

The t-test for one mean

January 9, 2021

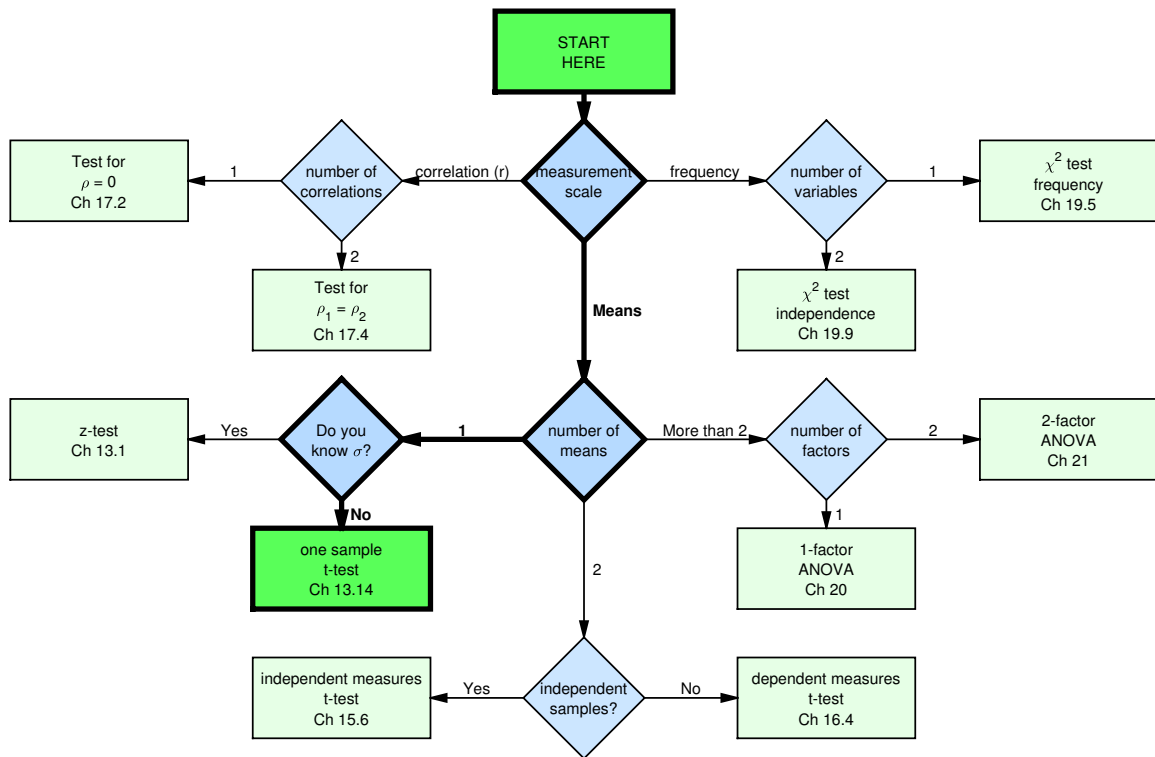
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Happy birthday to Nian Tong!

The one sample t-test is used to compare a **single mean** to an expected mean under the null hypothesis when you **don't know the standard deviation of the population**.

Here's the one sample t-test on our flow-chart:



Recall how for the z-test we used the population standard deviation to calculate the standard error of the mean:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

We then calculated our z-score by converting our observed mean into standard deviation units:

$$z = \frac{(\bar{x} - \mu_{hyp})}{\sigma_{\bar{x}}}$$

Usually we don't know the population standard deviation σ . But we do have the standard deviation of our sample, s_x which is our best guess (and an unbiased estimate) of σ .

It would be nice if we could simply replace the population standard deviation σ with our sample standard deviation s_x and run a z-test. But our sample standard deviation s_x (and the standard error $s_{\bar{x}}$) is a 'random variable', meaning that it varies from sample to sample, just like the mean. This means that if we substitute $s_{\bar{x}}$ for $\sigma_{\bar{x}}$ we won't get a z-distribution. Instead, it has it's own shape, called the **t-distribution**.

$$t = \frac{(\bar{x} - \mu_{hyp})}{s_{\bar{x}}}$$

I warned you earlier that the formula for the standard deviation was going to change half way through the class. Well, here we are. The original formula for the standard deviation

uses the sample size in the denominator:

$$s_x = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

This is the correct formula if we just want to know the standard deviation of our sample, and don't want to use it as a guess for the population standard deviation.

