Two Factor ANOVA

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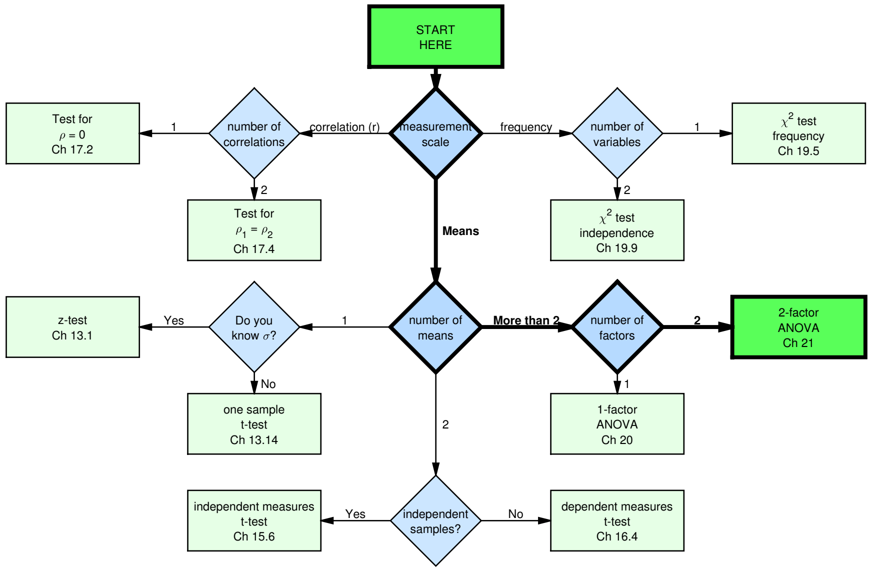
Two Factor ANOVA

A two factor ANOVA (sometimes called 'two-way' ANOVA) is a hypothesis test on means for which groups are placed in groups that vary along two factors instead of one.

For example, we might want to measure BMI for subjects with different diets AND for different levels of exercise. Or, we might measure the duration of hospital stays for patients undergoing different levels of medication AND different types of physical therapy. In these cases, we're interest in not only how the means vary across each factor (called the **main effects**), but also how these two dimensions interact with each other (called **interactions**).

Here's how to get to the 2-factor ANOVA with the flow chart:



For a two-factor ANOVA, each of the two **factors** (like 'diet' and 'exercise') are comprised of different **levels**, (like different diets, or different levels of exercise). The complete design involves having subjects placed into all possible combinations of levels for the two factors. For example, if there are two diets and three levels of exercise, there will be 2x3 = 6 groups, or **cells**.

Each cell should have the same number of subjects, which called a **balanced design**. There are ways to deal with unequal sample sizes, but we will not go there.

Example 1: peanut butter and jelly

Let's start with a simple example. Suppose you want to study the effects of adding peanut butter and jelly to bread to study how they affect taste preferences. You take 36 students and break them down into 4 groups of 9 in what's called a '2x2' design. One group will get just bread, another will get bread with peanut butter, another will get bread with jelly, and one group will get bread with both peanut butter and jelly.

We'll run our test with $\alpha = 0.05$.

Each student tastes their food and rates their preference on a scale from 1 (gross) to 10 (excellent). The raw data looks like this:

no jelly, no peanut butter	no jelly, peanut butter	jelly, no peanut butter	jelly and peanut butter
1	4	6	8
7	12	6	5
9	5	2	5
4	8	8	6
5	6	6	9
6	6	6	11
2	8	2	7
5	6	5	9
1	9	9	7

Here are the summary statistics from the experiment:

Means		
	no jelly	jelly
no peanut butter	4.4444	5.5556
peanut butter	7.1111	7.4444

SS$_{wc}$		
	no jelly	jelly
no peanut butter	60.2222	44.2222
peanut butter	46.8889	32.2222

Totals	
grand mean	6.1389
SS_{total}	236.3056

We've organized statistics in a 2x2 matrix for which the factor 'peanut butter' varies across the rows and the factor 'jelly' varies across the columnns. We therefore call 'peanut butter' the **row factor** and 'jelly' the **column factor**.

The table titled SS_{wc} contains the sums of squared deviation of each score from the mean of for the cell that it belongs to. For example, for the 'no peanut butter, no jelly' category, the mean for that cell is 4.4444, so the SS_{wc} for that cell is:

$$(1 - 4.4444)^2 + (7 - 4.4444)^2 + ... + (9 - 4.4444)^2 = 60.2222$$

SS_{total} is the sums of squared deviation of each score from the grand mean.

Before we run any statistical analyses, it's useful to plot the means with error bars representing the standard errors of the mean. Remember, standard errors can be calculated by first calculating standard deviations from SS_{wc}:

$$s_x = \sqrt{\frac{SS_{wc}}{n-1}}$$

And then dividing by by the square root of the sample size to get the standard error of the mean:

$s_{\bar{x}} = \frac{s_x}{\sqrt{n}}$

For example, for the 'no peanut butter, no jelly' category,

$s_x = \sqrt{\frac{60.2222}{9-1}} = 2.7437$

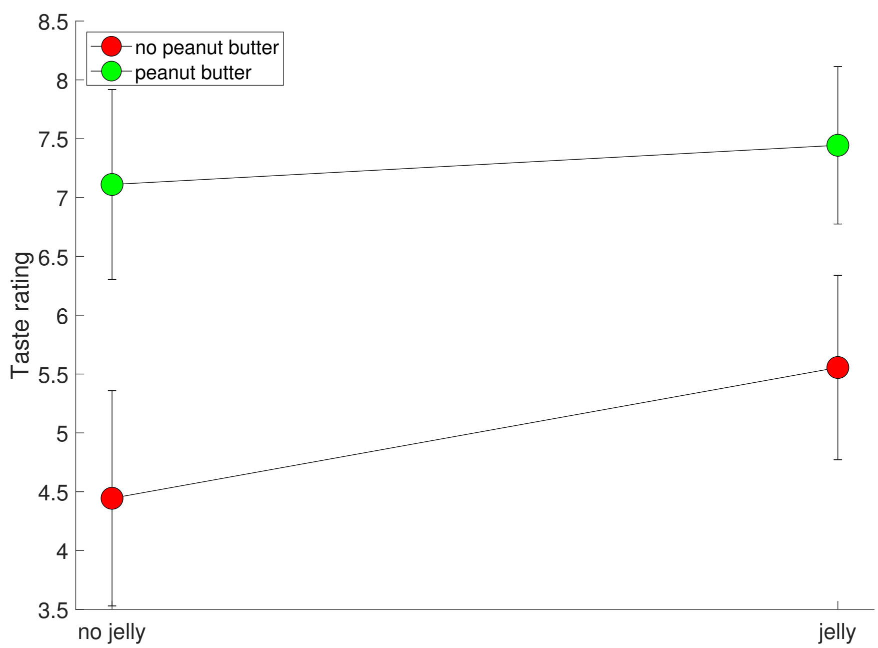
So

$s_{\bar{x}} = \frac{2.7437}{\sqrt{9}} = 0.9146$

Check for yourself that the standard errors of the mean turn out to be:

Standard errors		
	no jelly	jelly
no peanut butter	0.9146	0.7837
peanut butter	0.807	0.669

Here's the plot:



Visually inspecting the graph, you can see that the green points are shifted above the red ones. This means that adding peanut butter helps the taste for both the plain bread, and for bread with jelly. This is called a **main effect for rows**, where the rows correspond the factor 'peanut butter'.

Similarly, adding jelly increases the taste rating both for plain bread, and for bread with peanut butter. This is a **main effect for columns** where columns refer to the factor 'jelly'.

Notice also that the lines are roughly parallel. This means that the increase in the taste rating by adding peanut butter is about the same with and without jelly. We therefore say that there is no **interaction** between the factors of peanut butter and jelly.

The 2-factor ANOVA quantifies the statistical significance of these three observations: the main main effect for rows, the main effect for columns, and the interaction.

Within-cell variance - the denominator of all three F-tests

The statistical significance of the main effect for rows, the main effect for columns and the interaction will all be done with F-tests. Just as with the 1-factor ANOVA, the F-test will be a ratio of variances which are both estimates of the population variance under the null hypothesis.

Also, like the 1-factor ANOVA, the denominator will be an estimate of variance based on the variability within each group, and again we calculate it by summing up the sums of squared error between each score and its own cell's mean:

$SS_{wc} = \sum (X - \bar{X}_{cell})^2$

I've given you SS_{wc} for each cell, so the total SS_{wc} is just the sum of these four numbers:

$SS_{wc} = 60.2222 + 46.8889 + 44.2222 + 32.2222 = 183.5555$

Just as with the 1-way ANOVA, the degrees of freedom for SS_{wc} is the total number of scores minus the total number of groups, so df = 9 + 9 + 9 + 9 - 4 = 32

The variance within cells is SS_{wc} divided by its degrees of freedom:

$MS_{wc} = \frac{SS_{wc}}{df_{wc}} = \frac{183.5556}{32} = 5.7361$

Main effects for rows: Does peanut butter affect taste ratings?

