

# One Factor ANOVA

January 17, 2021

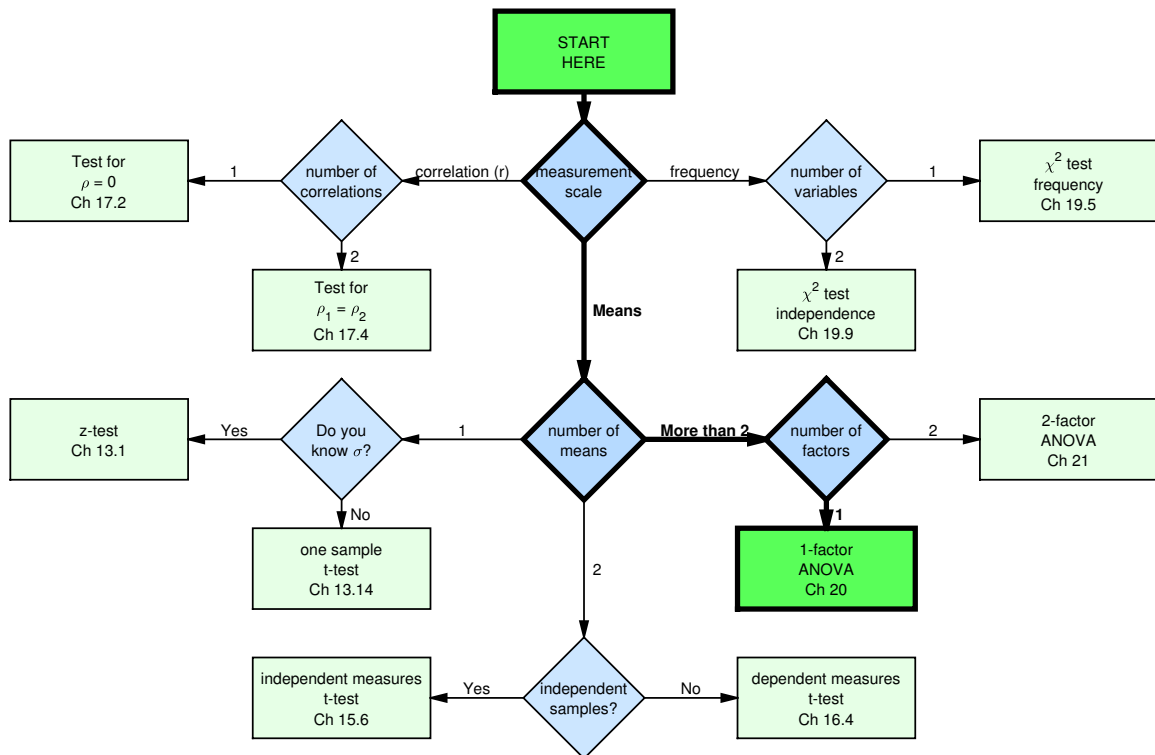
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## One-factor ANOVA

The ANOVA ("ANalysis Of VAriance") is a hypothesis test for the difference between two or more means. This tutorial will describe how to compute the simplest ANOVA - the '1-way' or '1-factor' ANOVA for independent measures.

Here's how to get to the 1-factor ANOVA with the flow chart:



The null hypothesis for the ANOVA is that that all samples are drawn from populations with equal means and equal variances. The alternative hypothesis is that the populations are not the same, but the variances are still the same.

The logic behind the ANOVA is to compare two estimates of the population variance. One is related to the variability between the means and the other is related to the variability of scores within each group. If the null hypothesis is true, these two estimates should be on average the same, so their ratio, called the F-ratio, should be around one. If the population means, differ, then the F-statistic will become larger than one on average. If the measured value of F exceeds a critical value determined by  $\alpha$ , then we reject the null hypothesis that the population means are the same.

We'll show how it works with an example.

### Example 1: Exam scores

Suppose a professor teaches the same small graduate course over three years ( 2019, 2020 and 2021). You want to know if the course grades have varied significantly over these 3 years. We'll run an ANOVA using alpha value of  $\alpha = 0.05$ . Here are the scores:

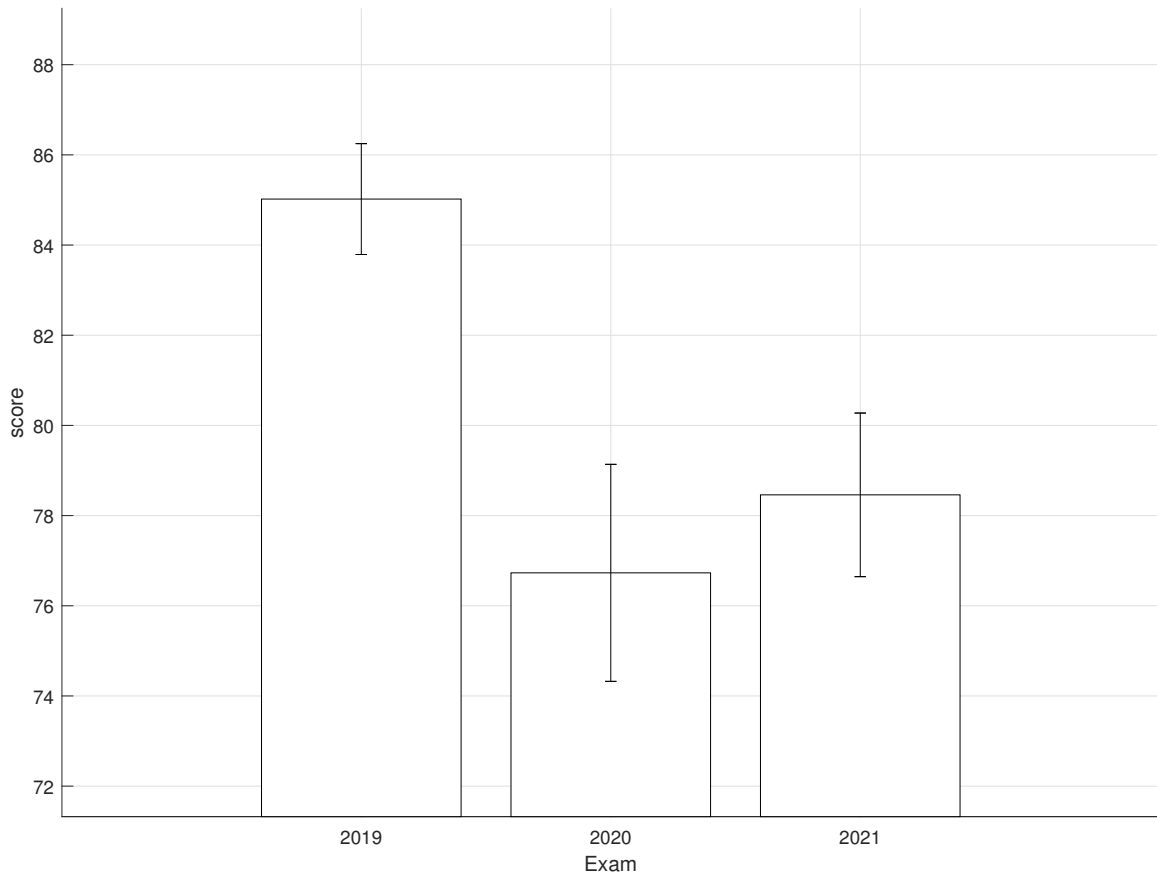
2019	2020	2021
88.4	84	76.6
77.5	81.3	69.6
83.5	80.2	85.4
88.2	72.3	84.4
84.7	87.3	74.3
86.2	67.5	78
82.1	66.8	80.7
89.3	78.2	83.5
85.3	73	73.6