For complicated (but interesting) reasons, we shouldn't use this formula if we want to use the sample standard deviation as an estimate for the standard deviation of the population that the sample came from. It turns out that this formula will give you a value that is, on average, too small. Instead, we need to replace the n with $n - 1$ in the denominator. The new formula is:

$$s_x = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

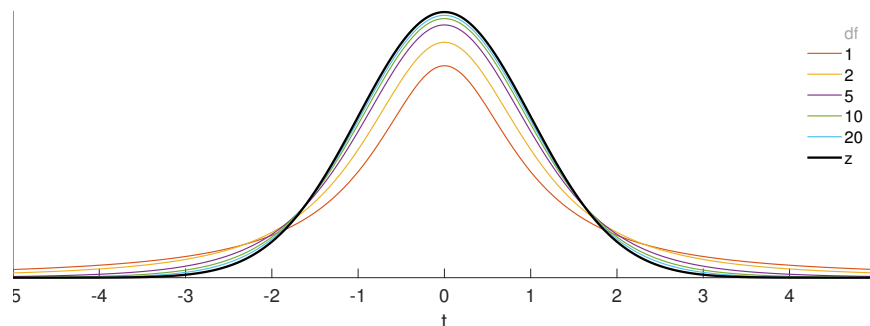
This formula is the more common formula used to calculate sample standard deviations. This is because we are usually using s_x as an estimate of the population standard deviation, σ_x . In fact, all software packages by default use this formula.

This formula for s_x is called the 'sample standard deviation' as opposed to the old formula which is called the 'population standard deviation'.

In any case, once sample sizes get large, the difference between n and $n-1$ becomes pretty small, so the differences in the formulas aren't large.

Using this new formula, the t-distribution looks a lot like the z-distribution, especially for large sample sizes when the standard deviation of our sample is a better estimation of σ . But for small sample sizes the random variable $s_{\bar{x}}$ in the denominator makes the tails in the t-distribution broader than for the z-distribution.

Since the t-distribution depends on the sample size, the t-distribution is actually a 'family' of distributions, one for each sample size. Specifically, we define each separate t-distribution by its 'degrees of freedom', df , which for a single mean is $df = n-1$. Here are some example t-distributions:



Notice how the t-distribution is very close to the z-distribution by the time you get to $df=20$.

Running t-tests follows the same steps as for z-tests, but we use the t-table (Table D) instead of the z-table (Table A) to calculate probabilities.

Example 1: (one tailed t-test)

Suppose you want to see if patients with PTSD have higher than normal systolic blood pressure. You sample 25 patients and obtain a sample mean Systolic BP of 131.66 and a sample standard deviation of 22.5352 mm Hg. Using an alpha value of 0.05 is this observed mean greater than a 'normal' Systolic BP of 120 mm Hg?

Like the z-test, we first compute the standard error of the mean, but using the sample standard deviation:

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{22.5352}{\sqrt{25}} = 4.507$$

We then calculate our t-statistic by subtracting the mean for the null hypothesis and divide by the estimated standard error of the mean. We'll call our observed value of t, t_{obs}

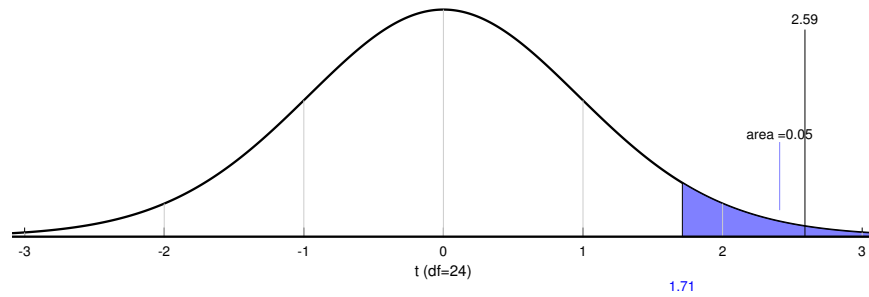
$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{131.66 - 120}{4.507} = 2.59$$