To quantify the main effect for rows (peanut butter) we average the means across the two levels for columns (jelly). We can show these two means in a third column in the matrix of means:

Means			
	no jelly	jelly	Row Means
no peanut butter	4.4444	5.5556	5
peanut butter	7.1111	7.4444	7.2778

The difference between these two row means indicates the effect of adding peanut butter after averaging across the two levels of jelly. We'll use these two means to calculate the numerator of the F-test for the main effect for rows.

We first calculate SS for rows much like we would if this were a 1-factor ANOVA. We calculate the sum of squared difference of the row means from the grand mean, scaled by the sample size for the number of scores for each mean. Since there are 2 groups for each mean, the sample size for each row mean is (2)(9) = 18. So,

$$SS_{row} = \sum (n_{row})(\bar{X}_{row} - \bar{\bar{X}})^2 =$$

$$SS_R = (2)(9)(5 - 6.1389)^2 + (2)(9)(7.2778 - 6.1389)^2 = 46.6945$$

The degrees of freedom for SS for rows is equal to the number of rows minus 1 (df = 2 - 1 = 1). The variance for rows is SS for rows (SS_{rows}) divided by its degrees of freedom:

$$MS_{rows} = \frac{SS_{rows}}{df_{rows}} = \frac{46.6945}{1} = 46.6945$$

The F-statistic for the main effect for rows is the variance for rows divided by the variance within cells:

$$F_{rows} = \frac{MS_{rows}}{MS_{wc}} = \frac{46.6945}{5.7361} = 8.14$$

Under the null hypothesis, the variance for rows is another estimate of the population variance. So if the null hypothesis is true, this F statistic should be around 1, on average. A large value of F indicates that the row means vary more than expected under the null hypothesis. If F is large enough, we conclude that this is too unusual for the null hypothesis to be true, so we reject it and conclude that the population means for the two rows are not the same.

The critical value for this F can be found in table E, using the degrees of freedom 1 and 32.

$df_w \| dfb$	1	2
31	4.16	3.3
	7.53	**5.36**
32	4.15	3.29
	7.5	**5.34**
33	4.14	3.28
	7.47	**5.31**

The critical value is 4.15. You can use the F-calculator to find that the p-value is 0.0075.

For our example, since our observed value of F (8.14) is greater than the critical value of F (4.15), we reject the null hypothesis. We can conclude using APA format: "There is a significant main effect of peanut butter on taste ratings, F(1,32) = 8.14, p = 0.0075"

Main effect for columns: does jelly affect taste ratings?

Conducting the F-test on the main effect for columns (jelly) is completely analogous to the main effect for rows. We first calculate the column means by calculating the mean for each of the two colummns. We can show this as a third row in the matrix of means:

Means		
	no jelly	jelly
no peanut butter	4.4444	5.5556
peanut butter	7.1111	7.4444
Column Means	5.7778	6.5

The difference between these two column means indicates the effect of adding jelly after averaging across the two levels of peanut butter.

As for rows, we'll calculate SS for columns, which is the sum of squared difference of the column means from the grand mean, scaled by the sample size for the number of scores for each mean:

$$SS_{col} = \sum (n_{col})(\bar{X}_{col} - \bar{\bar{X}})^2 =$$

$$(18)(5.7778 - 6.1389)^2 + (18)(6.5 - 6.1389)^2 = 4.6945$$

The degrees of freedom for SS for columns is equal to the number of columns minus 1 (df = 2 - 1 = 1). The variance for the main effect for columns is SS for columns divided by its degrees of freedom:

$$MS_{col} = \frac{SS_{col}}{df_{col}} = \frac{4.6945}{1} = 4.6945$$

The F-statistic for the main effect for columns is the variance for columns divided by the variance within cells:

$$F_{col} = \frac{MS_{col}}{MS_{wc}} = \frac{4.6945}{5.7361} = 0.82$$

The F-calculator shows that the p-value for the main effect for columns (the df's are 1 and 32) is 0.3719.

For our example, since our observed value of F (0.82) is not greater than the critical value of F (4.15), we fail to reject the null hypothesis. We can conclude using APA format: "There is not a significant main effect of jelly on taste ratings, F(1,32) = 0.82, p = 0.3719"

The interaction: does the effect of peanut butter on taste ratings depend on jelly?

This third F-test determines if there is a statistically significant interaction between our two factors on our dependent variable. For our example, an interaction would occur if the effect of adding peanut butter to the bread added a different amount to the taste rating depending upon whether or not there was jelly (and vice versa). Remember from the graph that the two lines were roughly parallel, indicating that peanut butter had a similar increase in taste ratings with and without jelly. **Parallel lines indicate no interaction between the two variables.**

To run this F-test, we need to calculate a third estimate of the population variance. The formula for the sums of squares for the interaction between rows and columns (SS_{RxC}) is messy and not very intuitive. Fortunately we don't need it because we can infer it by using the fact that the sums of squared deviation from the grand mean can be broken down into three components:

$SS_{total} = SS_{rows} + SS_{cols} + SS_{RxC} + SS_{wc}$

I've given you SS_{total} and we just calculated SS_{rows}, SS_{cols} and SS_{wc}. So

$SS_{RxC} = SS_{total} - SS_{rows} - SS_{cols} - SS_{wc} = 236.3056 - 46.6945 - 4.6945 - 183.5556 = 1.361$

The degrees of freedom for the row by column interaction is the number of rows minus 1 times the number of columns minus 1: (2-1)(2-1) = 1.

The variance for the row by column interaction is the SS divided by df:

$MS_{RxC} = \frac{SS_{RxC}}{df_{RxC}} = \frac{1.361}{1} = 1.361$

The F-statistic is:

$F_{RxC} = \frac{MS_{RxC}}{MS_{wc}} = \frac{1.361}{5.7361} = 0.24$

The F-calculator shows that the p-value for this value of F (the df's are 1 and 32) is 0.6275.

Since our p-value (0.6275) is greater than alpha (0.05), we fail to reject the null hypothesis. We can conclude using APA format: "There is not a significant interaction between the effects of peanut butter and jelly on taste ratings, F(1,32) = 0.24, p = 0.6275"

The Summary Table

Like for 1-factor ANOVAs, the results from 2-factor ANOVAs are often reported in a summary table like this:

	SS	df	MS	F	F_{crit}	p-value
Rows	46.6945	1	46.6945	8.14	4.15	0.0075
Columns	4.6945	1	4.6945	0.82	4.15	0.3719
R X C	1.361	1	1.361	0.24	4.15	0.6275
wc	183.5556	32	5.7361			
Total	236.3056	35				

Example 2: peanut butter and ham

Now suppose you want to study the effects of adding peanut butter and ham to bread. The

(2x2) design is like the previous example; you take 36 students and break them down into 4 groups.

Again, we'll run our test with $\alpha = 0.05$.

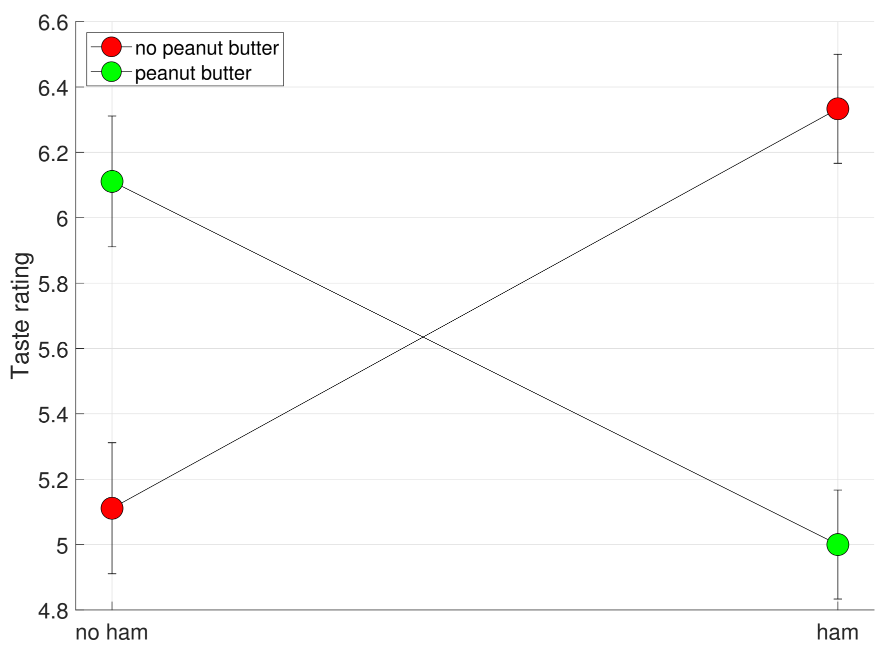
Here are the summary statistics from the experiment:

Means		
	no ham	ham
no peanut butter	5.1111	6.3333
peanut butter	6.1111	5

SS_{wc}		
	no ham	ham
no peanut butter	2.8889	2
peanut butter	2.8889	2

Totals	
grand mean	5.6389
SS_{total}	22.3056

Here's a plot the means with error bars representing the standard errors of the mean.