Here is a table of useful statistics from this list:

	2019	2020	2021
n	9	9	9
mean	85.02	76.73	78.46
SS	108.8156	417.0001	236.9624
s	3.6881	7.2198	5.4425
sem	1.2294	2.4066	1.8142

Also, the mean of all scores, called the 'grand mean', is 80.0704.

We'll use the numbers from this table to visualize the data using a bar graph with error bars representing standard errors of the mean, just like we did with the independent-measures t-test:



We can use the same rule of thumb as we did for the independent measures t-test to estimate the significance of the differences between pairs of means. Remember, if the error bars overlap, then we will almost always fail to reject the null hypothesis for a one-tailed test with  $\alpha = .05$ . Since this is the most liberal of all tests, we will also fail to reject for any other choice of tails or alphas. But remember, we can't make separate t-tests on each pair of means - the probability of at least one type I error will be greater than  $\alpha$ .

Looking at the graph, you might guess that there is a statistically significant difference between the means.

### The Numerator of the ANOVA - variability between the means

The numerator of the ANOVA's F-statistic reflects how different the means are from each other. Under the null hypothesis, this number, called the 'variance between', or  $MS_{bet}$  is an estimate of the population variance.

We first calculate the sums of squared deviation of each mean from the grand mean, scaled by the sample size. This is called  $SS_{bet}$ :

$$SS_{bet} = \sum n(\bar{X} - \bar{\bar{X}})^2 =$$

$$(9)(85.02 - 80.0704)^2 + (9)(76.73 - 80.0704)^2 + (9)(78.46 - 80.0704)^2 =$$

$$220.4869 + 100.4244 + 23.3405 = 344.2518$$

The degrees of freedom for  $SS_{bet}$  is the number of groups minus one:  $3 - 1 = 2$ .

We can now calculate  $MS_{bet}$  which is  $SS_{bet}$  divided by its degrees of freedom:

$$MS_{bet} = \frac{SS_{bet}}{df_{bet}} = \frac{344.2518}{2} = 172.1259$$

### The Denominator of the ANOVA - variability within each group

Another estimate of the population variance is the variance of the scores within each group. This number, called the 'variance within' or  $MS_w$  is calculated by first adding up the sum of the squared deviation of each score from it's own group mean:

$$SS_w = \sum_j \sum_i (X_i - \bar{X}_j)^2$$

The table above already gives us the sum of squared deviation for each group, so the total sums of squared within is:

$$SS_w = 108.8156 + 417.0001 + 236.9624 = 762.7781$$

The degrees of freedom for SS within is the number of scores minus the number of groups:  $9 + 9 + 9 - 3 = 24$ . The variance within is SS within divided by its degrees of freedom:

$$MS_w = \frac{SS_w}{df_w} = \frac{762.7781}{24} = 31.7824$$

### The F-statistic - the ratio of variability between and within.

Finally, we'll calculate the observed value of F which is the ratio of  $MS_{bet}$  and  $MS_w$ :

$$F = \frac{MS_{bet}}{MS_w} = \frac{172.1259}{31.7824} = 5.4158$$

### Finding the critical value of F with table E

The distribution of the F-statistic is a family of curves that varies both with  $df_{bet}$  and  $df_w$ . Table E provides critical values of F for  $\alpha = .05$  and  $\alpha = .01$  (bold). The rows of the table correspond to  $df_w$  and the columns are for  $df_{bet}$ . For our example,  $df_{bet}$  is 2 and  $df_w$  is 24. Here's the relevant part of the table:

$df_w   df_b$	1	2	3
23	4.28	3.42	3.03
	<b>7.88</b>	<b>5.66</b>	<b>4.76</b>
24	4.26	3.4	3.01
	<b>7.82</b>	<b>5.61</b>	<b>4.72</b>
25	4.24	3.39	2.99
	<b>7.77</b>	<b>5.57</b>	<b>4.68</b>

Looking at the row for  $df_{bet} = 2$  and the column for  $df_w = 24$ , we see that in bold the critical value for F with  $\alpha = 0.05$  is 3.4.

Remember, both the numerator and the denominator for the ANOVA are measures of the population variance under the null hypothesis. If the null hypothesis is false, then the means will have greater variability, so the numerator will grow large compared to the denominator. Therefore, a large value of F is evidence against the null hypothesis. Specifically, if our observed value of F is greater than the critical value, then we reject the null hypothesis and conclude that there is a significant difference between the means.

For our example, since our observed value of F (5.4158) is greater than the critical value of F (3.4), we reject the null hypothesis.

