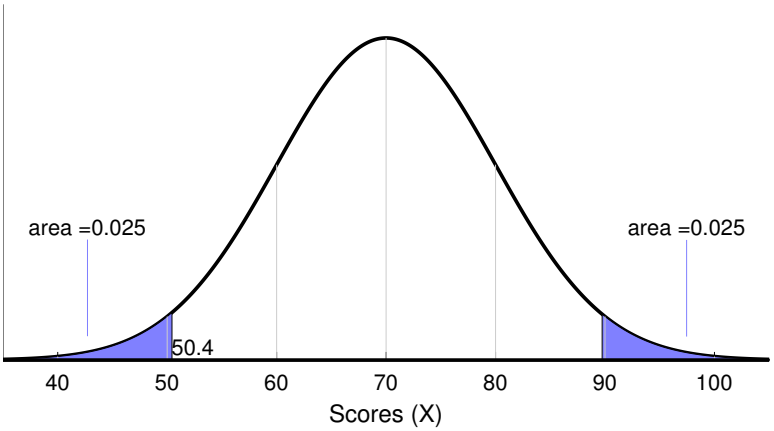
To go from areas to scores we first find the z-score for the area. Using the z-table we find that the z-score for which 5% of the area lies above is z = 1.64. To convert z-scores to test scores we multiply z by the population standard deviation and add the mean:

$X = (z)(\sigma) + \mu = (1.64)(10) + 70 = 86.45$



Example: What test scores bracket the middle 95% of all scores?





We first find the z-scores that bracket the middle 95% of all z scores. This corresponds to finding the z-score for which (100-95)/2 = 2.5% lies above and below. Using the table, we find that z = -1.96 and z = 1.96.

We then convert these two z-scores to test scores:

$X_1 = (z_1)(\sigma) + \mu = (-1.96)(10) + 70 = 50.4$

$X_2 = (z_2)(\sigma) + \mu = (1.96)(10) + 70 = 89.6$

So 95% of all test scores fall between 50.4 and 89.6.

Converting scores to areas and back in R

In the z_table_tutorial we saw how to use the R function 'pnorm' to convert from z-scores to areas and 'qnorm' to convert from areas to z-scores.

The R functions 'pnorm' and 'qnorm' can be also used to convert scores to areas and back (respectively) for any normal distribution with a given mean and standard deviation.

The R commands shown below can be found here: NormalDistribution.R

```
# The Normal Distribution
#
# In the tutorial we learned how to convert raw scores to z-scores
# and back again. For example, given a normal distribution with mean 70
# and standard deviation of 10, the z-score for a raw score of 80 is:
z <- (80-70)/10
print(z)
[1] 1

# We can define variables 'mu' and 'sigma' to make this more general:
mu <- 70
sigma <- 10
```

```
# and define X to be the raw score:
X <- 80
z <- (X-mu)/sigma
print(z)
[1] 1

# To find the area above this score, we can use 'pnorm':
1-pnorm(z)
[1] 0.1586553

# In R there's a shortcut.  'pnorm' (and qnorm) allow you to send in
# means and standard deviations instead of using the default 0 and 1.
# This will give you the same answer:
1-pnorm(80,70,10)
[1] 0.1586553

# or, equivalently:
1-pnorm(X,mu,sigma)
[1] 0.1586553

# Here are a couple more examples:
# The proportion of raw scores that fall below 55 is:
pnorm(55,mu,sigma)
[1] 0.0668072

# The proportion of raw scores that fall above 90 is:
1-pnorm(90,mu,sigma)
[1] 0.02275013

# Converting areas to scores is just as easy.  For example, for the
# normal distribution with mean 70 and standard deviation 10, we
# can calculate the score for which 5% lies above by first finding
# the z-score using qnorm:
z <- qnorm(1-.05)
print(z)
[1] 1.644854

# And then convert to raw scores using our formula:
X <- z*sigma + mu
print(X)
[1] 86.44854

# As with pnorm, there's a shortcut by giving qnorm the mean and
# standard deviation instead of the default 0 and 1.  This should
# give us the same answer:
qnorm(1-.05,mu,sigma)
[1] 86.44854
```

```
# The scores that bracket the middle 95% can be found similarly. The
# lower score is:
qnorm(.05/2,mu,sigma)
[1] 50.40036

# and the upper score is:
qnorm(1-.05/2,mu,sigma)
[1] 89.59964

# We can concatenate these into a single vector:
c(qnorm(.05/2,mu,sigma),qnorm(1-.05/2,mu,sigma))
[1] 50.40036 89.59964
```

Questions

Now it's your turn. Here are 30 problems with answers (including R commands). For all examples, assume that the population is normally distributed. Draw pictures if it helps!