Finally, we use the t-table to see if t_{obs} is far enough away from zero to be statistically significant. The t-table has a different format than the z-table, since the t-statistic is a family of curves. We can only find critical values of t for certain values of α . Here's a sample section from the table. We'll be using the row for $df = 24$:

df, one tail	0.25	0.1	0.05	0.025	0.01	0.005	0.0005
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
22	0.686	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.707
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

This is a one-tailed test with $df = 24$ and $\alpha = 0.05$. Our critical value of t is 1.711. Our observed value of t is 2.59 which is greater than the critical value of 1.711. We therefore reject H_0 and conclude that PTSD patients do have a Systolic BP that is greater than a normal value of 120.

Here's what the t-distribution looks like for $df = 24$, $t_{obs} = 2.59$ and $\alpha = 0.05$



The t-table is good only for specific values of α , so is not useful for calculating exact p-values. The best we can do is bracket the p-value between two values of α in the table. Instead we can use the t-calculator on the same page in the Excel spreadsheet, which gives us a p-value of 0.0081:

Convert t to α			
t	df	α (one tail)	α (two tail)
2.59	24	0.008	0.0161

Convert α to t			
α	df	t (one tail)	t (two tail)
0.05	24	1.7109	2.0639

To state our conclusions using APA format, we'd state:

The Systolic BP (mm Hg) of lovely skittles ($M = 131.66$, $SD = 22.54$) is significantly greater than 120 , $t(24) = 2.59$, $p = 0.0081$.

Example 2: (two tailed t-test)

Suppose you want to determine if the height of women with fathers 6 feet or taller in our class is significantly different from the average height of women in the US population. According to Wikipedia, the average height of women in the US is 64 inches, but the website does not provide standard deviations. The heights of the 34 women with fathers 6 feet or taller in our class has a mean of 66.62 and a standard deviation of 2.663 inches. Is this mean significantly different from 64 inches? Use an α value of 0.05.

Like for Example 1, we'll calculate our t-statistic:

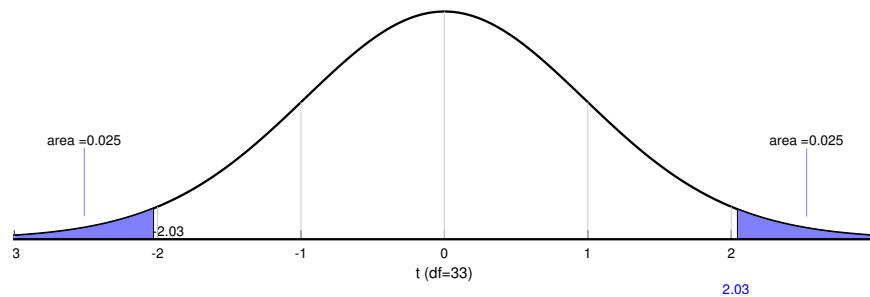
$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{2.663}{\sqrt{34}} = 0.4567$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{66.62 - 64}{0.4567} = 5.73$$

And use the t-table to see if this is statistically significant for $df = 34 - 1 = 33$:

df, one tail	0.25	0.1	0.05	0.025	0.01	0.005	0.0005
∴	∴	∴	∴	∴	∴	∴	∴
31	0.682	1.309	1.696	2.040	2.453	2.744	3.633
32	0.682	1.309	1.694	2.037	2.449	2.738	3.622
33	0.682	1.308	1.692	2.035	2.445	2.733	3.611
34	0.682	1.307	1.691	2.032	2.441	2.728	3.601
35	0.682	1.306	1.690	2.030	2.438	2.724	3.591
∴	∴	∴	∴	∴	∴	∴	∴

For a two-tailed test, we find the critical value of t in the table by looking under the column for $\frac{\alpha}{2} = 0.025$. This is because the total area under both tails to add up to α , so the area in each of the two tails is $\frac{\alpha}{2}$. Our critical value is 2.035:



Our observed value of t is 5.73 which is greater than the critical value of 2.035. We therefore reject H_0 and conclude that the mean height of the women with fathers 6 feet or taller in our class is statistically different from the population mean of 64 inches.