Using APA format, we state: "There is a significant difference between the means of the 3 exams,  $F(2,24) = 5.4158$ ,  $p < 0.05$ "

We can also use the F-calculator in the Excel spreadsheet to find the actual p-value:

Convert $\alpha$ to F			
$df_b$	$df_w$	$\alpha$	F
2	24	0.05	3.4

Convert F to $\alpha$			
$df_b$	$df_w$	F	$\alpha$
2	24	5.4158	0.0115

Using this p-value we can conclude using APA format: "There is a significant difference between the means of the 3 exams,  $F(2,24) = 5.4158$ ,  $p = 0.0115$ "

## The Summary Table

The statistics used to generate the F statistic for an ANOVA are traditionally reported in a summary table like this:

	SS	df	MS	F	$F_{crit}$	p-value
Between	344.2518	2	172.1259	5.4158	3.4	0.0115
Within	762.7781	24	31.7824			
Total	1107.1563	26				

In this next example we'll use this table to keep track of our values.

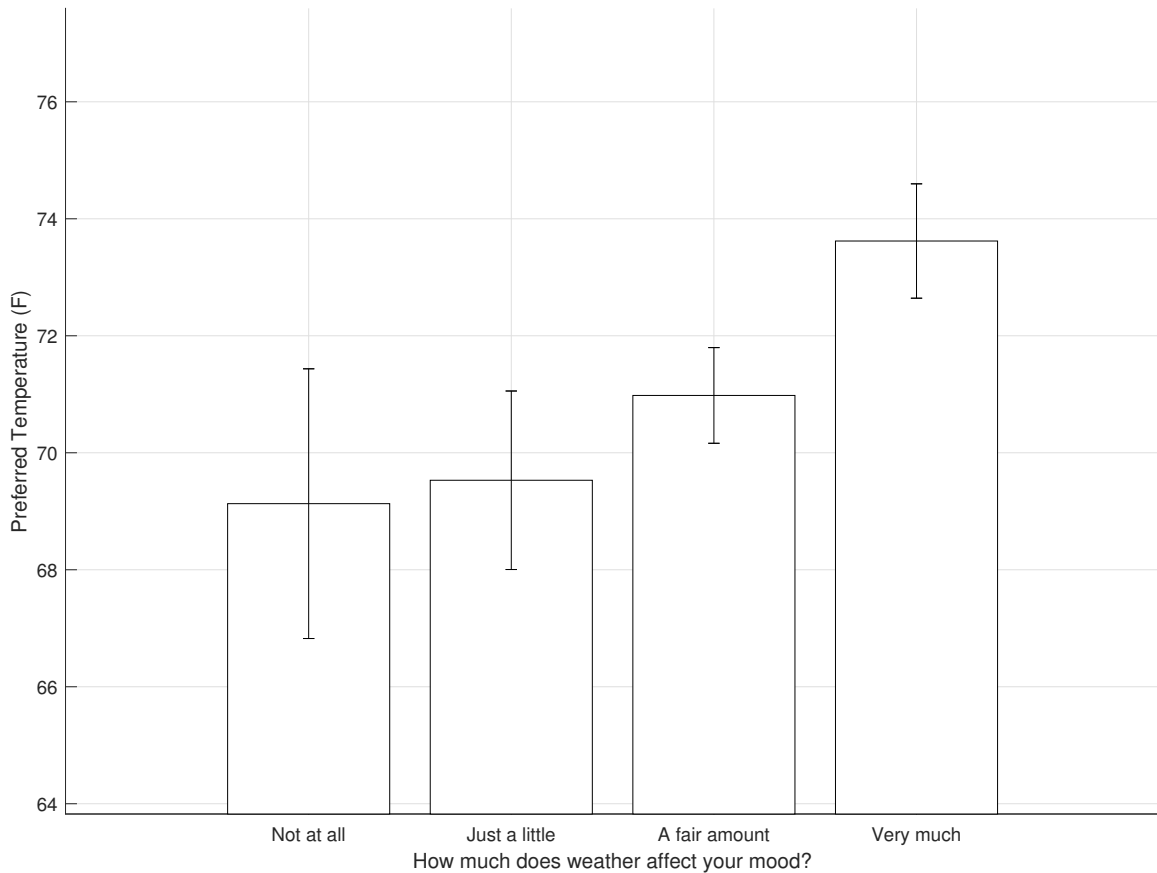
### Example 2: Preferred temperature for weather sensitivity

At the beginning of the quarter I surveyed you for your preferred outdoor temperature. I also asked you how much weather affected your mood with the options of Not at all, Just a little, A fair amount and Very much. Let's see if there is a significant difference between the preferred temperatures across these 4 options. We'll use  $\alpha = 0.05$  again. Here's a table of statistics:

	Not at all	Just a little	A fair amount	Very much
n	12	35	63	40
mean	69.13	69.53	70.98	73.62
SS	701.5468	2770.8755	2612.9852	1490.838
s	7.986	9.0275	6.4919	6.1828
sem	2.3054	1.5259	0.8179	0.9776

Totals:	
n	150
grand mean	71.1993
SS <sub>total</sub>	7961.6499

Here's a graph of the means with error bars representing the standard error of the means:



We can easily fill in some of the entries in the summary table. I've told you that  $SS_{total} = 7961.6499$ . The df for  $SS_{total}$  is the total sample size minus 1:  $150 - 1 = 149$ .

$SS_w$  is just the sum of SS<sub>w</sub> for each cell:  $701.5468 + 2770.8755 + 2612.9852 + 1490.838 = 7576.2455$ . The df for  $SS_w$  is the total sample size minus the number of groups:  $150 - 4 = 146$ .

	SS	df	MS	F	Fcrit	p-value
Between						
Within	7576.2455	146				
Total	7961.6499	149				

$SS_{bet}$  can be hard to calculate, but since we know that  $SS_{total} = SS_w + SS_{bet}$ , we know that  $SS_{bet} = 7961.6499 - 7576.2455 = 385.4044$ . The degrees of freedom is  $4 - 1 = 3$ .

Just like for the last example, the variance within and between are just the SS divided by their df's:

$$MS_{bet} = \frac{SS_{bet}}{df_{bet}} = \frac{386.3351}{3} = 128.7784$$



$$MS_w = \frac{SS_w}{df_w} = \frac{7576.2455}{146} = 51.8921$$

Finally, the F statistic is their ratio:

$$F = \frac{MS_{bet}}{MS_w} = \frac{128.7784}{51.8921} = 2.4817$$

The table now looks like this:

	SS	df	MS	F	$F_{crit}$	p-value
Between	386.3351	3	128.7784	2.4817		
Within	7576.2455	146	51.8921			
Total	7961.6499	149				

The critical value of F for  $\alpha = 0.05$  and  $df_{bet} = 3$  and  $df_w = 146$  can be found in table E.

