Two Factor ANOVA

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Two Factor ANOVA

A two factor ANOVA (sometimes called ’two-way’ ANOVA) is a hypothesis test on means for which groups are placed in groups that vary along two factors instead of one.

For example, we might want to measure BMI for subjects with different diets AND for different levels of exercise. Or, we might measure the duration of hospital stays for patients undergoing different levels of medication AND different types of physical therapy. In these cases, we’re interest in not only how the means vary across each factor (called the main effects), but also how these two dimensions interact with each other (called interactions).

Here’s how to get to the 2-factor ANOVA with the flow chart:
For a two-factor ANOVA, each of the two factors (like 'diet' and 'exercise') are comprised of different levels, (like different diets, or different levels of exercise). The complete design involves having subjects placed into all possible combinations of levels for the two factors. For example, if there are two diets and three levels of exercise, there will be $2 \times 3 = 6$ groups, or cells.

Each cell should have the same number of subjects, which called a balanced design. There are ways to deal with unequal sample sizes, but we will not go there.

**Example 1: peanut butter and jelly**

Let’s start with a simple example. Suppose you want to study the effects of adding peanut butter and jelly to bread to study how they affect taste preferences. You take 36 students and break them down into 4 groups of 9 in what’s called a ’2x2’ design. One group will get just bread, another will get bread with peanut butter, another will get bread with jelly, and one group will get bread with both peanut butter and jelly.

We’ll run our test with $\alpha = 0.05$.

Each student tastes their food and rates their preference on a scale from 1 (gross) to 10 (excellent). The raw data looks like this:
Here are the summary statistics from the experiment:

<table>
<thead>
<tr>
<th>no jelly, no peanut butter</th>
<th>no jelly, peanut butter</th>
<th>jelly, no peanut butter</th>
<th>jelly and peanut butter</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

Means

<table>
<thead>
<tr>
<th>no peanut butter</th>
<th>jelly</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2222</td>
<td>6.3333</td>
</tr>
<tr>
<td>6.2222</td>
<td>7.6667</td>
</tr>
</tbody>
</table>

SSwc

<table>
<thead>
<tr>
<th>no peanut butter</th>
<th>jelly</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.5556</td>
<td>50</td>
</tr>
<tr>
<td>25.5556</td>
<td>20</td>
</tr>
</tbody>
</table>

Totals

<table>
<thead>
<tr>
<th>grand mean</th>
<th>6.1111</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS_total</td>
<td>181.5556</td>
</tr>
</tbody>
</table>

We’ve organized statistics in a 2x2 matrix for which the factor 'peanut butter' varies across the rows and the factor 'jelly' varies across the columns. We therefore call 'peanut butter' the row factor and 'jelly' the column factor.

The table titled SSwc contains the sums of squared deviation of each score from the mean of the cell that it belongs to. For example, for the 'no peanut butter, no jelly' category, the mean for that cell is 4.2222, so the SSwc for that cell is:

$$(8 - 4.2222)^2 + (6 - 4.2222)^2 + ... + (5 - 4.2222)^2 = 31.5556$$

SS_total is the sums of squared deviation of each score from the grand mean.

Before we run any statistical analyses, it’s useful to plot the means with error bars representing the standard errors of the mean. Remember, standard errors can be calculated by first calculating standard deviations from SSwc:

$$s_x = \sqrt{\frac{SS_{wc}}{n-1}}$$

And then dividing by by the square root of the sample size to get the standard error of the mean:
For example, for the ‘no peanut butter, no jelly’ category,

\[ s_x = \sqrt{\frac{31.5556}{9-1}} = 1.9861 \]

So

\[ s_{\bar{x}} = \frac{1.9861}{\sqrt{9}} = 0.662 \]

Check for yourself that the standard errors of the mean turn out to be:

<table>
<thead>
<tr>
<th>Standard errors</th>
<th>no jelly</th>
<th>jelly</th>
</tr>
</thead>
<tbody>
<tr>
<td>no peanut butter</td>
<td>0.662</td>
<td>0.8333</td>
</tr>
<tr>
<td>peanut butter</td>
<td>0.5958</td>
<td>0.527</td>
</tr>
</tbody>
</table>

Visually inspecting the graph, you can see that the green points are shifted above the red ones. This means that adding peanut butter helps the taste for both the plain bread, and for bread with jelly. This is called a **main effect for rows**, where the rows correspond the factor ‘peanut butter’.
Similarly, adding jelly increases the taste rating both for plain bread, and for bread with peanut butter. This is a main effect for columns where columns refer to the factor 'jelly'.

Notice also that the lines are roughly parallel. This means that the increase in the taste rating by adding peanut butter is about the same with and without jelly. We therefore say that there is no interaction between the factors of peanut butter and jelly.

The 2-factor ANOVA quantifies the statistical significance of these three observations: the main main effect for rows, the main effect for columns, and the interaction.

**Within-cell variance - the denominator of all three F-tests**

The statistical significance of the main effect for rows, the main effect for columns and the interaction will all be done with F-tests. Just as with the 1-factor ANOVA, the F-test will be a ratio of variances which are both estimates of the population variance under the null hypothesis.