1) A friend tells you that the peace of brains has a mean of 22 and a standard deviation of 6.2,
find the proportion of the peace below 25.9

```
pnorm(25.9,22,6.2)
[1] 0.735336
```
Answer: 0.7353

2) If the health of children has a mean of 87 and a standard deviation of 5.1,
find value for which 63.79 percent of the health fall below.

```
qnorm(0.6379,87,5.1)
[1] 88.79954
```
Answer: 88.80

3) You discover that the softness of cats has a mean of 84 and a standard deviation of 4.3,
find the value for which 4.48 percent of the softness falls above.

```
qnorm(1-0.0448,84,4.3)
[1] 91.2993
```
Answer: 91.30

4) A friend tells you that the amplitude of Seattleites has a mean of 59 and a standard deviation of 5.1,
find the proportion of the amplitude between 60.3 and 68.2

```
pnorm(68.2,59,5.1) - pnorm(60.3,59,5.1)
[1] 0.3637776
```

Answer: 0.3638

5) A Google search shows that the piety of skittles has a mean of 67 and a standard deviation of 5.9,
find value for which 7.25 percent of the piety fall below.

```
qnorm(0.0725,67,5.9)
[1] 58.40121
```
Answer: 58.40

6) Suppose the knowledge of airlines has a mean of 58 and a standard deviation of 4.3,
find the proportion of the knowledge below 55.2 and above 63.4

```
pnorm(55.2,58,4.3) + (1- pnorm(63.4,58,4.3))
[1] 0.3620624
```
Answer: 0.3621

7) Let's assume that the news of weather events has a mean of 42 and a standard deviation of 5.6,
find the proportion of the news below 40 and above 41.2

```
pnorm(40,42,5.6) + (1- pnorm(41.2,42,5.6))
[1] 0.9172909
```
Answer: 0.9173

8) Suppose the importance of chickens has a mean of 90 and a standard deviation of 3.7,
find the proportion of the importance below 85.3

```
pnorm(85.3,90,3.7)
[1] 0.1019942
```
Answer: 0.1020

9) A friend tells you that the quantity of psychology classes has a mean of 21 and a standard deviation of 6,
find values that bracket the middle 2.66 percent of the quantity.

```
c(qnorm((1-0.0266)/2,21,6),qnorm(1-(1-0.0266)/2,21,6))
[1] 20.79993 21.20007
```
Answer: between 20.80 and 21.20

10) If you know that the wind of bananas has a mean of 11 and a standard deviation of 4.2,

find values that bracket the middle 55.39 percent of the wind.

```
c(qnorm((1-0.5539)/2,11,4.2),qnorm(1-(1-0.5539)/2,11,4.2))
[1]  7.799881 14.200119
```

Answer: between 7.80 and 14.20

11) Suppose the price of babies has a mean of 61 and a standard deviation of 5.9,
find the proportion of the price between 60.6 and 64

```
pnorm(64,61,5.9) - pnorm(60.6,61,5.9)
[1] 0.2214659
```
Answer: 0.2215

12) Suppose the culture of fathers has a mean of 38 and a standard deviation of 5.4,
find value for which 77.62 percent of the culture fall below.

```
qnorm(0.7762,38,5.4)
[1] 42.10088
```
Answer: 42.10

13) A friend tells you that the recognition of airlines has a mean of 85 and a standard deviation of 5.4,
find value for which 9.12 percent of the recognition fall below.

```
qnorm(0.0912,85,5.4)
[1] 77.79963
```
Answer: 77.80

14) Suppose the speed of UW undergraduates has a mean of 97 and a standard deviation of 5.4,
find the proportion of the speed below 100.9 and above 103.1

```
pnorm(100.9,97,5.4) + (1- pnorm(103.1,97,5.4))
[1] 0.8942372
```
Answer: 0.8942