The first thing you see is that the lines are not parallel. Looking at the red points (no peanut butter), you can see that adding ham to bread improves its taste. But, if you start with peanut butter on your bread, adding ham makes it taste worse (yuck). This is a classic 'X' shaped interaction, and we'll see below that it's statistically significant.

For a main effect for rows we'd need the overall rating to change when we add peanut butter. But adding peanut butter increases the rating for bread with no ham but decreases the rating with ham. Overall, the greeen points aren't higher or lower than the red points. It therefore looks like there is no main effect for peanut butter.

For a main main effect for columns (ham) we'd need the overall rating to change when we add ham. But you can see that the mean of the data point on the left (no ham are no higher or lower than the mean of the data points on the right (with jelly. So we don't expect a main effect for the column factor, ham.

Now we'll work through the problem and fill in the summary table at the end:

First, the within-cell variance:

$SS_{wc} = \sum (X - \bar{X}_{cell})^2$

$SS_{wc} = 2.8889 + 2.8889 + 2 + 2 = 9.7778$

df = 9 + 9 + 9 + 9 - 4 = 32

$MS_{wc} = \frac{SS_{wc}}{df_{wc}} = \frac{9.7778}{32} = 0.3056$

For the main effects we can create a new table with both the row and column means:

Means			
	no ham	ham	Row Means
no peanut butter	5.1111	6.3333	5.7222
peanut butter	6.1111	5	5.5556
Column Means	5.6111	5.6667	

Note that the two row means don't differ by much, nor do the two column means. This shows a lack of main effects for both rows and columns.

We'll now compute the SS, variance and F-statistic for the main effect for rows.

$SS_R = (2)(9)(5.7222 - 5.6389)^2 + (2)(9)(5.5556 - 5.6389)^2 = 0.25$

With $df_R = 2 - 1 = 1$

$MS_{rows} = \frac{SS_{rows}}{df_{rows}} = \frac{0.25}{1} = 0.25$

$F_{rows} = \frac{MS_{rows}}{MS_{wc}} = \frac{0.25}{0.3056} = 0.82$

The F-calculator shows that the p-value is 0.3719.

Since For our example, since our observed value of F (0.82) is not greater than the critical value of F (4.15), we fail to reject the null hypothesis. We can conclude using APA format: "There is not a significant main effect of peanut butter on taste ratings, F(1,32) = 0.82, p = 0.3719"

Now for the main effect for columns:

$SS_{col} = \sum (n_{col})(\bar{X}_{col} - \bar{\bar{X}})^2 =$

$(18)(5.6111 - 5.6389)^2 + (18)(5.6667 - 5.6389)^2 = 0.0278$

df = 2 - 1 = 1

$MS_{col} = \frac{SS_{col}}{df_{col}} = \frac{0.0278}{1} = 0.0278$

$F_{col} = \frac{MS_{col}}{MS_{wc}} = \frac{0.0278}{0.3056} = 0.09$

The F-calculator shows that the p-value for the main effect for columns (the df's are 1 and 32) is 0.7661.

Because our p-value (0.7661) is greater than alpha (0.05), we conclude "There is not a significant main effect of ham on taste ratings, F(1,32) = 0.82, p = 0.3719"

Finally, for the interaction:

$SS_{RxC} = SS_{total} - SS_{rows} - SS_{cols} - SS_{wc} = 22.3056 - 0.25 - 0.0278 - 9.7778 = 12.25$

df = (2-1)(2-1) = 1.

$MS_{RxC} = \frac{SS_{RxC}}{df_{RxC}} = \frac{12.25}{1} = 12.25$

$F_{RxC} = \frac{MS_{RxC}}{MS_{wc}} = \frac{12.25}{0.3056} = 40.09$

The F-calculator shows that the p-value for this value of F (the df's are 1 and 32) is 0.

Since our p-value (0) is less than alpha (0.05) We conclude: "There is a significant interaction between the effects of peanut butter and ham on test scores, F(1,32) = 40.09, p = 0"

The final summary table looks like this:

	SS	df	MS	F	F_{crit}	p-value
Rows	0.25	1	0.25	0.82	4.15	0.3719
Columns	0.0278	1	0.0278	0.09	4.15	0.7661
R X C	12.25	1	12.25	40.09	4.15	< 0.0001
wc	9.7778	32	0.3056			
Total	22.3056	35				

Example 3: A 2x3 ANOVA on study habits

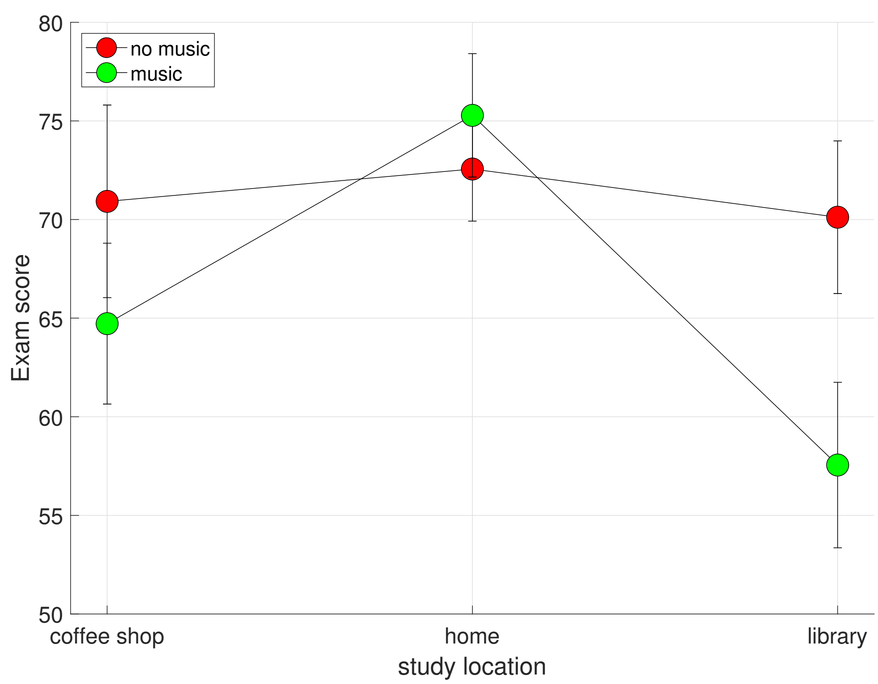
This third example is an two-factor ANOVA on study habbits. You manipulate two factors, 'music' and 'study location'. for the factor 'music', you have students either study in silence, or listen to the music of their choice. For the factor 'study location', you have groups of students study in one of 3 places: coffee shop, home and library. You find 25 students for each of the 4 groups. You generate the following summary statistics:

Means			
	coffee shop	home	library
no music	70.92	72.56	70.1168
music	64.72	75.28	57.552

SS_{wc}			
	coffee shop	home	library
no music	14309.84	4192.16	8994.1127
music	9979.04	5873.04	10561.2232

Totals	
grand mean	68.5248
SS_{total}	59036.0623

Here's a plot the means with error bars representing the standard errors of the mean.



You'll probably first notice that the red points are higher than the green points. This is a main effecct for the row factor, music, for which listening to music leads to lower scores.

Next you'll notice that the scores vary across the column factor, study location, with the best scores for students studying at home. Since there are three locations of study, we're looking for a difference across means - just like a 1-factor ANOVA with three levels. Here, we see that there does look like there is a main effect for study location. Now we'll work through the problem and fill in the summary table at the end:

The within-cell variance is calculated as in the examples above by adding up SS_{wc} across the 6 cells:

$SS_{wc} = \sum (X - \bar{X}_{cell})^2$

$SS_{wc} = 14309.84 + 9979.04 + 4192.16 + 5873.04 + 8994.1127 + 10561.2232 = 53909.4159$

$df = n_{total} - k = 150 - 6 = 144$

$MS_{wc} = \frac{SS_{wc}}{df_{wc}} = \frac{53909.4159}{144} = 374.3709$

For the main effects, again we'll create a new table with both the row and column means:

Means				
	coffee shop	home	library	column means
no music	70.92	72.56	70.1168	71.1989
music	64.72	75.28	57.552	65.8507
row means	67.82	73.92	63.8344	

Next we'll compute the F-statistic for the main effect of rows (music) Note that since there are three columns, there are three means contributing to each row mean, the sample size for each row mean is (25)(3) = 75.

$SS_{row} = \sum (n_{row})(\bar{X}_{row} - \bar{\bar{X}})^2 =$

$(75)(71.1989 - 68.5248)^2 + (75)(65.8507 - 68.5248)^2 = 1072.6484$

With df = 2 - 1 = 1.