Also, like the 1-factor ANOVA, the denominator will be an estimate of variance based on the variability within each group, and again we calculate it by summing up the sums of squared error between each score and its own cell’s mean:

\[ SS_{wc} = \sum (X - \bar{X}_{cell})^2 \]

I’ve given you \( SS_{wc} \) for each cell, so the total \( SS_{wc} \) is just the sum of these four numbers:

\[ SS_{wc} = 31.5556 + 25.5556 + 50 + 20 = 127.1112 \]

Just as with the 1-way ANOVA, the degrees of freedom for \( SS_{wc} \) is the total number of scores minus the total number of groups, so \( df = 9 + 9 + 9 + 9 - 4 = 32 \)

The variance within cells is \( SS_{wc} \) divided by its degrees of freedom:

\[ MS_{wc} = \frac{SS_{wc}}{df_{wc}} = \frac{127.1111}{32} = 3.9722 \]

**Main effects for rows: Does peanut butter affect taste ratings?**

To quantify the main effect for rows (peanut butter) we average the means across the two levels for columns (jelly). We can show these two means in a third column in the matrix of means:

<table>
<thead>
<tr>
<th></th>
<th>no jelly</th>
<th>jelly</th>
<th>Row Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>no peanut butter</td>
<td>4.2222</td>
<td>6.3333</td>
<td>5.2778</td>
</tr>
<tr>
<td>peanut butter</td>
<td>6.2222</td>
<td>7.6667</td>
<td>6.9445</td>
</tr>
</tbody>
</table>
The difference between these two row means indicates the effect of adding peanut butter after averaging across the two levels of jelly. We’ll use these two means to calculate the numerator of the F-test for the main effect for rows.

We first calculate SS for rows much like we would if this were a 1-factor ANOVA. We calculate the sum of squared difference of the row means from the grand mean, scaled by the sample size for the number of scores for each mean. Since there are 2 groups for each mean, the sample size for each row mean is (2)(9) = 18. So,

\[
SS_{row} = \sum (n_{row})(\bar{X}_{row} - \bar{X})^2 = 
\]

\[
\]

The degrees of freedom for SS for rows is equal to the number of rows minus 1 (df = 2 - 1 = 1). The variance for rows is SS for rows (\(SS_{rows}\)) divided by its degrees of freedom:

\[
MS_{rows} = \frac{SS_{rows}}{df_{rows}} = \frac{25}{1} = 25
\]

The F-statistic for the main effect for rows is the variance for rows divided by the variance within cells:

\[
F_{rows} = \frac{MS_{rows}}{MS_{wc}} = \frac{25}{3.9722} = 6.29
\]

Under the null hypothesis, the variance for rows is another estimate of the population variance. So if the null hypothesis is true, this F statistic should be around 1, on average. A large value of F indicates that the row means vary more than expected under the null hypothesis. If F is large enough, we conclude that this is too unusual for the null hypothesis to be true, so we reject it and conclude that the population means for the two rows are not the same.

The critical value for this F can be found in table E, using the degrees of freedom 1 and 32.

<table>
<thead>
<tr>
<th>(df_{ib})</th>
<th>(df_{b})</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>4.16</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>7.53</strong></td>
<td><strong>5.36</strong></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td><strong>4.15</strong></td>
<td>3.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>7.5</strong></td>
<td><strong>5.34</strong></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>4.14</td>
<td>3.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>7.47</strong></td>
<td><strong>5.31</strong></td>
<td></td>
</tr>
</tbody>
</table>

The critical value is 4.15. You can use the F-calculator to find that the p-value is 0.0174.

For our example, since our observed value of F (6.29) is greater than the critical value of F (4.15), we reject the null hypothesis. We can conclude using APA format: “There is a significant main effect of peanut butter on taste ratings, F(1,32) = 6.29, p = 0.0174”
Main effect for columns: does jelly affect taste ratings?

Conducting the F-test on the main effect for columns (jelly) is completely analogous to the main effect for rows. We first calculate the column means by calculating the mean for each of the two columns. We can show this as a third row in the matrix of means:

<table>
<thead>
<tr>
<th>Means</th>
<th>no jelly</th>
<th>jelly</th>
</tr>
</thead>
<tbody>
<tr>
<td>no peanut butter</td>
<td>4.2222</td>
<td>6.3333</td>
</tr>
<tr>
<td>peanut butter</td>
<td>6.2222</td>
<td>7.6667</td>
</tr>
<tr>
<td>Column Means</td>
<td>5.2222</td>
<td>7</td>
</tr>
</tbody>
</table>

The difference between these two column means indicates the effect of adding jelly after averaging across the two levels of peanut butter.

As for rows, we’ll calculate SS for columns, which is the sum of squared difference of the column means from the grand mean, scaled by the sample size for the number of scores for each mean:

\[
SS_{col} = \sum (n_{col})(\bar{X}_{col} - \bar{X})^2 =
\]

\[(18)(5.2222 - 6.1111)^2 + (18)(7 - 6.1111)^2 = 28.4445\]

The degrees of freedom for SS for columns is equal to the number of columns minus 1 (df = 2 - 1 = 1). The variance for the main effect for columns is SS for columns divided by its degrees of freedom:

\[
MS_{col} = \frac{SS_{col}}{df_{col}} = \frac{28.4445}{1} = 28.4445
\]

The F-statistic for the main effect for columns is the variance for columns divided by the variance within cells:

\[
F_{col} = \frac{MS_{col}}{MS_{wc}} = \frac{28.4445}{3.9722} = 7.16
\]

The F-calculator shows that the p-value for the main effect for columns (the df’s are 1 and 32) is 0.0117.

For our example, since our observed value of F (7.16) is greater than the critical value of F (4.15), we reject the null hypothesis. We can conclude using APA format: "There is a significant main effect of jelly on taste ratings, F(1,32) = 7.16, p = 0.0117”

The interaction: does the effect of peanut butter on taste ratings depend on jelly?