We can get a p-value from our calculator on the Excel spreadsheet. This gives us a p-value of $p < 0.0001$:

Convert t to α			
t	df	α (one tail)	α (two tail)
5.73	33	0	0

Convert α to t			
α	df	t (one tail)	t (two tail)
0.05	33	1.6924	2.0345

To conclude using APA format we'd state:

The height of fathers 6 feet or taller ($M = 66.62$, $SD = 2.66$) is significantly different than 64 inches, $t(33) = 5.73$, $p < 0.0001$.

Effect size (d)

The 'effect size' is a statistic that describes how far our observed mean is from the mean for the null hypothesis in standard deviation units. The most common measure is 'Cohen's d' which is used when we don't know the standard deviation of the population:

$$d = \frac{|\bar{x} - \mu_{hyp}|}{s_x}$$

A rule of thumb is that effect sizes around $d = .8$ or more are considered large, $d = .5$ is medium and $d = .2$ or less is small.

For the first example on Systolic BP (mm Hg),

$$d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|131.66 - 120|}{22.5352} = 0.52$$

This is considered to be a medium effect size.

For the second example on height,

$$d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|66.62 - 64|}{2.663} = 0.98$$

This is considered to be a large effect size.

One sample t-tests in R

If you already have summary statistics (mean, standard deviation sample size), then you can use R's 'pt' function, which is the t-distribution version of 'pnorm'.

If you have raw scores, then use the 't.test' function. The following script conducts t-tests for the two examples in this tutorial

The R commands shown below can be found here: [OneSampleTTest.R](#)

```
# OneSampleTTest.R
#
# There are two ways to calculate p-values in R for t-tests,
# depending on whether you're working with raw data or
# with summary statistics (mean and standard deviation)
#
# Load the survey data
survey <- read.csv("http://www.courses.washington.edu/psy315/datasets/Psych315W21survey.csv")
# If you have the mean and standard deviation of your sample (and degrees of freedom), then
# you can use R's 'pt' function to calculate the p-value. 'pt' is a lot like the 'pnorm' function
# for the z-distribution.
#
# Example 1 (from the t-test tutorial):
#
# Given sample of 25 systolic blood pressure scores with a mean of 131.66 and a standard
# deviation of 22.2532 mm Hg, is this mean significantly greater than a normal value of
# 120 mm HG?
#
# First, define our variables
m <- 131.66
s <- 22.2532
H0 <- 120
n <- 25
# Next, calculate the t-statistic:
sem <- s/sqrt(n)
t <- (m-H0)/sem
# Finally, calculate the p-value. We'll use the option 'lower.tail = FALSE' for
# a one-tailed test in the positive direction. Use 'TRUE' when rejecting for
# negative values of t:
p <- pt(t,n-1,lower.tail = FALSE)
p
[1] 0.007507206

# Effect size:
d <- abs(m-H0)/s
d
[1] 0.5239696
```



```

# Example 2: Are the heights of women with fathers 6 feet or taller significantly
# different from 64 inches?
H0 <- 64
# This example will conduct a t-test from the raw data instead of the summary statistics.
# We'll use R's 't.test' function.

# Load the survey data
survey <- read.csv("http://www.courses.washington.edu/psy315/datasets/Psych315W21survey.csv")
# Find the female students with fathers 6 feet (72 inches) or taller:
female.students <- survey$gender == "Female" & survey$pheight >= 72 & !is.na(survey$pheight)
female.height <- survey$height[female.students]
# Use t.test
out <- t.test(female.height,
              mu = H0,
              alternative = "two.sided")
# The option 'mu = H0' defines the mean for the null hypothesis (64 in this example)
# The option 'alternative = "two.sided"' is for a two-sided test (of course)
# Other options are "less" or "greater" for one sided tests.

# Here's the t-statistic from the output:
out$statistic
      t
5.731553

# Here are the degrees of freedom:
out$parameter
df
33

# Here's the p-value
out$p.value
[1] 2.127731e-06

# Calculate effect size:
d <- abs(mean(female.height) - H0) / sd(female.height)
d
[1] 0.9829532

# Using R to print the result in APA format
#
# You can use R's 'sprintf' function to output the results of your hypothesis test
# in APA format. This output can then be copied and pasted into your paper.

# 'sprintf' is an old command from the C programming language that prints out a formatted
# 'string', which is a list of characters. To stick a number into a string you can do this:
sprintf("My favorite number is %g.", pi)
[1] "My favorite number is 3.14159."