Since  $df_w = 146$  is not in the table, we'll use 125:

$df_w   df_b$	3
125	2.68
	<b>3.94</b>

If you use the F calculator the Excel spreadsheet you'll see that the p-value is 0.0633:

Convert $\alpha$ to F			
$df_b$	$df_w$	$\alpha$	F
3	146	0.05	2.67

Convert F to $\alpha$			
$df_b$	$df_w$	F	$\alpha$
3	146	2.4817	0.0633

The final summary table is:

	SS	df	MS	F	$F_{crit}$	p-value
Between	386.3351	3	128.7784	2.4817	2.68	0.0633
Within	7576.2455	146	51.8921			
Total	7961.6499	149				

The observed value of F (2.4817) is not greater than the critical value of F (2.68), we fail to reject the null hypothesis.

Using APA format, we state: "There is not a significant difference in preferred outdoor temperature across the 4 levels of the survey about how weather affects mood,  $F(3,146) = 2.4817$ ,  $p = 0.0633$ ."

Note that the ANOVA doesn't tell us anything about exactly how preferred temperature varies with how weather affects mood. Looking at the bar graph, it appears that there is an upward trend so that the more weather affects mood, the higher the preferred temperature. But to make statistical conclusions like this requires a post-hoc analysis that allows you to compare specific means to other means.

## Effect Size

There are several measures of effect size for an ANOVA, and most software packages will spit out more than one. Probably the most common is  $\eta^2$ , or 'eta squared', which is the proportion of total variance in your data that is attributed to the effect driving the difference between the means. It's simply the ratio of  $SS_{bet}$  to  $SS_{total}$ :

$$\eta^2 = \frac{SS_{bet}}{SS_{total}}$$

From our last example:

$$\eta^2 = \frac{386.335}{7961.65} = 0.0485$$

If  $SS_{bet} = 0$ , then  $\eta^2 = 0$ . This is small as  $\eta^2$  can get. It happens when there are no differences between the means, so all of the variability in your data is attributed to variability within each group.

If  $SS_w = 0$ , then  $SS_{bet} = SS_{total}$ , which makes  $\eta^2 = 1$ . This is as big as  $\eta^2$  can get. It happens when the means are different, but there is no variability within each group.

$\eta^2$  is simple, commonly used, but tends to overestimate effect size for larger number of treatments. Just so you know,  $\eta^2$  is sometimes called 'partial eta-squared' for some reason.

A rule of thumb for interpreting the size of  $\eta^2$  is:

0.01	small
0.06	medium
0.14	large

Note that these values of small, medium and large are very different than those used to interpret effect size for t-tests (Cohen's d).

## Using R to conduct a one factor ANOVA

Computing a one factor ANOVA is easy using the 'aov' function. The tricky part is pulling out the values from the output, and using ggplot to plot bar graphs and error bars. The following R script conducts the test in Example 2 on preferred temperature and how much weather affects mood.

The R commands shown below can be found here: [OneFactorANOVA.R](#)

```
# 1-factor ANOVA
#
# Conducting a 1-factor ANOVA from our survey data is easy using the 'aov' function.
# Interpreting and plotting the results will require two libraries, 'ggplot' for
# plotting and 'broom' for cleaning up the output of 'aov'.
# install.packages("ggplot2")
```

```

library(ggplot2)
Warning message:
package 'ggplot2' was built under R version 4.0.3

# Here we'll run the ANOVA from the 'one_factor_ANOVA_tutorial' which looked at the mean
# preferred temperature across the levels of how much weather affects your mood.
rm(list = ls())
# load in the data. Note the new option 'na.strings = ""'. This converts missing
# responses in nominal data to 'NA's.
survey <- read.csv("http://www.courses.washington.edu/psy315/datasets/Psych315W21survey.csv",
                  na.strings = "")
# To run a one-factor ANOVA using the function 'lm' which stands for 'linear model'. It's
# beyond the scope of this class, but if you're interested, an ANOVA can be run as a fit
# of a 'linear model' to the data. 'lm' is a powerful function in R that can do much more
# than ANOVA.

# We give 'lm' a dependent (ratio scale) variable and tell it to sort this variable across
# 'levels' of a nominal scale variable. For this example, the ratio scale variable is
# 'temperature' and the nominal scale variable is 'weather'.
#
# The second input is the data itself, which is a data frame that should at least have variables
# that match the ratio and nominal scale names. The third input tells 'lm' what to do
# with 'NA's. Here we'll use 'na.action = na.omit' which removes data with NA's in either
# the ratio scale or nominal scale variables.
out <- lm(temperature ~ weather, data = survey, na.action = na.omit)
# It's hard to find the p-value and other statistics in this output, but there's
# a function called 'anova' that takes the output of 'lm' and gives you a standard
# anova table:
anova.out <- anova(out)
# Hopefully these numbers match the values in the tutorial!

# Useful fields in anova.out are 'Df', 'F value', and 'Pr(>F)' which is the p-value.
# We can use this output and 'sprintf' to present the results in APA format:
sprintf('F(%g,%g) = %0.2f, p = %0.4f', anova.out$Df[1], anova.out$Df[2],
        anova.out$`F value`[1], anova.out$`Pr(>F)`[1])
[1] "F(3,146) = 2.48, p = 0.0638"

# Effect size: eta-squared is the ratio of SS between and SS total
# The output 'anova.out' contains a field 'Sum Sq' that contains SS bet and SS within:
anova.out$`Sum Sq`
[1] 385.406 7576.244

# This is how you can calculate eta-squared:
SS <- anova.out$`Sum Sq`
# Remember, SS total is SS between + SS within, so:
eta.squared <- SS[1]/(SS[1]+SS[2])
# Displaying the result:
sprintf('Eta-squared = %0.4f', eta.squared)

```

```

[1] "Eta-squared = 0.0484"

# Next we'll make a bar plot of means across levels with error bars representing the
# standard error of the mean. This will use 'ggplot', much like we did in
# 'TwoSampleIndependentTTest.R'.

# This generates a data frame containing statistics for each level of 'weather'. Note
# the way we're removing NA's
summary <- data.frame(
  mean <- tapply(survey$temperature,survey$weather,mean,na.rm = TRUE),
  n <- tapply(survey$temperature,survey$weather,function(x) sum(!is.na(x))),
  sd <- tapply(survey$temperature,survey$weather,sd,na.rm = TRUE))
summary$sem <- summary$sd/sqrt(summary$n)
colnames(summary) = c("mean","n","sd","sem")
summary