$MS_{rows} = \frac{SS_{rows}}{df_{rows}} = \frac{1072.6484}{1} = 1072.6484$

$F_{rows} = \frac{MS_{rows}}{MS_{wc}} = \frac{1072.6484}{374.3709} = 2.87$

The F-calculator shows that the critical value is 3.91 and the p-value is 0.0924.

Our observed value of F (2.87) is not greater than the critical value of F (3.91), we fail to reject the null hypothesis. We can conclude using APA format: "There is not a significant main effect of music on test scores, F(1,144) = 2.87, p = 0.0924"

Now for the main effect for columns (study location):

$SS_{col} = \sum (n_{col})(\bar{X}_{col} - \bar{\bar{X}})^2 =$

$(50)(67.82 - 68.5248)^2 + (50)(73.92 - 68.5248)^2 + (50)(63.8344 - 68.5248)^2 = 2580.239$

df = 3 - 1 = 2

$$MS_{col} = \frac{SS_{col}}{df_{col}} = \frac{2580.239}{2} = 1290.1195$$

$$F_{col} = \frac{MS_{col}}{MS_{wc}} = \frac{1290.1195}{374.3709} = 3.45$$

The F-calculator shows that the p-value for the main effect for columns (the df's are 2 and 144) is 0.0344.

Because our p-value (0.0344) is less than alpha (0.05), we conclude "There is a significant main effect of study location on taste ratings, F(1,144) = 2.87, p = 0.0924"

Finally, for the interaction:

$SS_{RxC} = SS_{total} - SS_{rows} - SS_{cols} - SS_{wc} = 59036.0623 - 1072.6484 - 2580.239 - 53909.4159 = 1473.759$

df = (3-1)(2-1) = 2.

$$MS_{RxC} = \frac{SS_{RxC}}{df_{RxC}} = \frac{1473.759}{2} = 736.8795$$

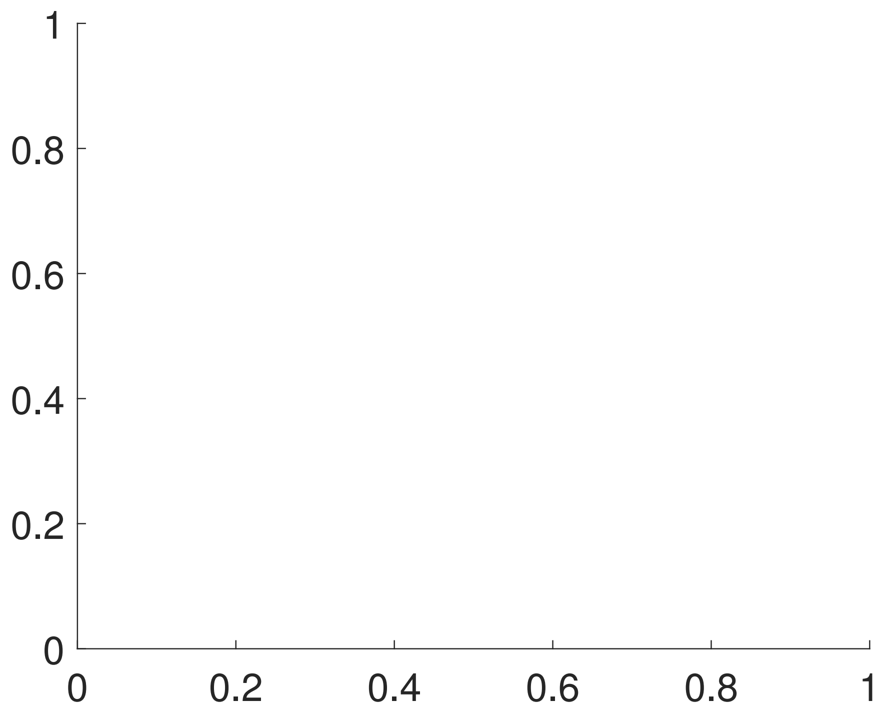
$$F_{RxC} = \frac{MS_{RxC}}{MS_{wc}} = \frac{736.8795}{374.3709} = 1.97$$

The F-calculator shows that the p-value for this value of F (the df's are 2 and 144) is 0.1432.

Since our p-value (0.1432) is greater than alpha (0.05) We conclude: "There is not a significant interaction between the effects of music and study location on taste ratings, F(2,144) = 1.97, p = 0.1432"

The final summary table looks like this:

	SS	df	MS	F	F_{crit}	p-value
Rows	1072.6484	1	1072.6484	2.87	3.91	0.0924
Columns	2580.239	2	1290.1195	3.45	3.06	0.0344
R X C	1473.759	2	736.8795	1.97	3.06	0.1432
wc	53909.4159	144	374.3709			
Total	59036.0623	149				



Questions

Your turn again. Here are 10 random practice questions followed by their answers.

1) You ask a friend to measure the effect of 2 kinds of underwear and 4 kinds of friends on the shopping of oranges. You find 23 oranges for each group and measure shopping for each of the 2x4=8 groups.

You generate the following table of means:

Means				
	friends: 1	friends: 2	friends: 3	friends: 4
underwear: 1	63.4387	59.7226	61.6848	64.3609
underwear: 2	64.3135	64.6678	64.1152	65.8161

and the following table for SS_{wc}:

SS_{wc}				
	friends: 1	friends: 2	friends: 3	friends: 4
underwear: 1	272.7455	279.9746	250.851	373.228
underwear: 2	317.9483	307.9582	315.3024	381.9149

The grand mean is 63.5149 and SS_{total} is 3099.6498.
Make a plot of the data with error bars representing standard errors of the mean.
Using an alpha value of α=0.05, test for a main effect of underwear, a main effect of friends and an interaction on the shopping of oranges.

2) I measure the effect of 2 kinds of underwear and 4 kinds of undergraduates on the response time of telephones. You find 13 telephones for each group and measure response time for each of the 2x4=8 groups.

You generate the following table of means:

Means				
	undergraduates: 1	undergraduates: 2	undergraduates: 3	undergraduates: 4
underwear: 1	68.25	64.9162	62.9923	68.5569
underwear: 2	66.7515	66.3408	66.8008	67.0146

and the following table for SS_{wc}:

SS_{wc}				
	undergraduates: 1	undergraduates: 2	undergraduates: 3	undergraduates: 4
underwear: 1	281.3152	253.1687	413.7418	190.6365
underwear: 2	350.9958	233.3293	408.1977	148.2673

The grand mean is 66.4529 and SS_{total} is 2572.5691.
Make a plot of the data with error bars representing standard errors of the mean.
Using an alpha value of α=0.01, test for a main effect of underwear, a main effect of undergraduates and an interaction on the response time of telephones.

3) Suppose you measure the effect of 2 kinds of poo and 2 kinds of antidepressants on the homework of pants. You find 11 pants for each group and measure homework for each of the 2x2=4 groups.

You generate the following table of means:

Means		
	antidepressants: 1	antidepressants: 2
poo: 1	14.1509	21.3418
poo: 2	13.8518	16.1427

and the following table for SS_{wc}:

SS_{wc}		
	antidepressants: 1	antidepressants: 2
poo: 1	765.6953	334.9758
poo: 2	1082.1334	1170.719

The grand mean is 16.3718 and SS_{total} is 3749.9219.
Using an alpha value of α=0.01, test for a main effect of poo, a main effect of antidepressants and an interaction on the homework of pants.

4) I measure the effect of 2 kinds of bananas and 4 kinds of elections on the size of chickens. You find 7 chickens for each group and measure size for each of the 2x4=8 groups.

You generate the following table of means:

Means				
	elections: 1	elections: 2	elections: 3	elections: 4
bananas: 1	67.2471	71.28	64.3529	64.6171
bananas: 2	68.8486	67.0557	75.3486	66.3329

and the following table for SS_{wc}:

SS_{wc}				
	elections: 1	elections: 2	elections: 3	elections: 4
bananas: 1	92.9621	69.5252	66.8865	64.4799
bananas: 2	67.6313	145.7754	43.9509	175.4179

The grand mean is 68.1354 and SS_{total} is 1386.8456.
Using an alpha value of α=0.01, test for a main effect of bananas, a main effect of elections and an interaction on the size of chickens.

5) Because you don't have anything better to do you measure the effect of 2 kinds of skittles and 2 kinds of students on the frequency of fingers. You find 19 fingers for each group and measure frequency for each of the 2x2=4 groups.

You generate the following table of means:

Means		
	students: 1	students: 2
skittles: 1	11.1879	13.2584
skittles: 2	12.7137	8.1158

and the following table for SS_{wc}:

SS_{wc}		
	students: 1	students: 2
skittles: 1	1546.2063	2460.2319
skittles: 2	764.9536	1355.3583

The grand mean is 11.3189 and SS_{total} is 6430.4507.
Make a plot of the data with error bars representing standard errors of the mean.
Using an alpha value of α=0.01, test for a main effect of skittles, a main effect of students and an interaction on the frequency of fingers.

6) Your stats professor asks you to measure the effect of 2 kinds of sororities and 3 kinds of skittles on the heaviness of bus riders. You find 12 bus riders for each group and measure heaviness for each of the 2x3=6 groups.