```

```

# the '%g' tells sprintf to replace '%g' with the number supplied afterward.  If there is
# more than one '%g' then replace '%g' with numbers in the order supplied:
sprintf('My favorite number is %g, and the square root of that number is %g',pi,sqrt(pi))
[1] "My favorite number is 3.14159, and the square root of that number is 1.77245"

# If you want to put in characters instead of numbers, use '%s':
sprintf('It is a %s day for statistics!','great')
[1] "It is a great day for statistics!"

# If you want to control the number of decimal points, use '%a.bf' where a is the total
# number of digits (including decimal points) and b is the number of digits to the right
# of the decimal point.
sprintf('pi to the nearest 2 digits is %3.2f',pi)
[1] "pi to the nearest 2 digits is 3.14"

# Putting this together, we can report the results of our last t-test in APA format:
sprintf('t(%g) = %4.2f, p = %5.4f',out$parameter,out$statistic,out$p.value)
[1] "t(33) = 5.73, p = 0.0000"

# Even if you don't understand 'sprintf' well, you can just cut and paste the line above
# with 'out' as the output of the t-test and it'll give you what you want.

```

Questions

Here are 10 practice t-test questions followed by their answers.

1) Your friend gets you to sample the amplitude of 74 psych 315 students from a population and obtain a mean amplitude of 74.75 and a standard deviation of 6.2287 . Using an alpha value of $\alpha = 0.01$, is this observed mean significantly less than an expected amplitude of 77? What is the effect size? Is the effect size small, medium or large?

2) Let's pretend that you sample the traffic of 13 cooperative hair styles from a population and obtain a mean traffic of 28.27 and a standard deviation of 2.1473 . Using an alpha value of $\alpha = 0.05$, is this observed mean significantly less than an expected traffic of 31? What is the effect size? Is the effect size small, medium or large?

3) In the pursuit of science, you sample the work of 89 invincible bus riders from a population and obtain a mean work of 58.56 and a standard deviation of 5.8534 . Using an alpha value of $\alpha = 0.05$, is this observed mean significantly less than an expected work of 61?

What is the effect size?
Is the effect size small, medium or large?

4) For your first year project you sample the piety of 74 far-flung Americans from a population and obtain a mean piety of 96.43 and a standard deviation of 4.7922 .
Using an alpha value of $\alpha = 0.01$, is this observed mean significantly greater than an expected piety of 95?
What is the effect size?
Is the effect size small, medium or large?

5) You go out and sample the weight of 103 colors from a population and obtain a mean weight of 5.48 and a standard deviation of 8.7789 .
Using an alpha value of $\alpha = 0.01$, is this observed mean significantly different than an expected weight of 8?
What is the effect size?
Is the effect size small, medium or large?

6) Tomorrow you sample the evil of 50 eggs from a population and obtain a mean evil of 95.43 and a standard deviation of 5.165 .
Using an alpha value of $\alpha = 0.01$, is this observed mean significantly different than an expected evil of 93?
What is the effect size?
Is the effect size small, medium or large?