```

	mean	n	sd	sem
A fair amount	70.98413	63	6.491911	0.8179039
Just a little	69.53143	35	9.027532	1.5259314
Not at all	69.13333	12	7.986048	2.3053736
Very much	73.61750	40	6.182767	0.9775813

```

# The levels in 'weather' come out in alphabetical order. To re-order them
# from 'Not at all' to 'Very much' we'll define a list in right order
# and use this list in ggplot
levels <- row.names(summary)
levels <- levels[c(3,2,1,4)]
# Bar graph:
# Define y limits for the bar graph
ylimlimit <- c(min(summary$mean-1.5*summary$sem),
               max(summary$mean+1.5*summary$sem))
# Plot bar graph with error bar as one standard error (standard error of the mean/SEM)
ggplot(summary, aes(x = row.names(summary), y = mean)) +
  xlab("How does weather affect your mood?") +
  geom_bar(position = position_dodge(), stat="identity", fill="blue") +
  geom_errorbar(aes(ymin=mean-sem, ymax=mean+sem),width = .5) +
  scale_y_continuous(name = "Preferred Temperature (F)") +
  scale_x_discrete(limits = levels) +
  coord_cartesian(ylim=ylimlimit)

```

## Questions

Your turn again. Here are 7 practice questions based on the class survey, followed by their answers.

1) From our survey we can calculate Caffeine consumption as a function of how much you said you liked math. Here is a table of statistics based on our survey:

	Not at all	Just a little	A fair amount	Very much
n	14	56	75	6
mean	1.21	1.01	1.41	3
$SS$	7.8574	41.7456	354.1363	108

$n_{total}$	151
grand mean	1.3065
$SS_{total}$	534.834

Calculate the standard errors of the mean for each of the 4 groups.

Make a bar graph of the means for each of the 4 groups with error bars as the standard error of the means.

Using an alpha value of  $\alpha = 0.05$ , is there difference in Caffeine consumption across the 4 groups of students who vary their preference for math?

2) From our survey we can calculate the preferred outdoor temperature as a function of how much weather affects your mood. Here is a table of statistics based on our survey:

	agree	disagree	neutral	strongly agree	strongly disagree
n	47	37	47	15	6
mean	1.38	1.11	1.57	1.13	2
$SS$	73.1068	15.5677	157.4903	45.7335	6

$n_{total}$	152
grand mean	1.375
$SS_{total}$	305.625

Calculate the standard errors of the mean for each of the 5 groups.

Make a bar graph of the means for each of the 5 groups with error bars as the standard error of the means.

Using an alpha value of  $\alpha = 0.05$ , is there difference in number of siblings across the 5 groups of how siblings varies with introversion?

3) From our survey we can calculate how much you drink across voting preferences. Here is a table of statistics based on our survey:

	Democrat	I never (or can't) vote	Independent
n	101	39	7
mean	2.09	0.59	2.57
<i>SS</i>	1230.3581	67.4359	69.7143
<i>n</i> <sub>total</sub>	147		
grand mean	1.7177		
<i>SS</i> <sub>total</sub>	1436.534		

Calculate the standard errors of the mean for each of the 3 groups.

Make a bar graph of the means for each of the 3 groups with error bars as the standard error of the means.

Using an alpha value of  $\alpha = 0.01$ , is there difference in drink per week across the 3 groups of voting preference for students?

4) From our survey we can calculate how well you think you'll do on Exam 1 as a function of how much you think you'll like this class. Here is a table of statistics based on our survey:

	Not at all	Just a little	A fair amount	Very much
n	14	56	75	6
mean	81.64	84.04	86.32	93.67
<i>SS</i>	1127.2144	2873.9296	4184.32	259.3334

<i>n</i> <sub>total</sub>	151
grand mean	85.3311
<i>SS</i> <sub>total</sub>	9219.4437

Calculate the standard errors of the mean for each of the 4 groups.

Make a bar graph of the means for each of the 4 groups with error bars as the standard error of the means.

Using an alpha value of  $\alpha = 0.01$ , is there difference in predicted Exam 1 score across the 4 groups of how much you think you'll like Psych 315?

5) Suppose you want to know if how much students drink varies with how much they exercise. Here is a table of statistics based on our survey:

	Not at all	Just a little	A fair amount	Very much
n	8	61	52	30
mean	0.75	1.51	1.55	2.6
<i>SS</i>	9.5	699.2461	369.63	327.2

$n_{total}$	151
grand mean	1.6987
$SS_{total}$	1440.5397

Calculate the standard errors of the mean for each of the 4 groups.

Make a bar graph of the means for each of the 4 groups with error bars as the standard error of the means.

Using an alpha value of  $\alpha = 0.01$ , is there difference in alcoholic drinks across the 4 groups of students who exercise?

6) From our survey we can compute how many hours per week students play video games as a function of how much they exercise. Here is a table of statistics based on our survey:

	Not at all	Just a little	A fair amount	Very much
n	8	61	52	30
mean	2.06	1.13	1.04	0.57
<i>SS</i>	24.2188	172.9189	108.4232	36.867

$n_{total}$	151
grand mean	1.0344
$SS_{total}$	357.9609

Calculate the standard errors of the mean for each of the 4 groups.

Make a bar graph of the means for each of the 4 groups with error bars as the standard error of the means.

Using an alpha value of  $\alpha = 0.05$ , is there difference in hours of video game playing per week across the 4 groups of students who exercise?

7) From our survey we can see if there is a difference in the heights of students across different levels of exercise. Here is a table of statistics based on our survey:

	Just a little	A fair amount	Very much
n	61	52	30
mean	65.33	66	66.73
<i>SS</i>	745.4429	638	435.867

$n_{total}$	143
grand mean	65.8671
$SS_{total}$	1860.4755

Calculate the standard errors of the mean for each of the 3 groups.

Make a bar graph of the means for each of the 3 groups with error bars as the standard error of the means.

Using an alpha value of  $\alpha = 0.05$ , is there difference in height across the 3 groups of students who exercise?