You generate the following table of means:

Means			
	skittles: 1	skittles: 2	skittles: 3
sororities: 1	15.1117	13.7758	17.6742
sororities: 2	14.3525	16.4342	13.685

and the following table for SS_{wc}:

SS_{wc}			
	skittles: 1	skittles: 2	skittles: 3
sororities: 1	59.9932	161.1537	123.3445
sororities: 2	74.2512	268.1741	102.0113

The grand mean is 15.1722 and SS_{total} is 941.2028.
Using an alpha value of α=0.05, test for a main effect of sororities, a main effect of skittles and an interaction on the heaviness of bus riders.

7) Your friend gets you to measure the effect of 2 kinds of skittles and 4 kinds of oceans on the laughter of brain images. You find 9 brain images for each group and measure laughter for each of the 2x4=8 groups.

You generate the following table of means:

Means				
	oceans: 1	oceans: 2	oceans: 3	oceans: 4
skittles: 1	25.7144	21.3844	26.4878	24.4533
skittles: 2	24.8467	23.0922	22.8056	18.5944

and the following table for SS_{wc}:

SS_{wc}				
	oceans: 1	oceans: 2	oceans: 3	oceans: 4
skittles: 1	85.5756	467.779	155.8816	417.59
skittles: 2	223.2644	241.8902	478.1048	653.3286

The grand mean is 23.4224 and SS_{total} is 3134.6539.
Using an alpha value of α=0.01, test for a main effect of skittles, a main effect of oceans and an interaction on the laughter of brain images.

8) You ask a friend to measure the effect of 2 kinds of colors and 3 kinds of psychology classes on the equipment of daughters. You find 22 daughters for each group and measure equipment for each of the 2x3=6 groups.

You generate the following table of means:

Means			
	psychology classes: 1	psychology classes: 2	psychology classes: 3
colors: 1	19.4745	18.2109	11.2064
colors: 2	18.92	17.4259	16.5236

and the following table for SS_{wc}:

SS_{wc}			
	psychology classes: 1	psychology classes: 2	psychology classes: 3
colors: 1	1854.8853	1814.7262	1733.3709
colors: 2	2314.9356	1669.1377	1474.6489

The grand mean is 16.9602 and SS_{total} is 11857.0095.
Make a plot of the data with error bars representing standard errors of the mean.
Using an alpha value of α=0.01, test for a main effect of colors, a main effect of psychology classes and an interaction on the equipment of daughters.

9) We measure the effect of 2 kinds of cows and 3 kinds of chickens on the violence of potatoes. You find 7 potatoes for each group and measure violence for each of the 2x3=6 groups.

You generate the following table of means:

Means			
	chickens: 1	chickens: 2	chickens: 3
cows: 1	79.7229	82.6943	85.3986
cows: 2	81.9714	86.7971	91.7886

and the following table for SS_{wc}:

SS_{wc}			
	chickens: 1	chickens: 2	chickens: 3
cows: 1	192.4777	87.453	55.1483
cows: 2	257.4609	152.9059	287.2733

The grand mean is 84.7288 and SS_{total} is 1672.3008.
Using an alpha value of α=0.01, test for a main effect of cows, a main effect of chickens and an interaction on the violence of potatoes.

10) I measure the effect of 2 kinds of oranges and 4 kinds of poo on the distance of bus riders. You find 13 bus riders for each group and measure distance for each of the 2x4=8 groups.

You generate the following table of means:

Means				
	poo: 1	poo: 2	poo: 3	poo: 4
oranges: 1	87.5569	87.4331	85.7592	88.4985
oranges: 2	79.2508	85.8685	88.8854	92.47

and the following table for SS_{wc}:

SS_{wc}				
	poo: 1	poo: 2	poo: 3	poo: 4
oranges: 1	11.2101	31.9781	18.6283	31.7274
oranges: 2	17.3251	15.0128	14.5249	24.9446

The grand mean is 86.9653 and SS_{total} is 1453.3848.
Using an alpha value of α=0.01, test for a main effect of oranges, a main effect of poo and an interaction on the distance of bus riders.

Answers

1)

Within cell:

$SS_{wc} = 272.7455+317.9483+279.9746+307.9582+250.851+315.3024+373.228+381.9149 = 2499.9229$

$df_{wc} = 184 - (2)(4) = 176$

$MS_{wc} = \frac{2499.92}{176} = 14.2041$

Rows: underwear

mean row 1 = $\frac{63.4387+59.7226+61.6848+64.3609}{4} = 62.3018$

mean row 2 = $\frac{64.3135+64.6678+64.1152+65.8161}{4} = 64.7282$

$SS_R = (4)(23)(62.3018 - 63.5149)^2 + (4)(23)(64.7282 - 63.5149)^2 = 270.8241$

$df_R = 2 - 1 = 1$

$MS_R = \frac{270.8241}{1} = 270.8241$

$F_R(1, 176) = \frac{270.8241}{14.2041} = 19.07$

Columns: friends

mean col 1 : $\frac{63.4387+64.3135}{2} = 63.8761$

mean col 2 : $\frac{59.7226+64.6678}{2} = 62.1952$

mean col 3 : $\frac{61.6848+64.1152}{2} = 62.9$

mean col 4 : $\frac{64.3609+65.8161}{2} = 65.0885$

$SS_C = (2)(23)(63.8761 - 63.5149)^2 + (2)(23)(62.1952 - 63.5149)^2 + (2)(23)(62.9 - 63.5149)^2 + (2)(23)(65.0885 - 63.5149)^2 = 217.408$

$df_C = 4 - 1 = 3$

$MS_C = \frac{217.4084}{3} = 72.4695$

$F_C(3, 176) = \frac{72.4695}{14.2041} = 5.102$

Interaction:

$SS_{RXC} = 3099.65 - (2499.9229 + 270.8241 + 217.4084) = 111.494$

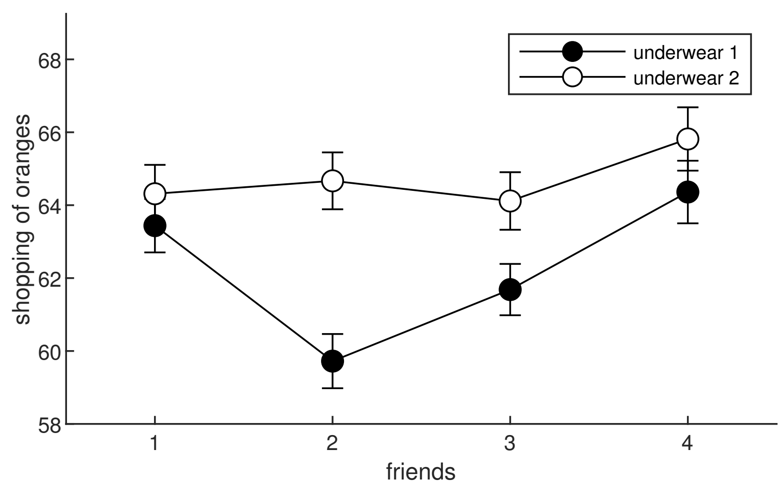
$df_{RXC} = (2-1)(4-1) = 3$

$MS_{RXC} = \frac{111.4944}{3} = 37.1648$

$F_{RXC}(3, 176) = \frac{37.1648}{14.2041} = 2.6165$

	SS	df	MS	F	F_{crit}	p-value
Rows	270.8241	1	270.8241	19.07	3.89	< 0.0001
Columns	217.4084	3	72.4695	5.1	2.66	0.0021
R X C	111.4944	3	37.1648	2.62	2.66	0.0524
wc	2499.9229	176	14.2041			
Total	3099.6498	183				

There is a significant main effect for underwear (rows) on the shopping of oranges, F(1,176) = 19.070, p = 0.000.
There is a significant main effect for friends (columns), F(3,176) = 5.100, p = 0.002.
There is not a significant interaction between underwear and friends, F(3,176) = 2.620, p = 0.052.