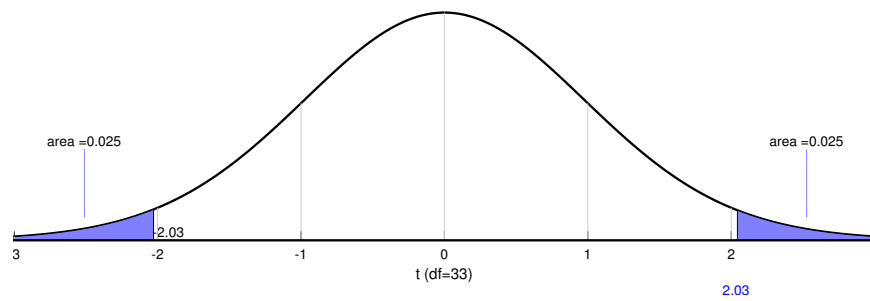
7) For a 499 project you sample the damage of 20 yielding chickens from a population and obtain a mean damage of 35.43 and a standard deviation of 9.2076 .
Using an alpha value of $\alpha = 0.01$, is this observed mean significantly greater than an expected damage of 33?
What is the effect size?
Is the effect size small, medium or large?

8) One day you sample the knowledge of 31 utopian dinosaurs from a population and obtain a mean knowledge of 45.64 and a standard deviation of 7.407 .
Using an alpha value of $\alpha = 0.01$, is this observed mean significantly less than an expected knowledge of 50?
What is the effect size?
Is the effect size small, medium or large?

9) I go and sample the recognition of 38 macabre candy bars from a population and obtain a mean recognition of 88.59 and a standard deviation of 3.957 .
Using an alpha value of $\alpha = 0.05$, is this observed mean significantly greater than an expected recognition of 88?

What is the effect size?
Is the effect size small, medium or large?

10) For some reason you sample the response time of 66 alluring bananas from a population and obtain a mean response time of 68.5 and a standard deviation of 4.8465 .
Using an alpha value of $\alpha = 0.01$, is this observed mean significantly different than an expected response time of 68?
What is the effect size?
Is the effect size small, medium or large?



Answers

1) amplitude of psych 315 students

$$\bar{x} = 74.75, s_x = 6.2287, n = 74$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{6.2287}{\sqrt{74}} = 0.7241$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{74.75 - 77}{0.7241} = -3.11$$

$$df = (n-1) = (74-1) = 73$$

$$t_{crit} = -2.38$$

We reject H_0 .

The amplitude of psych 315 students ($M = 74.75$, $SD = 6.23$) is significantly less than 77 , $t(73) = -3.11$, $p = 0.0013$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|74.75 - 77|}{6.2287} = 0.3618$$

This is a medium effect size.

Using R:

```
sem <- 6.2287/sqrt(74)
```

```
t <- (74.75-77)/0.7241
```

```
t
```

```
[1] -3.107306
```

```
p <- pt(t,73,lower.tail = TRUE)
```

```
# APA format:
```

```
sprintf('t(73) = %4.2f, p = %5.4f',t,p)
```

```
[1] "t(73) = -3.11, p = 0.0013"
```

```
# Effect size:
```

```
abs(74.75 - 77)/6.2287
```

```
[1] 0.3612311
```

2) traffic of hair styles

$$\bar{x} = 28.27, s_x = 2.1473, n = 13$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{2.1473}{\sqrt{13}} = 0.5956$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{28.27 - 31}{0.5956} = -4.58$$

$$df = (n-1) = (13-1) = 12$$

$$t_{crit} = -1.78$$

We reject H_0 .

The traffic of cooperative hair styles ($M = 28.27$, $SD = 2.15$) is significantly less than 31 , $t(12) = -4.58$, $p = 0.0003$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|28.27 - 31|}{2.1473} = 1.2716$$

This is a large effect size.

```
# Using R:
sem <- 2.1473/sqrt(13)
t <- (28.27-31)/0.5956
t
[1] -4.583613
p <- pt(t,12,lower.tail = TRUE)
# APA format:
sprintf('t(12) = %4.2f, p = %5.4f',t,p)
[1] "t(12) = -4.58, p = 0.0003"
# Effect size:
abs(28.27 - 31)/2.1473
[1] 1.271364
```

3) work of bus riders

$$\bar{x} = 58.56, s_x = 5.8534, n = 89$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{5.8534}{\sqrt{89}} = 0.6205$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{58.56 - 61}{0.6205} = -3.93$$

$$df = (n-1) = (89-1) = 88$$

$$t_{crit} = -1.66$$

We reject H_0 .