## Answers

1)

$$SS_{bet} = (14)(1.21 - 1.3065)^2 + (56)(1.01 - 1.3065)^2 + (75)(1.41 - 1.3065)^2 + (6)(3 - 1.3065)^2 = 23.0645$$

$$MS_{bet} = \frac{23.0645}{3} = 7.6882$$

$$SS_w = SS_{total} - SS_{bet} = 534.834 - 23.0645 = 511.7393$$

$$MS_w = \frac{511.7393}{147} = 3.48$$

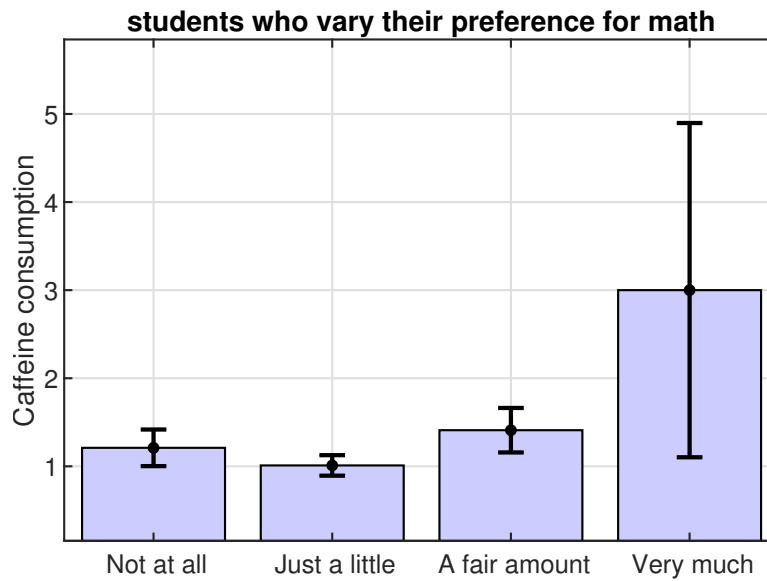
$$F = \frac{7.6882}{3.4812} = 2.21$$

$$F_{crit} = 2.68 \text{ (with } df_{bet} = 3, df_w = 125 \text{ and } \alpha = 0.05)$$

	SS	df	MS	F	$F_{crit}$	p-value
Between	23.0645	3	7.6882	2.2085	2.68	0.0896
Within	511.7393	147	3.4812			
Total	534.834	150				

We fail to reject  $H_0$ .

There is not a significant difference in mean Caffeine consumption across the 4 groups of students who vary their preference for math,  $F(3,147) = 2.21$ ,  $p = 0.0896$ .



	SS	df	MS	F	$F_{crit}$	p-value
Between	23.0645	3	7.6882	2.2085	2.68	0.0896
Within	511.7393	147	3.4812			
Total	534.834	150				

2)

$$SS_{bet} = (47)(1.38 - 1.375)^2 + (37)(1.11 - 1.375)^2 + (47)(1.57 - 1.375)^2 + (15)(1.13 - 1.375)^2 + (6)(2 - 1.375)^2 = 7.6308$$

$$MS_{bet} = \frac{7.6308}{4} = 1.9077$$

$$SS_w = SS_{total} - SS_{bet} = 305.625 - 7.6308 = 297.898$$

$$MS_w = \frac{297.8983}{147} = 2.03$$

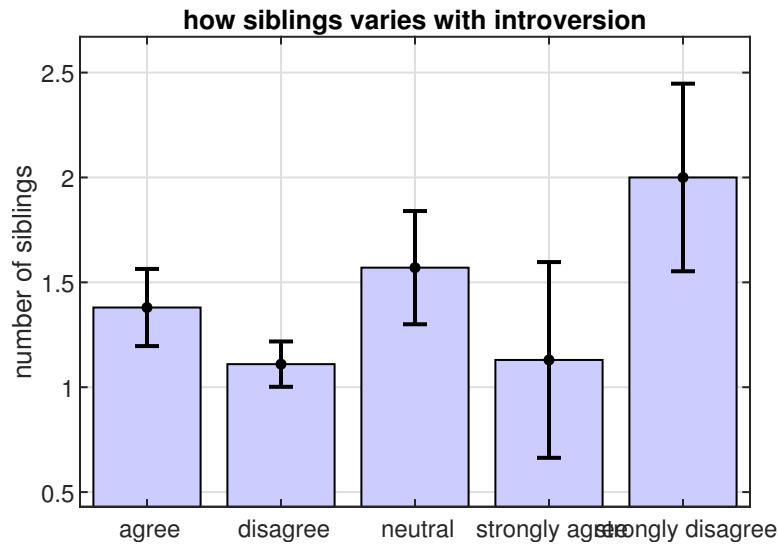
$$F = \frac{1.9077}{2.0265} = 0.94$$

$F_{crit} = 2.44$  (with  $df_{bet} = 4$ ,  $df_w = 125$  and  $\alpha = 0.05$ )

	SS	df	MS	F	$F_{crit}$	p-value
Between	7.6308	4	1.9077	0.9414	2.44	0.4419
Within	297.8983	147	2.0265			
Total	305.625	151				

We fail to reject  $H_0$ .

There is not a significant difference in mean number of siblings across the 5 groups of how siblings varies with introversion,  $F(4,147) = 0.94$ ,  $p = 0.4419$ .



	SS	df	MS	F	$F_{crit}$	p-value
Between	7.6308	4	1.9077	0.9414	2.44	0.4419
Within	297.8983	147	2.0265			
Total	305.625	151				

3)

$$SS_{bet} = (101)(2.09 - 1.7177)^2 + (39)(0.59 - 1.7177)^2 + (7)(2.57 - 1.7177)^2 = 68.6808$$