2)

Within cell:

$SS_{wc} = 281.3152 + 350.9958 + 253.1687 + 233.3293 + 413.7418 + 408.1977 + 190.6365 + 148.2673 = 2279.6523$

$df_{wc} = 104 - (2)(4) = 96$

$MS_{wc} = \frac{2279.65}{96} = 23.7464$

Rows: underwear

mean row 1 = $\frac{68.25+64.9162+62.9923+68.5569}{4} = 66.1789$

mean row 2 = $\frac{66.7515+66.3408+66.8008+67.0146}{4} = 66.7269$

$SS_R = (4)(13)(66.1789 - 66.4529)^2 + (4)(13)(66.7269 - 66.4529)^2 = 7.8101$

$df_R = 2 - 1 = 1$

$MS_R = \frac{7.8101}{1} = 7.8101$

$F_R(1, 96) = \frac{7.8101}{23.7464} = 0.33$

Columns: undergraduates

mean col 1 : $\frac{68.25+66.7515}{2} = 67.5008$

mean col 2 : $\frac{64.9162+66.3408}{2} = 65.6285$

mean col 3 : $\frac{62.9923+66.8008}{2} = 64.8965$

mean col 4 : $\frac{68.5569+67.0146}{2} = 67.7858$

$SS_C = (2)(13)(67.5008 - 66.4529)^2 + (2)(13)(65.6285 - 66.4529)^2 + (2)(13)(64.8965 - 66.4529)^2 + (2)(13)(67.7858 - 66.4529)^2 = 155.39$

$df_C = 4 - 1 = 3$

$MS_C = \frac{155.3899}{3} = 51.7966$

$F_C(3, 96) = \frac{51.7966}{23.7464} = 2.1812$

Interaction:

$SS_{RXC} = 2572.57 - (2279.6523 + 7.8101 + 155.3899) = 129.717$

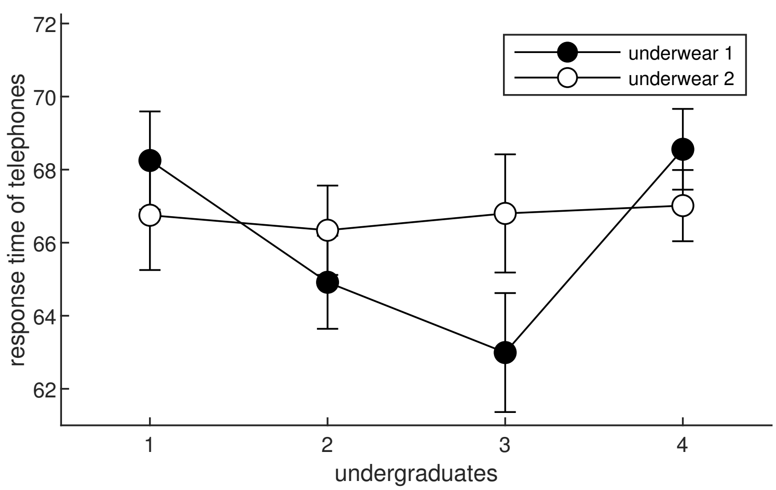
$df_{RXC} = (2-1)(4-1) = 3$

$MS_{RXC} = \frac{129.7168}{3} = 43.2389$

$F_{RXC}(3, 96) = \frac{43.2389}{23.7464} = 1.8209$

	SS	df	MS	F	F_{crit}	p-value
Rows	7.8101	1	7.8101	0.33	6.91	0.567
Columns	155.3899	3	51.7966	2.18	3.99	0.0954
R X C	129.7168	3	43.2389	1.82	3.99	0.1487
wc	2279.6523	96	23.7464			
Total	2572.5691	103				

There is not a significant main effect for underwear (rows) on the response time of telephones, F(1,96) = 0.330, p = 0.567.
There is not a significant main effect for undergraduates (columns), F(3,96) = 2.180, p = 0.095.
There is not a significant interaction between underwear and undergraduates, F(3,96) = 1.820, p = 0.149.



3)

Within cell:

$SS_{wc} = 765.6953 + 1082.1334 + 334.9758 + 1170.719 = 3353.5235$

$df_{wc} = 44 - (2)(2) = 40$

$MS_{wc} = \frac{3353.52}{40} = 83.8381$

Rows: poo

mean row 1 $= \frac{14.1509+21.3418}{2} = 17.7464$

mean row 2 $= \frac{13.8518+16.1427}{2} = 14.9973$

$SS_R = (2)(11)(17.7464 - 16.3718)^2 + (2)(11)(14.9973 - 16.3718)^2 = 83.1326$

$df_R = 2 - 1 = 1$

$MS_R = \frac{83.1326}{1} = 83.1326$

$F_R(1, 40) = \frac{83.1326}{83.8381} = 0.99$

Columns: antidepressants

mean col 1 : $\frac{14.1509+13.8518}{2} = 14.0014$

mean col 2 : $\frac{21.3418+16.1427}{2} = 18.7422$

$SS_C = (2)(11)(14.0014 - 16.3718)^2 + (2)(11)(18.7422 - 16.3718)^2 = 247.238$

$df_C = 2 - 1 = 1$

$MS_C = \frac{247.2384}{1} = 247.2384$

$F_C(1, 40) = \frac{247.2384}{83.8381} = 2.949$

Interaction:

$SS_{RXC} = 3749.92 - (3353.5234 + 83.1326 + 247.2384) = 66.0275$

$df_{RXC} = (2 - 1)(2 - 1) = 1$

$MS_{RXC} = \frac{66.0275}{1} = 66.0275$

$F_{RXC}(1,40) = \frac{66.0275}{83.8381} = 0.7876$

	SS	df	MS	F	F_{crit}	p-value
Rows	83.1326	1	83.1326	0.99	7.31	0.3257
Columns	247.2384	1	247.2384	2.95	7.31	0.0936
R X C	66.0275	1	66.0275	0.79	7.31	0.3794
wc	3353.5234	40	83.8381			
Total	3749.9219	43				

There is not a significant main effect for poo (rows) on the homework of pants, F(1,40) = 0.990, p = 0.326.
There is not a significant main effect for antidepressants (columns), F(1,40) = 2.950, p = 0.094.
There is not a significant interaction between poo and antidepressants, F(1,40) = 0.790, p = 0.379.

4)

Within cell:

$SS_{wc} = 92.9621 + 67.6313 + 69.5252 + 145.7754 + 66.8865 + 43.9509 + 64.4799 + 175.4179 = 726.6292$

$df_{wc} = 56 - (2)(4) = 48$

$MS_{wc} = \frac{726.629}{48} = 15.1381$

Rows: bananas

mean row 1 $= \frac{67.2471+71.28+64.3529+64.6171}{4} = 66.8743$

mean row 2 $= \frac{68.8486+67.0557+75.3486+66.3329}{4} = 69.3965$

$SS_R = (4)(7)(66.8743 - 68.1354)^2 + (4)(7)(69.3965 - 68.1354)^2 = 89.0569$

$df_R = 2 - 1 = 1$

$MS_R = \frac{89.0569}{1} = 89.0569$

$F_R(1, 48) = \frac{89.0569}{15.1381} = 5.88$

Columns: elections

mean col 1 : $\frac{67.2471+68.8486}{2} = 68.0479$

mean col 2 : $\frac{71.28+67.0557}{2} = 69.1679$

mean col 3 : $\frac{64.3529+75.3486}{2} = 69.8508$

mean col 4 : $\frac{64.6171+66.3329}{2} = 65.475$

$SS_C = (2)(7)(68.0479 - 68.1354)^2 + (2)(7)(69.1679 - 68.1354)^2 + (2)(7)(69.8508 - 68.1354)^2 + (2)(7)(65.475 - 68.1354)^2 = 155.311$

$df_C = 4 - 1 = 3$

$MS_C = \frac{155.3112}{3} = 51.7704$

$F_C(3, 48) = \frac{51.7704}{15.1381} = 3.4199$

Interaction:

$SS_{RXC} = 1386.85 - (726.6293 + 89.0569 + 155.3112) = 415.848$

$df_{RXC} = (2 - 1)(4 - 1) = 3$

$MS_{RXC} = \frac{415.8482}{3} = 138.6161$

$F_{RXC}(3, 48) = \frac{138.6161}{15.1381} = 9.1568$

	SS	df	MS	F	F_{crit}	p-value
Rows	89.0569	1	89.0569	5.88	7.19	0.0191
Columns	155.3112	3	51.7704	3.42	4.22	0.0245
R X C	415.8482	3	138.6161	9.16	4.22	0.0001
wc	726.6293	48	15.1381			
Total	1386.8456	55				

There is not a significant main effect for bananas (rows) on the size of chickens, F(1,48) = 5.880, p = 0.019.
There is not a significant main effect for elections (columns), F(3,48) = 3.420, p = 0.025.
There is a significant interaction between bananas and elections, F(3,48) = 9.160, p = 0.000.