This third F-test determines if there is a statistically significant interaction between our two factors on our dependent variable. For our example, an interaction would occur if the effect of adding peanut butter to the bread added a different amount to the taste rating depending upon whether or not there was jelly (and vice versa). Remember from the graph
that the two lines were roughly parallel, indicating that peanut butter had a similar increase in taste ratings with and without jelly. **Parallel lines indicate no interaction between the two variables.**

To run this F-test, we need to calculate a third estimate of the population variance. The formula for the sums of squares for the interaction between rows and columns (\( \text{SS}_{R\times C} \)) is messy and not very intuitive. Fortunately we don’t need it because we can infer it by using the fact that the sums of squared deviation from the grand mean can be broken down into three components:

\[
\text{SS}_{\text{total}} = \text{SS}_{\text{rows}} + \text{SS}_{\text{cols}} + \text{SS}_{R\times C} + \text{SS}_{wc}
\]

I’ve given you \( \text{SS}_{\text{total}} \) and we just calculated \( \text{SS}_{\text{rows}}, \text{SS}_{\text{cols}} \) and \( \text{SS}_{wc} \). So

\[
\text{SS}_{R\times C} = \text{SS}_{\text{total}} - \text{SS}_{\text{rows}} - \text{SS}_{\text{cols}} - \text{SS}_{wc} = 181.5556 - 25 - 28.4445 - 127.1111 = 1
\]

The degrees of freedom for the row by column interaction is the number of rows minus 1 times the number of columns minus 1: \((2-1)(2-1) = 1\).

The variance for the row by column interaction is the SS divided by df:

\[
\text{MS}_{R\times C} = \frac{\text{SS}_{R\times C}}{df_{R\times C}} = \frac{1}{1} = 1
\]

The F-statistic is:

\[
F_{R\times C} = \frac{\text{MS}_{R\times C}}{\text{MS}_{wc}} = \frac{1}{3.9722} = 0.25
\]

The F-calculator shows that the p-value for this value of F (the df’s are 1 and 32) is 0.6205.

Since our p-value (0.6205) is greater than alpha (0.05), we fail to reject the null hypothesis. We can conclude using APA format: "There is not a significant interaction between the effects of peanut butter and jelly on taste ratings, \( F(1,32) = 0.25, \text{p} = 0.6205 \)"

**The Summary Table**

Like for 1-factor ANOVAs, the results from 2-factor ANOVAs are often reported in a summary table like this:

<table>
<thead>
<tr>
<th></th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>( F_{\text{crit}} )</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rows</td>
<td>25</td>
<td>1</td>
<td>25</td>
<td>6.29</td>
<td>4.15</td>
<td>0.0174</td>
</tr>
<tr>
<td>Columns</td>
<td>28.4445</td>
<td>1</td>
<td>28.4445</td>
<td>7.16</td>
<td>4.15</td>
<td>0.0117</td>
</tr>
<tr>
<td>R X C</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.25</td>
<td>4.15</td>
<td>0.6205</td>
</tr>
<tr>
<td>wc</td>
<td>127.1111</td>
<td>32</td>
<td>3.9722</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>181.5556</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 2: peanut butter and ham

Now suppose you want to study the effects of adding peanut butter and ham to bread. The (2x2) design is like the previous example; you take 36 students and break them down into 4 groups.

Again, we’ll run our test with $\alpha = 0.05$.

Here are the summary statistics from the experiment:

<table>
<thead>
<tr>
<th>Means</th>
<th>no ham</th>
<th>ham</th>
</tr>
</thead>
<tbody>
<tr>
<td>no peanut butter</td>
<td>4.8889</td>
<td>5.8889</td>
</tr>
<tr>
<td>peanut butter</td>
<td>5.6667</td>
<td>4.8889</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$SS_{wc}$</th>
<th>no ham</th>
<th>ham</th>
</tr>
</thead>
<tbody>
<tr>
<td>no peanut butter</td>
<td>2.8889</td>
<td>2.8889</td>
</tr>
<tr>
<td>peanut butter</td>
<td>4</td>
<td>4.8889</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Totals</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>grand mean</td>
<td>5.3333</td>
</tr>
<tr>
<td>$SS_{total}$</td>
<td>22</td>
</tr>
</tbody>
</table>

Here’s a plot the means with error bars representing the standard errors of the mean.
The first thing you see is that the lines are not parallel. Looking at the red points (no peanut butter), you can see that adding ham to bread improves its taste. But, if you start with peanut butter on your bread, adding ham makes it taste worse (yuck). This is a classic 'X' shaped interaction, and we’ll see below that it’s statistically significant.

For a main effect for rows we’d need the overall rating to change when we add peanut butter. But adding peanut butter increases the rating for bread with no ham but decreases the rating with ham. Overall, the green points aren’t higher or lower than the red points. It therefore looks like there is no main effect for peanut butter.

For a main main effect for columns (ham) we’d need the overall rating to change when we add ham. But you can see that the mean of the data point on the left (no ham are no higher or lower than the mean of the data points on the right (with jelly. So we don’t expect a main effect for the column factor, ham.