The work of invincible bus riders ($M = 58.56$, $SD = 5.85$) is significantly less than 61 , $t(88) = -3.93$, $p = 0.0001$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|58.56 - 61|}{5.8534} = 0.4168$$

This is a medium effect size.

```

# Using R:
sem <- 5.8534/sqrt(89)
t <- (58.56-61)/0.6205
t
[1] -3.932313
p <- pt(t,88,lower.tail = TRUE)
# APA format:
sprintf('t(88) = %4.2f, p = %5.4f',t,p)
[1] "t(88) = -3.93, p = 0.0001"
# Effect size:
abs(58.56 - 61)/5.8534
[1] 0.4168517

```

4) piety of Americans

$$\bar{x} = 96.43, s_x = 4.7922, n = 74$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{4.7922}{\sqrt{74}} = 0.5571$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{96.43 - 95}{0.5571} = 2.57$$

$$df = (n-1) = (74-1) = 73$$

$$t_{crit} = 2.38$$

We reject H_0 .

The piety of far-flung Americans ($M = 96.43$, $SD = 4.79$) is significantly greater than 95 , $t(73) = 2.57$, $p = 0.0061$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|96.43 - 95|}{4.7922} = 0.2991$$

This is a small effect size.

```

# Using R:
sem <- 4.7922/sqrt(74)
t <- (96.43-95)/0.5571
t
[1] 2.566864
p <- pt(t,73,lower.tail = FALSE)
# APA format:
sprintf('t(73) = %4.2f, p = %5.4f',t,p)
[1] "t(73) = 2.57, p = 0.0062"
# Effect size:
abs(96.43 - 95)/4.7922
[1] 0.2984016

```

5) weight of colors

$$\bar{x} = 5.48, s_x = 8.7789, n = 103$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{8.7789}{\sqrt{103}} = 0.865$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{5.48 - 8}{0.865} = -2.91$$

$$df = (n-1) = (103-1) = 102$$

$$t_{crit} = \pm 2.63$$

We reject H_0 .

The weight of colors ($M = 5.48$, $SD = 8.78$) is significantly different than 8, $t(102) = -2.91$, $p = 0.0044$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|5.48 - 8|}{8.7789} = 0.2868$$

This is a small effect size.

Using R:

```
sem <- 8.7789/sqrt(103)
```

```
t <- (5.48-8)/0.865
```

```
t
```

```
[1] -2.913295
```

```
# Since this is a two-tailed test, use abs(t) and double the p-value
```

```
p <- 2*pt(abs(t),102,lower.tail = FALSE)
```

```
# APA format:
```

```
sprintf('t(102) = %4.2f, p = %5.4f',t,p)
```

```
[1] "t(102) = -2.91, p = 0.0044"
```

```
# Effect size:
```

```
abs(5.48 - 8)/8.7789
```

```
[1] 0.2870519
```

6) evil of eggs

$$\bar{x} = 95.43, s_x = 5.165, n = 50$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{5.165}{\sqrt{50}} = 0.7304$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{95.43 - 93}{0.7304} = 3.33$$

$$df = (n-1) = (50-1) = 49$$

$$t_{crit} = \pm 2.68$$

We reject H_0 .

The evil of eggs ($M = 95.43$, $SD = 5.17$) is significantly different than 93, $t(49) = 3.33$, $p = 0.0017$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|95.43 - 93|}{5.165} = 0.471$$

This is a medium effect size.

```
# Using R:
sem <- 5.165/sqrt(50)
t <- (95.43-93)/0.7304
t
[1] 3.326944
# Since this is a two-tailed test, use abs(t) and double the p-value
p <- 2*pt(abs(t),49,lower.tail = FALSE)
# APA format:
sprintf('t(49) = %4.2f, p = %5.4f',t,p)
[1] "t(49) = 3.33, p = 0.0017"
# Effect size:
abs(95.43 - 93)/5.165
[1] 0.4704743
```

7) damage of chickens

$$\bar{x} = 35.43, s_x = 9.2076, n = 20$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{9.2076}{\sqrt{20}} = 2.0589$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{35.43 - 33}{2.0589} = 1.18$$

$$df = (n-1) = (20-1) = 19$$

$$t_{crit} = 2.54$$

We fail to reject H_0 .