15) A Google search shows that the speed of apartments has a mean of 95 and a standard deviation of 2.6,
find value for which 39.39 percent of the speed fall below.

```
qnorm(0.3939,95,2.6)
[1] 94.30016
```
Answer: 94.30

16) Let's assume that the IQ of monkeys has a mean of 9 and a standard deviation of 4.7,
find the proportion of the IQ below 10.5

```
pnorm(10.5,9,4.7)
[1] 0.6251932
```
Answer: 0.6252

17) You discover that the equipment of interest rates has a mean of 6 and a standard deviation of 4.5,
find the proportion of the equipment below 6

```
pnorm(6,6,4.5)
[1] 0.5
```
Answer: 0.5000

18) A friend tells you that the arousal of oceans has a mean of 51 and a standard deviation of 6.6,
find value for which 80.19 percent of the arousal fall below.

```
qnorm(0.8019,51,6.6)
[1] 56.59962
```
Answer: 56.60

19) If you know that the soap of republicans has a mean of 93 and a standard deviation of 5.3,
find the proportion of the soap between 88.1 and 92.6

```
pnorm(92.6,93,5.3) - pnorm(88.1,93,5.3)
[1] 0.292314
```
Answer: 0.2923

20) If the shopping of ping pong balls has a mean of 15 and a standard deviation of 5.1,
find the proportion of the shopping below 12.1

```
pnorm(12.1,15,5.1)
[1] 0.2848045
```
Answer: 0.2848

21) A friend tells you that the piety of bananas has a mean of 59 and a standard deviation of 4.7,
find value for which 66.48 percent of the piety fall below.

```
qnorm(0.6648,59,4.7)
[1] 61.00032
```
Answer: 61.00

22) Suppose the time of brothers has a mean of 83 and a standard deviation of 6.6,
find values that bracket the middle 98.53 percent of the time.

```
c(qnorm((1-0.9853)/2,83,6.6),qnorm(1-(1-0.9853)/2,83,6.6))
[1] 66.89807 99.10193
```
Answer: between 66.90 and 99.10

23) Suppose the importance of oranges has a mean of 73 and a standard deviation of 3.9,
find the proportion of the importance below 77.4

```
pnorm(77.4,73,3.9)
[1] 0.8703834
```
Answer: 0.8704

24) If the importance of daughters has a mean of 33 and a standard deviation of 5.6,
find the value for which 60.56 percent of the importance falls above.

```
qnorm(1-0.6056,33,5.6)
[1] 31.49993
```
Answer: 31.50

25) Suppose the distance of Europeans has a mean of 50 and a standard deviation of 5.4,
find the value for which 83.68 percent of the distance falls above.

```
qnorm(1-0.8368,50,5.4)
[1] 44.70049
```
Answer: 44.70

26) If you know that the safety of computers has a mean of 29 and a standard deviation of 4,
find value for which 62.74 percent of the safety fall below.

```
qnorm(0.6274,29,4)
[1] 30.2999
```
Answer: 30.30

27) Let's assume that the music of laboratory rats has a mean of 89 and a standard deviation of 6.5,
find the proportion of the music below 93.9

```
pnorm(93.9,89,6.5)
[1] 0.7745292
```
Answer: 0.7745

28) Let's assume that the money of psychology classes has a mean of 92 and a standard deviation of 4.5,
find the proportion of the money below 90.9 and above 96

```
pnorm(90.9,92,4.5) + (1- pnorm(96,92,4.5))
[1] 0.5904747
```
Answer: 0.5905

29) A Google search shows that the happiness of underwear has a mean of 92 and a standard deviation of 4.3,
find the proportion of the happiness below 89.4 and above 90.1

```
pnorm(89.4,92,4.3) + (1- pnorm(90.1,92,4.3))
```

```
[1] 0.9434103
```
Answer: 0.9434

30) A Google search shows that the peace of oranges has a mean of 38 and a standard deviation of 4.8,
find the proportion of the peace below 32.9 and above 40.7

```
pnorm(32.9,38,4.8) + (1- pnorm(40.7,38,4.8))
[1] 0.4308921
```
Answer: 0.4309