Now we’ll work through the problem and fill in the summary table at the end:

First, the within-cell variance:

\[ SS_{wc} = \sum (X - \bar{X}_{cell})^2 \]

\[ SS_{wc} = 2.8889 + 4 + 2.8889 + 4.8889 = 14.6667 \]

\[ df = 9 + 9 + 9 + 9 - 4 = 32 \]

\[ MS_{wc} = \frac{SS_{wc}}{df_{wc}} = \frac{14.6667}{32} = 0.4583 \]

For the main effects we can create a new table with both the row and column means:

<table>
<thead>
<tr>
<th></th>
<th>no ham</th>
<th>ham</th>
<th>Row Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>no peanut butter</td>
<td>4.8889</td>
<td>5.8889</td>
<td>5.3889</td>
</tr>
<tr>
<td>peanut butter</td>
<td>5.6667</td>
<td>4.8889</td>
<td>5.2778</td>
</tr>
<tr>
<td>Column Means</td>
<td>5.2778</td>
<td>5.3889</td>
<td></td>
</tr>
</tbody>
</table>

Note that the two row means don’t differ by much, nor do the two column means. This shows a lack of main effects for both rows and columns.

We’ll now compute the SS, variance and F-statistic for the main effect for rows.

\[ SS_R = (2)(9)(5.3889 - 5.3333)^2 + (2)(9)(5.2778 - 5.3333)^2 = 0.1111 \]

With \( df_R = 2 - 1 = 1 \)

\[ MS_{rows} = \frac{SS_{rows}}{df_{rows}} = \frac{0.1111}{1} = 0.1111 \]

\[ F_{rows} = \frac{MS_{rows}}{MS_{wc}} = \frac{0.1111}{0.4583} = 0.24 \]

The F-calculator shows that the p-value is 0.6275.
Since for our example, since our observed value of F (0.24) is not greater than the critical value of F (4.15), we fail to reject the null hypothesis. We can conclude using APA format: "There is not a significant main effect of peanut butter on taste ratings, F(1,32) = 0.24, p = 0.6275"

Now for the main effect for columns:

\[ SS_{col} = \sum (n_{col})(\bar{X}_{col} - \bar{X})^2 = \]

\[ (18)(5.2778 - 5.3333)^2 + (18)(5.3889 - 5.3333)^2 = 0.1111 \]

\[ df = 2 - 1 = 1 \]

\[ MS_{col} = \frac{SS_{col}}{df_{col}} = \frac{0.1111}{1} = 0.1111 \]

\[ F_{col} = \frac{MS_{col}}{MS_{wc}} = \frac{0.1111}{0.4583} = 0.24 \]

The F-calculator shows that the p-value for the main effect for columns (the df’s are 1 and 32) is 0.6275.

Because our p-value (0.6275) is greater than alpha (0.05), we conclude "There is not a significant main effect of ham on taste ratings, F(1,32) = 0.24, p = 0.6275"

Finally, for the interaction:

\[ SS_{RxC} = SS_{total} - SS_{rows} - SS_{cols} - SS_{wc} = 22 - 0.1111 - 0.1111 - 14.6667 = 7.1111 \]

\[ df = (2-1)(2-1) = 1 \]

\[ MS_{RxC} = \frac{SS_{RxC}}{df_{RxC}} = \frac{7.1111}{1} = 7.1111 \]

\[ F_{RxC} = \frac{MS_{RxC}}{MS_{wc}} = \frac{7.1111}{0.4583} = 15.52 \]

The F-calculator shows that the p-value for this value of F (the df’s are 1 and 32) is 0.0004.

Since our p-value (0.0004) is less than alpha (0.05) We conclude: "There is a significant interaction between the effects of peanut butter and ham on test scores, F(1,32) = 15.52, p = 0.0004"

The final summary table looks like this:
### Example 3: A 2x3 ANOVA on study habits

This third example is an two-factor ANOVA on study habits. You manipulate two factors, ‘music’ and ‘study location’. For the factor ‘music’, you have students either study in silence, or listen to the music of their choice. For the factor ‘study location’, you have groups of students study in one of 3 places: coffee shop, home and library. You find 25 students for each of the 4 groups. You generate the following summary statistics:

#### Means

<table>
<thead>
<tr>
<th></th>
<th>coffee shop</th>
<th>home</th>
<th>library</th>
</tr>
</thead>
<tbody>
<tr>
<td>no music</td>
<td>72.68</td>
<td>75.64</td>
<td>65.21</td>
</tr>
<tr>
<td>music</td>
<td>62.12</td>
<td>70.88</td>
<td>60.02</td>
</tr>
</tbody>
</table>

#### SS<sub>WC</sub>

<table>
<thead>
<tr>
<th></th>
<th>coffee shop</th>
<th>home</th>
<th>library</th>
</tr>
</thead>
<tbody>
<tr>
<td>no music</td>
<td>9961.44</td>
<td>6109.76</td>
<td>9688.36</td>
</tr>
<tr>
<td>music</td>
<td>8454.64</td>
<td>10880.64</td>
<td>8265.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>grand mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>SStotal</td>
<td>58216.7671</td>
</tr>
</tbody>
</table>

Here’s a plot the means with error bars representing the standard errors of the mean.
You’ll probably first notice that the red points are higher than the green points. This is a main effect for the row factor, music, for which listening to music leads to lower scores.