5)

Within cell:

$SS_{wc} = 1546.2063 + 764.9536 + 2460.2319 + 1355.3583 = 6126.7501$

$df_{wc} = 76 - (2)(2) = 72$

$MS_{wc} = \frac{6126.75}{72} = 85.0938$

Rows: skittles

mean row 1 $= \frac{11.1879+13.2584}{2} = 12.2232$

mean row 2 $= \frac{12.7137+8.1158}{2} = 10.4148$

$SS_R = (2)(19)(12.2232 - 11.3189)^2 + (2)(19)(10.4148 - 11.3189)^2 = 62.1373$

$df_R = 2 - 1 = 1$

$MS_R = \frac{62.1373}{1} = 62.1373$

$F_R(1, 72) = \frac{62.1373}{85.0938} = 0.73$

Columns: students

mean col 1 : $\frac{11.1879+12.7137}{2} = 11.9508$

mean col 2 : $\frac{13.2584+8.1158}{2} = 10.6871$

$SS_C = (2)(19)(11.9508 - 11.3189)^2 + (2)(19)(10.6871 - 11.3189)^2 = 30.3411$

$df_C = 2 - 1 = 1$

$MS_C = \frac{30.3411}{1} = 30.3411$

$F_C(1, 72) = \frac{30.3411}{85.0938} = 0.3566$

Interaction:

$SS_{RXC} = 6430.45 - (6126.7501 + 62.1373 + 30.3411) = 211.222$

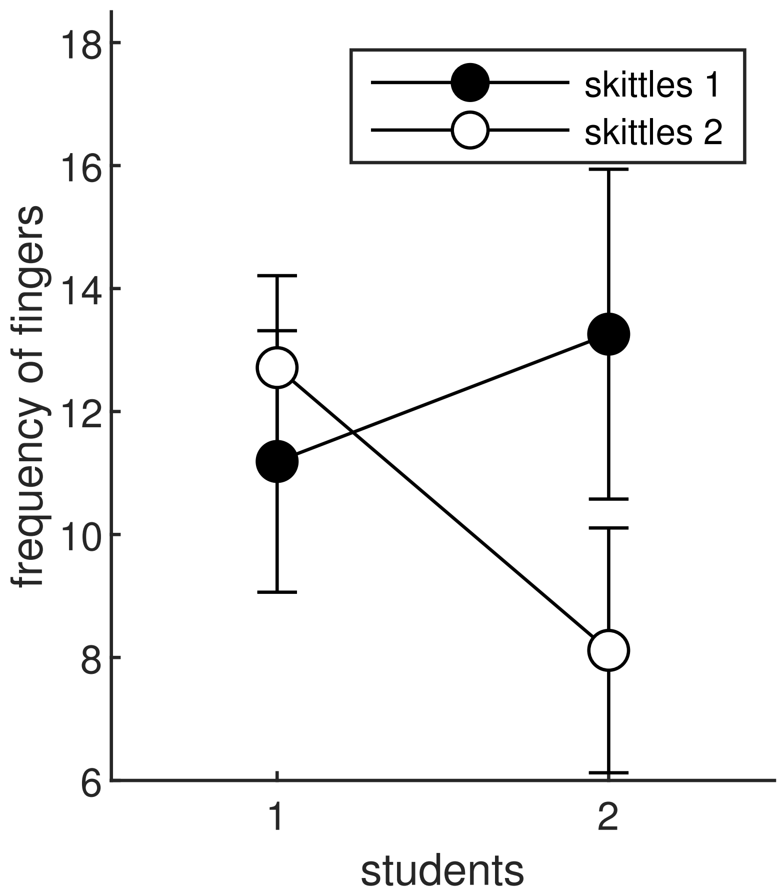
$df_{RXC} = (2 - 1)(2 - 1) = 1$

$MS_{RXC} = \frac{211.2222}{1} = 211.2222$

$F_{RXC}(1,72) = \frac{211.2222}{85.0938} = 2.4822$

	SS	df	MS	F	F_{crit}	p-value
Rows	62.1373	1	62.1373	0.73	7	0.3957
Columns	30.3411	1	30.3411	0.36	7	0.5504
R X C	211.2222	1	211.2222	2.48	7	0.1197
wc	6126.7501	72	85.0938			
Total	6430.4507	75				

There is not a significant main effect for skittles (rows) on the frequency of fingers, F(1,72) = 0.730, p = 0.396.
There is not a significant main effect for students (columns), F(1,72) = 0.360, p = 0.550.
There is not a significant interaction between skittles and students, F(1,72) = 2.480, p = 0.120.



6)

Within cell:

$SS_{wc} = 59.9932 + 74.2512 + 161.1537 + 268.1741 + 123.3445 + 102.0113 = 788.928$

$df_{wc} = 72 - (2)(3) = 66$

$MS_{wc} = \frac{788.928}{66} = 11.9535$

Rows: sororities

mean row 1 $= \frac{15.1117+13.7758+17.6742}{3} = 15.5206$

mean row 2 $= \frac{14.3525+16.4342+13.685}{3} = 14.8239$

$SS_R = (3)(12)(15.5206 - 15.1722)^2 + (3)(12)(14.8239 - 15.1722)^2 = 8.7362$

$df_R = 2 - 1 = 1$

$MS_R = \frac{8.7362}{1} = 8.7362$

$F_R(1, 66) = \frac{8.7362}{11.9535} = 0.73$

Columns: skittles

mean col 1 : $\frac{15.1117+14.3525}{2} = 14.7321$

mean col 2 : $\frac{13.7758+16.4342}{2} = 15.105$

mean col 3 : $\frac{17.6742+13.685}{2} = 15.6796$

$SS_C = (2)(12)(14.7321 - 15.1722)^2 + (2)(12)(15.105 - 15.1722)^2 + (2)(12)(15.6796 - 15.1722)^2 = 10.9358$

$df_C = 3 - 1 = 2$

$MS_C = \frac{10.9358}{2} = 5.4679$

$F_C(2, 66) = \frac{5.4679}{11.9535} = 0.4574$

Interaction:

$SS_{RXC} = 941.203 - (788.928 + 8.7362 + 10.9358) = 132.603$

$df_{RXC} = (2 - 1)(3 - 1) = 2$

$MS_{RXC} = \frac{132.6028}{2} = 66.3014$

$F_{RXC}(2, 66) = \frac{66.3014}{11.9535} = 5.5466$

	SS	df	MS	F	F_{crit}	p-value
Rows	8.7362	1	8.7362	0.73	3.99	0.396
Columns	10.9358	2	5.4679	0.46	3.14	0.6333
R X C	132.6028	2	66.3014	5.55	3.14	0.0059
wc	788.928	66	11.9535			
Total	941.2028	71				

There is not a significant main effect for sororities (rows) on the heaviness of bus riders, F(1,66) = 0.730, p = 0.396.
There is not a significant main effect for skittles (columns), F(2,66) = 0.460, p = 0.633.
There is a significant interaction between sororities and skittles, F(2,66) = 5.550, p = 0.006.

7)

Within cell:

$SS_{wc} = 85.5756 + 223.2644 + 467.779 + 241.8902 + 155.8816 + 478.1048 + 417.59 + 653.3286 = 2723.4142$

$df_{wc} = 72 - (2)(4) = 64$

$MS_{wc} = \frac{2723.41}{64} = 42.5533$

Rows: skittles

mean row 1 = $\frac{25.7144+21.3844+26.4878+24.4533}{4} = 24.51$

mean row 2 = $\frac{24.8467+23.0922+22.8056+18.5944}{4} = 22.3347$

$SS_R = (4)(9)(24.51 - 23.4224)^2 + (4)(9)(22.3347 - 23.4224)^2 = 85.173$

$df_R = 2 - 1 = 1$

$MS_R = \frac{85.173}{1} = 85.173$

$F_R(1, 64) = \frac{85.173}{42.5533} = 2$

Columns: oceans

mean col 1 : $\frac{25.7144+24.8467}{2} = 25.2805$

mean col 2 : $\frac{21.3844+23.0922}{2} = 22.2383$

mean col 3 : $\frac{26.4878+22.8056}{2} = 24.6467$

mean col 4 : $\frac{24.4533+18.5944}{2} = 21.5239$

$SS_C = (2)(9)(25.2805 - 23.4224)^2 + (2)(9)(22.2383 - 23.4224)^2 + (2)(9)(24.6467 - 23.4224)^2 + (2)(9)(21.5239 - 23.4224)^2 = 179.243$

$df_C = 4 - 1 = 3$

$MS_C = \frac{179.2427}{3} = 59.7476$

$F_C(3, 64) = \frac{59.7476}{42.5533} = 1.4041$

Interaction:

$SS_{RXC} = 3134.65 - (2723.4142 + 85.173 + 179.2427) = 146.824$

$df_{RXC} = (2 - 1)(4 - 1) = 3$

$MS_{RXC} = \frac{146.824}{3} = 48.9413$

$F_{RXC}(3, 64) = \frac{48.9413}{42.5533} = 1.1501$

	SS	df	MS	F	F_{crit}	p-value
Rows	85.173	1	85.173	2	7.05	0.1621
Columns	179.2427	3	59.7476	1.4	4.1	0.2509
R X C	146.824	3	48.9413	1.15	4.1	0.3358
wc	2723.4142	64	42.5533			
Total	3134.6539	71				

There is not a significant main effect for skittles (rows) on the laughter of brain images, F(1,64) = 2.000, p = 0.162.
There is not a significant main effect for oceans (columns), F(3,64) = 1.400, p = 0.251.
There is not a significant interaction between skittles and oceans, F(3,64) = 1.150, p = 0.336.