The damage of yielding chickens ($M = 35.43$, $SD = 9.21$) is not significantly greater than 33, $t(19) = 1.18$, $p = 0.1262$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|35.43 - 33|}{9.2076} = 0.2639$$

This is a small effect size.

```

# Using R:
sem <- 9.2076/sqrt(20)
t <- (35.43-33)/2.0589
t
[1] 1.180242
p <- pt(t,19,lower.tail = FALSE)
# APA format:
sprintf('t(19) = %4.2f, p = %5.4f',t,p)
[1] "t(19) = 1.18, p = 0.1262"
# Effect size:
abs(35.43 - 33)/9.2076
[1] 0.2639124

```

8) knowledge of dinosaurs

$$\bar{x} = 45.64, s_x = 7.407, n = 31$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{7.407}{\sqrt{31}} = 1.3303$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{45.64 - 50}{1.3303} = -3.28$$

$$df = (n-1) = (31-1) = 30$$

$$t_{crit} = -2.46$$

We reject H_0 .

The knowledge of utopian dinosaurs ($M = 45.64$, $SD = 7.41$) is significantly less than 50 , $t(30) = -3.28$, $p = 0.0013$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|45.64 - 50|}{7.407} = 0.5891$$

This is a medium effect size.

```

# Using R:
sem <- 7.407/sqrt(31)
t <- (45.64-50)/1.3303
t
[1] -3.277456
p <- pt(t,30,lower.tail = TRUE)
# APA format:
sprintf('t(30) = %4.2f, p = %5.4f',t,p)
[1] "t(30) = -3.28, p = 0.0013"
# Effect size:
abs(45.64 - 50)/7.407

```

[1] 0.5886324

9) recognition of candy bars

$$\bar{x} = 88.59, s_x = 3.957, n = 38$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{3.957}{\sqrt{38}} = 0.6419$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{88.59 - 88}{0.6419} = 0.92$$

$$df = (n-1) = (38-1) = 37$$

$$t_{crit} = 1.69$$

We fail to reject H_0 .

The recognition of macabre candy bars ($M = 88.59$, $SD = 3.96$) is not significantly greater than 88, $t(37) = 0.92$, $p = 0.1811$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|88.59 - 88|}{3.957} = 0.1496$$

This is a small effect size.

Using R:

```
sem <- 3.957/sqrt(38)
```

```
t <- (88.59-88)/0.6419
```

```
t
```

```
[1] 0.9191463
```

```
p <- pt(t,37,lower.tail = FALSE)
```

```
# APA format:
```

```
sprintf('t(37) = %4.2f, p = %5.4f',t,p)
```

```
[1] "t(37) = 0.92, p = 0.1820"
```

```
# Effect size:
```

```
abs(88.59 - 88)/3.957
```

```
[1] 0.1491029
```

10) response time of bananas

$$\bar{x} = 68.5, s_x = 4.8465, n = 66$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{4.8465}{\sqrt{66}} = 0.5966$$

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}} = \frac{68.5 - 68}{0.5966} = 0.84$$

$$df = (n-1) = (66-1) = 65$$

$$t_{crit} = \pm 2.65$$

We fail to reject H_0 .

The response time of alluring bananas ($M = 68.5$, $SD = 4.85$) is not significantly different than 68, $t(65) = 0.84$, $p = 0.4025$.

$$\text{Effect size: } d = \frac{|\bar{x} - \mu_{hyp}|}{s_x} = \frac{|68.5 - 68|}{4.8465} = 0.1037$$

This is a small effect size.

```
# Using R:
sem <- 4.8465/sqrt(66)
t <- (68.5-68)/0.5966
t
[1] 0.8380825
# Since this is a two-tailed test, use abs(t) and double the p-value
p <- 2*pt(abs(t),65,lower.tail = FALSE)
# APA format:
sprintf('t(65) = %4.2f, p = %5.4f',t,p)
[1] "t(65) = 0.84, p = 0.4051"
# Effect size:
abs(68.5 - 68)/4.8465
[1] 0.1031672
```