Next you’ll notice that the scores vary across the column factor, study location, with the best scores for students studying at home. Since there are three locations of study, we’re looking for a difference across means - just like a 1-factor ANOVA with three levels. Here, we see that there does look like there is a main effect for study location. Now we’ll work through the problem and fill in the summary table at the end:

The within-cell variance is calculated as in the examples above by adding up $SS_{wc}$ across the 6 cells:

$$SS_{wc} = \sum (X - \bar{X}_{cell})^2$$

$$SS_{wc} = 9961.44 + 8454.64 + 6109.76 + 10880.64 + 9688.3641 + 8265.0847 = 53359.9288$$

$$df = n_{total} - k = 150 - 6 = 144$$

$$MS_{wc} = \frac{SS_{wc}}{df_{wc}} = \frac{53359.9288}{144} = 370.5551$$

For the main effects, again we’ll create a new table with both the row and column means:
Means

<table>
<thead>
<tr>
<th></th>
<th>coffee shop</th>
<th>home</th>
<th>library</th>
<th>column means</th>
</tr>
</thead>
<tbody>
<tr>
<td>no music</td>
<td>72.68</td>
<td>75.64</td>
<td>65.2112</td>
<td>71.1771</td>
</tr>
<tr>
<td>music</td>
<td>62.12</td>
<td>70.88</td>
<td>60.0184</td>
<td>64.3395</td>
</tr>
<tr>
<td>row means</td>
<td>67.4</td>
<td>73.26</td>
<td>62.6148</td>
<td></td>
</tr>
</tbody>
</table>

Next we’ll compute the F-statistic for the main effect of rows (music) Note that since there are three columns, there are three means contributing to each row mean, the sample size for each row mean is $(25)(3) = 75$.

$$SS_{row} = \sum (n_{row})(\bar{X}_{row} - \bar{X})^2 =$$

$$(75)(71.1771 - 67.7583)^2 + (75)(64.3395 - 67.7583)^2 = 1753.229$$

With $df = 2 - 1 = 1$.

$$MS_{rows} = \frac{SS_{rows}}{df_{rows}} = \frac{1753.229}{1} = 1753.229$$

$$F_{rows} = \frac{MS_{rows}}{MS_{wc}} = \frac{1753.229}{370.5551} = 4.73$$

The F-calculator shows that the critical value is 3.91 and the p-value is 0.0313.

Our observed value of F (4.73) is greater than the critical value of F (3.91), we reject the null hypothesis. We can conclude using APA format: "There is a significant main effect of music on test scores, $F(1,144) = 4.73, p = 0.0313$".

Now for the main effect for columns (study location):

$$SS_{col} = \sum (n_{col})(\bar{X}_{col} - \bar{X})^2 =$$

$$(50)(67.4 - 67.7583)^2 + (50)(73.26 - 67.7583)^2 + (50)(62.6148 - 67.7583)^2 = 2842.6336$$

$df = 3 - 1 = 2$

$$MS_{col} = \frac{SS_{col}}{df_{col}} = \frac{2842.6336}{2} = 1421.3168$$

$$F_{col} = \frac{MS_{col}}{MS_{wc}} = \frac{1421.3168}{370.5551} = 3.84$$

The F-calculator shows that the p-value for the main effect for columns (the df’s are 2 and 144) is 0.0237.

Because our p-value (0.0237) is less than alpha (0.05), we conclude "There is a significant main effect of study location on taste ratings, $F(1,144) = 4.73, p = 0.0313$".

Finally, for the interaction:

$$SS_{RxC} = SS_{total} - SS_{rows} - SS_{cols} - SS_{wc} = 58216.7671 - 1753.229 - 2842.6336 - 53359.9288 = 260.9757$$
df = (3-1)(2-1) = 2.

\[ MS_{RxC} = \frac{SS_{RxC}}{df_{RxC}} = \frac{260.9757}{2} = 130.4878 \]

\[ F_{RxC} = \frac{MS_{RxC}}{MS_{wc}} = \frac{130.4878}{370.5551} = 0.35 \]

The F-calculator shows that the p-value for this value of F (the df’s are 2 and 144) is 0.7053.

Since our p-value (0.7053) is greater than alpha (0.05) We conclude: "There is not a significant interaction between the effects of music and study location on taste ratings, F(2,144) = 0.35, p = 0.7053”

The final summary table looks like this:

<table>
<thead>
<tr>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>F_{crit}</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rows 1753.229</td>
<td>1</td>
<td>1753.229</td>
<td>4.73</td>
<td>3.91</td>
<td>0.0313</td>
</tr>
<tr>
<td>Columns 2842.6336</td>
<td>2</td>
<td>1421.3168</td>
<td>3.84</td>
<td>3.06</td>
<td>0.0237</td>
</tr>
<tr>
<td>R X C 260.9757</td>
<td>2</td>
<td>130.4878</td>
<td>0.35</td>
<td>3.06</td>
<td>0.7053</td>
</tr>
<tr>
<td>wc 53359.9288</td>
<td>144</td>
<td>370.5551</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total 58216.7671</td>
<td>149</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Questions**

Your turn again. Here are 10 random practice questions followed by their answers.

1) For a 499 project you measure the effect of 2 kinds of musical groups and 4 kinds of grad students on the warmth of republicans. You find 13 republicans for each group and measure warmth for each of the 2x4=8 groups.

You generate the following table of means:

<table>
<thead>
<tr>
<th>Means</th>
<th>grad students: 1</th>
<th>grad students: 2</th>
<th>grad students: 3</th>
<th>grad students: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>musical groups: 1</td>
<td>35.7092</td>
<td>38.7877</td>
<td>30.6754</td>
<td>35.9208</td>
</tr>
<tr>
<td>musical groups: 2</td>
<td>37.5123</td>
<td>33.53</td>
<td>38.1569</td>
<td>34.8408</td>
</tr>
</tbody>
</table>

and the following table for SS_{wc}:
SSwc

<table>
<thead>
<tr>
<th></th>
<th>grad students: 1</th>
<th>grad students: 2</th>
<th>grad students: 3</th>
<th>grad students: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>musical groups: 1</td>
<td>382.9805</td>
<td>869.2682</td>
<td>269.4553</td>
<td>373.3895</td>
</tr>
<tr>
<td>musical groups: 2</td>
<td>405.0636</td>
<td>494.8676</td>
<td>894.7525</td>
<td>352.8769</td>
</tr>
</tbody>
</table>

The grand mean is 35.6416 and \( SS_{total} \) is 4687.0678.