$$MS_{bet} = \frac{68.6808}{2} = 34.3404$$

$$SS_w = SS_{total} - SS_{bet} = 1436.53 - 68.6808 = 1367.51$$

$$MS_w = \frac{1367.5083}{144} = 9.5$$

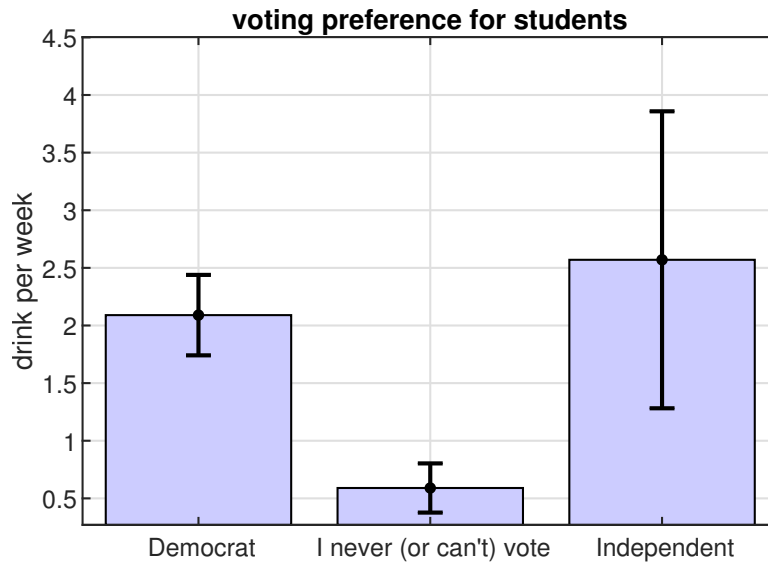
$$F = \frac{34.3404}{9.4966} = 3.62$$

$$F_{crit} = 4.78 \text{ (with } df_{bet} = 2, df_w = 125 \text{ and } \alpha = 0.01)$$

	SS	df	MS	F	$F_{crit}$	p-value
Between	68.6808	2	34.3404	3.6161	4.78	0.0294
Within	1367.5083	144	9.4966			
Total	1436.534	146				

We fail to reject  $H_0$ .

There is not a significant difference in mean drink per week across the 3 groups of voting preference for students,  $F(2,144) = 3.62$ ,  $p = 0.0294$ .



	SS	df	MS	F	$F_{crit}$	p-value
Between	68.6808	2	34.3404	3.6161	4.78	0.0294
Within	1367.5083	144	9.4966			
Total	1436.534	146				

4)

$$SS_{bet} = (14)(81.64 - 85.3311)^2 + (56)(84.04 - 85.3311)^2 + (75)(86.32 - 85.3311)^2 + (6)(93.67 - 85.3311)^2 = 774.655$$

$$MS_{bet} = \frac{774.6554}{3} = 258.2185$$

$$SS_w = SS_{total} - SS_{bet} = 9219.44 - 774.655 = 8444.8$$

$$MS_w = \frac{8444.7974}{147} = 57.45$$

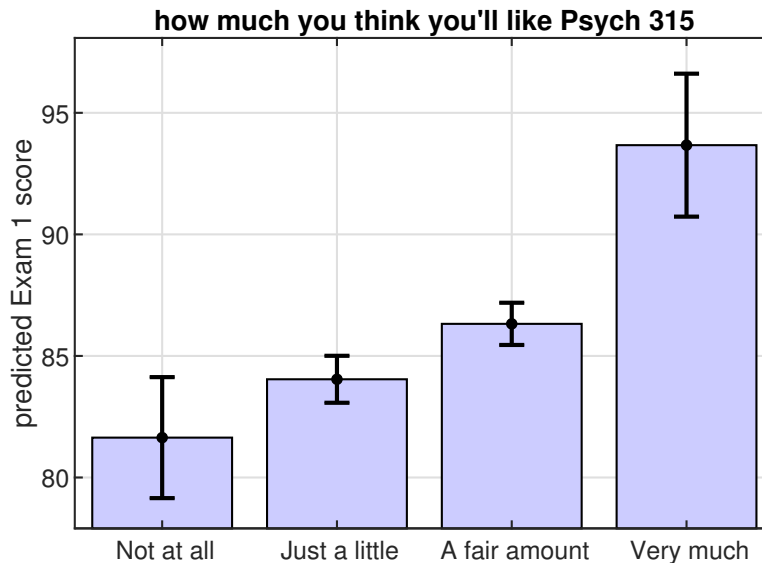
$$F = \frac{258.2185}{57.4476} = 4.49$$

$F_{crit} = 3.94$  (with  $df_{bet} = 3$ ,  $df_w = 125$  and  $\alpha = 0.01$ )

	SS	df	MS	F	$F_{crit}$	p-value
Between	774.6554	3	258.2185	4.4949	3.94	0.0048
Within	8444.7974	147	57.4476			
Total	9219.4437	150				

We reject  $H_0$ .

There is a significant difference in mean predicted Exam 1 score across the 4 groups of how much you think you'll like Psych 315,  $F(3,147) = 4.49$ ,  $p = 0.0048$ .



	SS	df	MS	F	$F_{crit}$	p-value
Between	774.6554	3	258.2185	4.4949	3.94	0.0048
Within	8444.7974	147	57.4476			
Total	9219.4437	150				



5)

$$SS_{bet} = (8)(0.75 - 1.6987)^2 + (61)(1.51 - 1.6987)^2 + (52)(1.55 - 1.6987)^2 + (30)(2.6 - 1.6987)^2 = 34.8924$$

$$MS_{bet} = \frac{34.8924}{3} = 11.6308$$

$$SS_w = SS_{total} - SS_{bet} = 1440.54 - 34.8924 = 1405.58$$

$$MS_w = \frac{1405.5761}{147} = 9.56$$

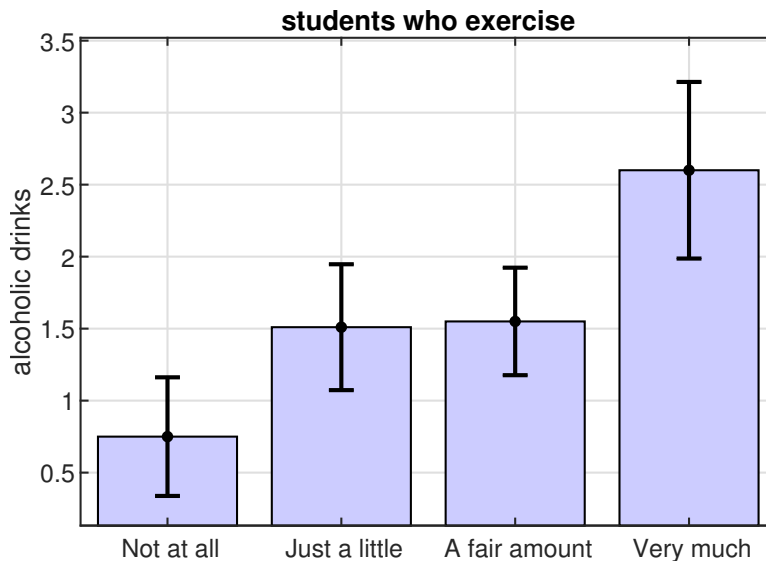
$$F = \frac{11.6308}{9.5617} = 1.22$$

$F_{crit} = 3.94$  (with  $df_{bet} = 3$ ,  $df_w = 125$  and  $\alpha = 0.01$ )

	SS	df	MS	F	$F_{crit}$	p-value
Between	34.8924	3	11.6308	1.2164	3.94	0.3059
Within	1405.5761	147	9.5617			
Total	1440.5397	150				

We fail to reject  $H_0$ .