8)

Within cell:

$SS_{wc} = 1854.8853 + 2314.9356 + 1814.7262 + 1669.1377 + 1733.3709 + 1474.6489 = 10861.7046$

$df_{wc} = 132 - (2)(3) = 126$

$MS_{wc} = \frac{10861.7}{126} = 86.204$

Rows: colors

mean row 1 $= \frac{19.4745+18.2109+11.2064}{3} = 16.2973$

mean row 2 $= \frac{18.92+17.4259+16.5236}{3} = 17.6232$

$SS_R = (3)(22)(16.2973 - 16.9602)^2 + (3)(22)(17.6232 - 16.9602)^2 = 58.0152$

$df_R = 2 - 1 = 1$

$MS_R = \frac{58.0152}{1} = 58.0152$

$F_R(1, 126) = \frac{58.0152}{86.204} = 0.67$

Columns: psychology classes

mean col 1 : $\frac{19.4745+18.92}{2} = 19.1973$

mean col 2 : $\frac{18.2109+17.4259}{2} = 17.8184$

mean col 3 : $\frac{11.2064+16.5236}{2} = 13.865$

$SS_C = (2)(22)(19.1973 - 16.9602)^2 + (2)(22)(17.8184 - 16.9602)^2 + (2)(22)(13.865 - 16.9602)^2 = 674.136$

$df_C = 3 - 1 = 2$

$MS_C = \frac{674.1364}{2} = 337.0682$

$F_C(2, 126) = \frac{337.0682}{86.204} = 3.9101$

Interaction:

$SS_{RXC} = 11857 - (10861.7047 + 58.0152 + 674.1364) = 263.153$

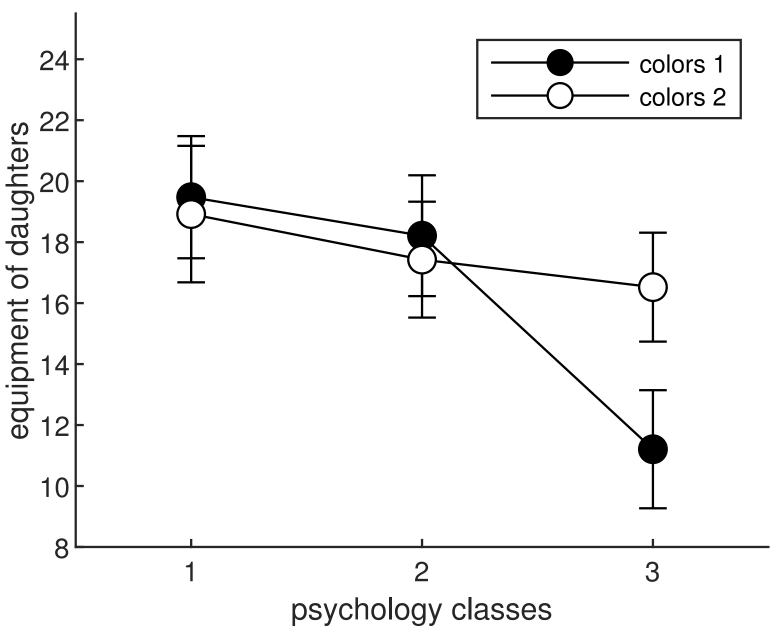
$df_{RXC} = (2-1)(3-1) = 2$

$MS_{RXC} = \frac{263.1532}{2} = 131.5766$

$F_{RXC}(2,126) = \frac{131.5766}{86.204} = 1.5263$

	SS	df	MS	F	F_{crit}	p-value
Rows	58.0152	1	58.0152	0.67	6.84	0.4146
Columns	674.1364	2	337.0682	3.91	4.78	0.0225
R X C	263.1532	2	131.5766	1.53	4.78	0.2205
wc	10861.7047	126	86.204			
Total	11857.0095	131				

There is not a significant main effect for colors (rows) on the equipment of daughters, F(1,126) = 0.670, p = 0.415.
There is not a significant main effect for psychology classes (columns), F(2,126) = 3.910, p = 0.022.
There is not a significant interaction between colors and psychology classes, F(2,126) = 1.530, p = 0.221.



9)

Within cell:

$SS_{wc} \;=\; 192.4777 \;+\; 257.4609 \;+\; 87.453 \;+\; 152.9059 \;+\; 55.1483 \;+\; 287.2733 \;=\; 1032.7191$

$df_{wc} = 42 - (2)(3) = 36$

$MS_{wc} = \frac{1032.72}{36} = 28.6866$

Rows: cows

mean row 1 = $\frac{79.7229+82.6943+85.3986}{3} = 82.6053$

mean row 2 = $\frac{81.9714+86.7971+91.7886}{3} = 86.8524$

$SS_R = (3)(7)(82.6053 - 84.7288)^2 + (3)(7)(86.8524 - 84.7288)^2 = 189.4013$

$df_R = 2 - 1 = 1$

$MS_R = \frac{189.4013}{1} = 189.4013$

$F_R(1, 36) = \frac{189.4013}{28.6866} = 6.6$

Columns: chickens

mean col 1 : $\frac{79.7229+81.9714}{2} = 80.8472$

mean col 2 : $\frac{82.6943+86.7971}{2} = 84.7457$

mean col 3 : $\frac{85.3986+91.7886}{2} = 88.5936$

$SS_C = (2)(7)(80.8472-84.7288)^2+(2)(7)(84.7457-84.7288)^2+(2)(7)(88.5936-84.7288)^2 =$ 420.056

$df_C = 3 - 1 = 2$

$MS_C = \frac{420.0561}{2} = 210.0281$

$F_C(2, 36) = \frac{210.0281}{28.6866} = 7.3215$

Interaction:

$SS_{RXC} = 1672.3 - (1032.7191 + 189.4013 + 420.0561) = 30.1243$

$df_{RXC} = (2-1)(3-1) = 2$

$MS_{RXC} = \frac{30.1243}{2} = 15.0621$

$F_{RXC}(2,36) = \frac{15.0621}{28.6866} = 0.5251$

	SS	df	MS	F	F_{crit}	p-value
Rows	189.4013	1	189.4013	6.6	7.4	0.0145
Columns	420.0561	2	210.0281	7.32	5.25	0.0022
R X C	30.1243	2	15.0621	0.53	5.25	0.5931
wc	1032.7191	36	28.6866			
Total	1672.3008	41				

There is not a significant main effect for cows (rows) on the violence of potatoes, F(1,36) = 6.600, p = 0.015.
There is a significant main effect for chickens (columns), F(2,36) = 7.320, p = 0.002.
There is not a significant interaction between cows and chickens, F(2,36) = 0.530, p = 0.593.

10)

Within cell:

$SS_{wc} = 11.2101 + 17.3251 + 31.9781 + 15.0128 + 18.6283 + 14.5249 + 31.7274 + 24.9446 = 165.3513$

$df_{wc} = 104 - (2)(4) = 96$

$MS_{wc} = \frac{165.351}{96} = 1.7224$

Rows: oranges

mean row 1 = $\frac{87.5569+87.4331+85.7592+88.4985}{4} = 87.3119$

mean row 2 = $\frac{79.2508+85.8685+88.8854+92.47}{4} = 86.6187$

$SS_R = (4)(13)(87.3119 - 86.9653)^2 + (4)(13)(86.6187 - 86.9653)^2 = 12.4962$

$df_R = 2 - 1 = 1$

$MS_R = \frac{12.4962}{1} = 12.4962$

$F_R(1, 96) = \frac{12.4962}{1.7224} = 7.26$

Columns: poo

mean col 1 : $\frac{87.5569+79.2508}{2} = 83.4039$

mean col 2 : $\frac{87.4331+85.8685}{2} = 86.6508$

mean col 3 : $\frac{85.7592+88.8854}{2} = 87.3223$

mean col 4 : $\frac{88.4985+92.47}{2} = 90.4843$

$SS_C = (2)(13)(83.4039 - 86.9653)^2 + (2)(13)(86.6508 - 86.9653)^2 + (2)(13)(87.3223 - 86.9653)^2 + (2)(13)(90.4843 - 86.9653)^2 = 657.624$

$df_C = 4 - 1 = 3$

$MS_C = \frac{657.6235}{3} = 219.2078$

$F_C(3, 96) = \frac{219.2078}{1.7224} = 127.2688$

Interaction:

$SS_{RXC} = 1453.38 - (165.3512 + 12.4962 + 657.6235) = 617.914$

$df_{RXC} = (2 - 1)(4 - 1) = 3$

$MS_{RXC} = \frac{617.9139}{3} = 205.9713$

$F_{RXC}(3, 96) = \frac{205.9713}{1.7224} = 119.5839$

	SS	df	MS	F	F_{crit}	p-value
Rows	12.4962	1	12.4962	7.26	6.91	0.0083
Columns	657.6235	3	219.2078	127.27	3.99	< 0.0001
R X C	617.9139	3	205.9713	119.58	3.99	< 0.0001
wc	165.3512	96	1.7224			
Total	1453.3848	103				

There is a significant main effect for oranges (rows) on the distance of bus riders, F(1,96) = 7.260, p = 0.008.
There is a significant main effect for poo (columns), F(3,96) = 127.270, p = 0.000.
There is a significant interaction between oranges and poo, F(3,96) = 119.580, p = 0.000.