Make a plot of the data with error bars representing standard errors of the mean.

Using an alpha value of \( \alpha = 0.05 \), test for a main effect of musical groups, a main effect of grad students and an interaction on the warmth of republicans.

2) Your advisor asks you to measure the effect of 2 kinds of psych 315 students and 4 kinds of facial expressions on the recognition of exams. You find 19 exams for each group and measure recognition for each of the 2x4=8 groups.

You generate the following table of means:

<table>
<thead>
<tr>
<th></th>
<th>facial expressions: 1</th>
<th>facial expressions: 2</th>
<th>facial expressions: 3</th>
<th>facial expressions: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>psych 315 students: 1</td>
<td>9.3226</td>
<td>8.8842</td>
<td>5.1553</td>
<td>6.7453</td>
</tr>
<tr>
<td>psych 315 students: 2</td>
<td>16.5774</td>
<td>11.3858</td>
<td>12.9679</td>
<td>11.1963</td>
</tr>
</tbody>
</table>

and the following table for \( SS_{wc} \):

<table>
<thead>
<tr>
<th></th>
<th>facial expressions: 1</th>
<th>facial expressions: 2</th>
<th>facial expressions: 3</th>
<th>facial expressions: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>psych 315 students: 1</td>
<td>1581.0928</td>
<td>1046.1547</td>
<td>899.8963</td>
<td>2152.7349</td>
</tr>
<tr>
<td>psych 315 students: 2</td>
<td>1496.6466</td>
<td>886.3517</td>
<td>1408.0069</td>
<td>1368.2592</td>
</tr>
</tbody>
</table>

The grand mean is 10.2793 and \( SS_{total} \) is 12559.8989.

Make a plot of the data with error bars representing standard errors of the mean.

Using an alpha value of \( \alpha = 0.05 \), test for a main effect of psych 315 students, a main effect of facial expressions and an interaction on the recognition of exams.

3) I go and measure the effect of 2 kinds of poo and 4 kinds of monkeys on the smell of potatoes. You find 20 potatoes for each group and measure smell for each of the 2x4=8 groups.

You generate the following table of means:
and the following table for $SS_{wc}$:

<table>
<thead>
<tr>
<th></th>
<th>monkeys: 1</th>
<th>monkeys: 2</th>
<th>monkeys: 3</th>
<th>monkeys: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>poo: 1</td>
<td>483.0966</td>
<td>464.487</td>
<td>506.1125</td>
<td>488.0813</td>
</tr>
<tr>
<td>poo: 2</td>
<td>245.3403</td>
<td>498.5705</td>
<td>647.5841</td>
<td>443.5555</td>
</tr>
</tbody>
</table>

The grand mean is 85.5447 and $SS_{total}$ is 4316.1448. Using an alpha value of $\alpha=0.05$, test for a main effect of poo, a main effect of monkeys and an interaction on the smell of potatoes.

4) Your advisor asks you to measure the effect of 2 kinds of teenagers and 4 kinds of examples on the farts of statistics problems. You find 8 statistics problems for each group and measure farts for each of the 2x4=8 groups.

You generate the following table of means:

<table>
<thead>
<tr>
<th></th>
<th>examples: 1</th>
<th>examples: 2</th>
<th>examples: 3</th>
<th>examples: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>teenagers: 1</td>
<td>64.7863</td>
<td>61.2563</td>
<td>64.0325</td>
<td>62.9838</td>
</tr>
<tr>
<td>teenagers: 2</td>
<td>65.8513</td>
<td>61.4375</td>
<td>60.34</td>
<td>63.0813</td>
</tr>
</tbody>
</table>

and the following table for $SS_{wc}$:

<table>
<thead>
<tr>
<th></th>
<th>examples: 1</th>
<th>examples: 2</th>
<th>examples: 3</th>
<th>examples: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>teenagers: 1</td>
<td>104.082</td>
<td>83.1262</td>
<td>65.4205</td>
<td>91.9814</td>
</tr>
<tr>
<td>teenagers: 2</td>
<td>155.6483</td>
<td>32.8122</td>
<td>215.3636</td>
<td>32.8705</td>
</tr>
</tbody>
</table>

The grand mean is 62.9711 and $SS_{total}$ is 980.8584. Using an alpha value of $\alpha=0.05$, test for a main effect of teenagers, a main effect of examples and an interaction on the farts of statistics problems.

5) I go and measure the effect of 2 kinds of fingers and 3 kinds of photoreceptors on the happiness of personalities. You find 22 personalities for each group and measure happiness for each of the 2x3=6 groups.
You generate the following table of means:

<table>
<thead>
<tr>
<th></th>
<th>photoreceptors: 1</th>
<th>photoreceptors: 2</th>
<th>photoreceptors: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>fingers: 1</td>
<td>64.375</td>
<td>70.7568</td>
<td>65.7986</td>
</tr>
<tr>
<td>fingers: 2</td>
<td>62.8423</td>
<td>63.5882</td>
<td>62.4109</td>
</tr>
</tbody>
</table>

and the following table for $SS_{wc}$:

<table>
<thead>
<tr>
<th></th>
<th>photoreceptors: 1</th>
<th>photoreceptors: 2</th>
<th>photoreceptors: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>fingers: 1</td>
<td>413.7444</td>
<td>385.2423</td>
<td>410.3429</td>
</tr>
<tr>
<td>fingers: 2</td>
<td>296.404</td>
<td>234.1379</td>
<td>379.7682</td>
</tr>
</tbody>
</table>

The grand mean is 64.962 and $SS_{total}$ is 3164.9285.