There is not a significant difference in mean alcoholic drinks across the 4 groups of students who exercise,  $F(3,147) = 1.22$ ,  $p = 0.3059$ .



	SS	df	MS	F	$F_{crit}$	p-value
Between	34.8924	3	11.6308	1.2164	3.94	0.3059
Within	1405.5761	147	9.5617			
Total	1440.5397	150				

6)

$$SS_{bet} = (8)(2.06 - 1.0344)^2 + (61)(1.13 - 1.0344)^2 + (52)(1.04 - 1.0344)^2 + (30)(0.57 - 1.0344)^2 = 15.444$$

$$MS_{bet} = \frac{15.444}{3} = 5.148$$

$$SS_w = SS_{total} - SS_{bet} = 357.961 - 15.444 = 342.428$$

$$MS_w = \frac{342.4279}{147} = 2.33$$

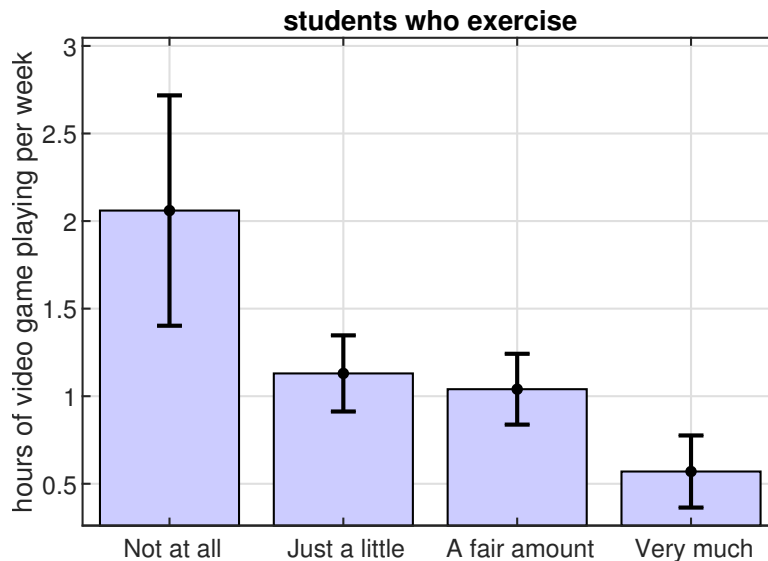
$$F = \frac{5.148}{2.3294} = 2.21$$

$F_{crit} = 2.68$  (with  $df_{bet} = 3$ ,  $df_w = 125$  and  $\alpha = 0.05$ )

	SS	df	MS	F	$F_{crit}$	p-value
Between	15.444	3	5.148	2.21	2.68	0.0894
Within	342.4279	147	2.3294			
Total	357.9609	150				

We fail to reject  $H_0$ .

There is not a significant difference in mean hours of video game playing per week across the 4 groups of students who exercise,  $F(3,147) = 2.21$ ,  $p = 0.0894$ .



	SS	df	MS	F	$F_{crit}$	p-value
Between	15.444	3	5.148	2.21	2.68	0.0894
Within	342.4279	147	2.3294			
Total	357.9609	150				

7)

$$SS_{bet} = (61)(65.33 - 65.8671)^2 + (52)(66 - 65.8671)^2 + (30)(66.73 - 65.8671)^2 = 40.8534$$

$$MS_{bet} = \frac{40.8534}{2} = 20.4267$$

$$SS_w = SS_{total} - SS_{bet} = 1860.48 - 40.8534 = 1819.31$$

$$MS_w = \frac{1819.3099}{140} = 13$$

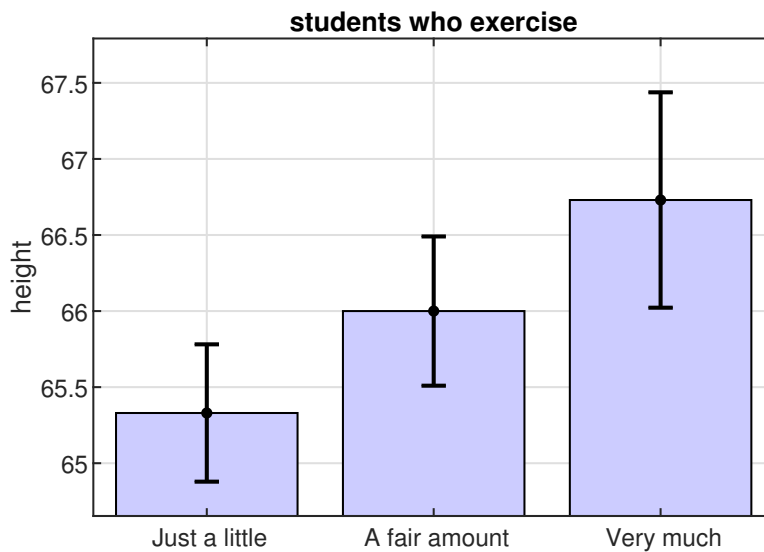
$$F = \frac{20.4267}{12.9951} = 1.57$$

$F_{crit} = 3.07$  (with  $df_{bet} = 2$ ,  $df_w = 125$  and  $\alpha = 0.05$ )

	SS	df	MS	F	$F_{crit}$	p-value
Between	40.8534	2	20.4267	1.5719	3.07	0.2113
Within	1819.3099	140	12.9951			
Total	1860.4755	142				

We fail to reject  $H_0$ .

There is not a significant difference in mean height across the 3 groups of students who exercise,  $F(2,140) = 1.57$ ,  $p = 0.2113$ .



	SS	df	MS	F	$F_{crit}$	p-value
Between	40.8534	2	20.4267	1.5719	3.07	0.2113
Within	1819.3099	140	12.9951			
Total	1860.4755	142				