Make a plot of the data with error bars representing standard errors of the mean.

Using an alpha value of $\alpha=0.01$, test for a main effect of fingers, a main effect of photoreceptors and an interaction on the happiness of personalities.

6) Let’s measure the effect of 2 kinds of sponges and 3 kinds of elbows on the education of Americans. You find 19 Americans for each group and measure education for each of the $2 \times 3 = 6$ groups.

You generate the following table of means:

<table>
<thead>
<tr>
<th></th>
<th>elbows: 1</th>
<th>elbows: 2</th>
<th>elbows: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>sponges: 1</td>
<td>74.3616</td>
<td>79.6484</td>
<td>71.9221</td>
</tr>
<tr>
<td>sponges: 2</td>
<td>77.76</td>
<td>75.2526</td>
<td>82.4047</td>
</tr>
</tbody>
</table>

and the following table for $SS_{wc}$:

<table>
<thead>
<tr>
<th></th>
<th>elbows: 1</th>
<th>elbows: 2</th>
<th>elbows: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>sponges: 1</td>
<td>1841.4889</td>
<td>1335.6235</td>
<td>1114.4215</td>
</tr>
<tr>
<td>sponges: 2</td>
<td>1644.2684</td>
<td>1962.2844</td>
<td>2270.9023</td>
</tr>
</tbody>
</table>

The grand mean is 76.8916 and $SS_{total}$ is 11547.0961.

Make a plot of the data with error bars representing standard errors of the mean.

Using an alpha value of $\alpha=0.01$, test for a main effect of sponges, a main effect of elbows and an interaction on the education of Americans.

7) You measure the effect of 2 kinds of eggs and 2 kinds of monkeys on the health
of teams. You find 23 teams for each group and measure health for each of the 2x2=4 groups.

You generate the following table of means:

<table>
<thead>
<tr>
<th>Means</th>
<th>monkeys: 1</th>
<th>monkeys: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>eggs: 1</td>
<td>57.2491</td>
<td>56.4335</td>
</tr>
<tr>
<td>eggs: 2</td>
<td>56.8817</td>
<td>53.3413</td>
</tr>
</tbody>
</table>

and the following table for SS_{wc}:

<table>
<thead>
<tr>
<th>SS_{wc}</th>
<th>monkeys: 1</th>
<th>monkeys: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>eggs: 1</td>
<td>1245.9752</td>
<td>2229.8991</td>
</tr>
<tr>
<td>eggs: 2</td>
<td>842.0207</td>
<td>1800.0023</td>
</tr>
</tbody>
</table>

The grand mean is 55.9764 and SS_{total} is 6338.5163.

Using an alpha value of α=0.01, test for a main effect of eggs, a main effect of monkeys and an interaction on the health of teams.

8) You measure the effect of 2 kinds of video games and 2 kinds of otter pops on the taste of fathers. You find 17 fathers for each group and measure taste for each of the 2x2=4 groups.

You generate the following table of means:

<table>
<thead>
<tr>
<th>Means</th>
<th>otter pops: 1</th>
<th>otter pops: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>video games: 1</td>
<td>78.8247</td>
<td>82.0406</td>
</tr>
<tr>
<td>video games: 2</td>
<td>80.1094</td>
<td>80.9159</td>
</tr>
</tbody>
</table>

and the following table for SS_{wc}:

<table>
<thead>
<tr>
<th>SS_{wc}</th>
<th>otter pops: 1</th>
<th>otter pops: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>video games: 1</td>
<td>5.1546</td>
<td>3.5921</td>
</tr>
<tr>
<td>video games: 2</td>
<td>2.9919</td>
<td>7.0072</td>
</tr>
</tbody>
</table>

The grand mean is 80.4726 and SS_{total} is 112.2891.

Using an alpha value of α=0.05, test for a main effect of video games, a main effect of otter pops and an interaction on the taste of fathers.

9) For some reason you measure the effect of 2 kinds of spaghetti and 4 kinds of dinosaurs on the warmth of grandmothers. You find 7 grandmothers for each group and measure warmth for each of the 2x4=8 groups.
You generate the following table of means:

<table>
<thead>
<tr>
<th></th>
<th>dinosaurs: 1</th>
<th>dinosaurs: 2</th>
<th>dinosaurs: 3</th>
<th>dinosaurs: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>spaghetti: 1</td>
<td>52.6229</td>
<td>56.8143</td>
<td>64.4486</td>
<td>55.9614</td>
</tr>
<tr>
<td>spaghetti: 2</td>
<td>53.3371</td>
<td>59.9571</td>
<td>52.26</td>
<td>57.5457</td>
</tr>
</tbody>
</table>

and the following table for $SS_{wc}$:

<table>
<thead>
<tr>
<th></th>
<th>dinosaurs: 1</th>
<th>dinosaurs: 2</th>
<th>dinosaurs: 3</th>
<th>dinosaurs: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>spaghetti: 1</td>
<td>415.0559</td>
<td>1049.8392</td>
<td>322.9629</td>
<td>815.3837</td>
</tr>
<tr>
<td>spaghetti: 2</td>
<td>834.9543</td>
<td>89.5251</td>
<td>517.8356</td>
<td>284.7882</td>
</tr>
</tbody>
</table>

The grand mean is 56.6184 and $SS_{total}$ is 5166.9524.

Make a plot of the data with error bars representing standard errors of the mean.

Using an alpha value of $\alpha=0.05$, test for a main effect of spaghetti, a main effect of dinosaurs and an interaction on the warmth of grandmothers.

10) You want to measure the effect of 2 kinds of brains and 3 kinds of cartoon characters on the clothing of beers. You find 10 beers for each group and measure clothing for each of the 2x3=6 groups.

You generate the following table of means:

<table>
<thead>
<tr>
<th></th>
<th>cartoon characters: 1</th>
<th>cartoon characters: 2</th>
<th>cartoon characters: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>brains: 1</td>
<td>35.114</td>
<td>34.326</td>
<td>34.669</td>
</tr>
<tr>
<td>brains: 2</td>
<td>33.507</td>
<td>37.875</td>
<td>34.402</td>
</tr>
</tbody>
</table>

and the following table for $SS_{wc}$:

<table>
<thead>
<tr>
<th></th>
<th>cartoon characters: 1</th>
<th>cartoon characters: 2</th>
<th>cartoon characters: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>brains: 1</td>
<td>22.209</td>
<td>39.4826</td>
<td>77.0077</td>
</tr>
<tr>
<td>brains: 2</td>
<td>89.7686</td>
<td>17.856</td>
<td>107.3566</td>
</tr>
</tbody>
</table>

The grand mean is 34.9822 and $SS_{total}$ is 467.9526.

Using an alpha value of $\alpha=0.05$, test for a main effect of brains, a main effect of cartoon characters and an interaction on the clothing of beers.
Answers
1)

Within cell:

\[
SS_{wc} = 382.9805 + 405.0636 + 869.2682 + 494.8676 + 269.4553 + 894.7525 + 373.3895 + 352.8769 = 4042.6541
\]

\[
df_{wc} = 104 - (2)(4) = 96
\]

\[
MS_{wc} = \frac{4042.65}{96} = 42.111
\]

Rows: musical groups

mean row 1 = \[
\frac{35.7092 + 38.7877 + 30.6754 + 35.9208}{4} = 35.2733
\]

mean row 2 = \[
\frac{37.5123 + 33.53 + 38.1569 + 34.8408}{4} = 36.01
\]

\[
SS_R = (4)(13)(35.2733 - 35.6416)^2 + (4)(13)(36.01 - 35.6416)^2 = 14.1121
\]

\[
df_R = 2 - 1 = 1
\]

\[
MS_R = \frac{14.1121}{1} = 14.1121
\]

\[
F_R(1, 96) = \frac{14.1121}{42.111} = 0.34
\]

Columns: grad students

mean col 1 : \[
\frac{35.7092 + 37.5123}{2} = 36.6108
\]

mean col 2 : \[
\frac{38.7877 + 33.53}{2} = 36.1589
\]

mean col 3 : \[
\frac{30.6754 + 38.1569}{2} = 34.4162
\]

mean col 4 : \[
\frac{35.9208 + 34.8408}{2} = 35.3808
\]

\[
\]

\[
df_C = 4 - 1 = 3
\]

\[
MS_C = \frac{72.1911}{3} = 24.0637
\]

\[
F_C(3, 96) = \frac{24.0637}{42.111} = 0.5714
\]

Interaction:
\[ SS_{RXC} = 4687.07 - (4042.6541 + 14.1121 + 72.1911) = 558.11 \]

\[ df_{RXC} = (2 - 1)(4 - 1) = 3 \]

\[ MS_{RXC} = \frac{558.1105}{3} = 186.0368 \]

\[ F_{RXC}(3, 96) = \frac{186.0368}{42.111} = 4.4178 \]

<table>
<thead>
<tr>
<th></th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>( F_{crit} )</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rows</td>
<td>14.1121</td>
<td>1</td>
<td>14.1121</td>
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There is not a significant main effect for musical groups (rows) on the warmth of republicans, \( F(1,96) = 0.340, p = 0.561 \).
There is not a significant main effect for grad students (columns), \( F(3,96) = 0.570, p = 0.636 \).
There is a significant interaction between musical groups and grad students, \( F(3,96) = 4.420, p = 0.006 \).
Within cell:

\[ SS_{wc} = 1581.0928 + 1496.6466 + 1046.1547 + 886.3517 + 899.8963 + 1408.0069 + 2152.7349 + 1368.2592 = 10839.1431 \]

\[ df_{wc} = 152 - (2)(4) = 144 \]

\[ MS_{wc} = \frac{10839.144}{144} = 75.2718 \]

Rows: psych 315 students

mean row 1 = \( \frac{9.3226+8.8842+5.1553+6.7453}{4} = 7.5269 \)

mean row 2 = \( \frac{16.5774+11.3858+12.9679+11.1963}{4} = 13.0319 \)

\[ SS_R = (4)(19)(7.5269 - 10.2793)^2 + (4)(19)(13.0319 - 10.2793)^2 = 1151.591 \]

\[ df_R = 2 - 1 = 1 \]

\[ MS_R = \frac{1151.591}{1} = 1151.591 \]

\[ F_R(1,144) = \frac{1151.591}{75.2718} = 15.3 \]

Columns: facial expressions

mean col 1 : \( \frac{9.3226+16.5774}{2} = 12.95 \)

mean col 2 : \( \frac{8.8842+11.3858}{2} = 10.135 \)

mean col 3 : \( \frac{5.1553+12.9679}{2} = 9.0616 \)

mean col 4 : \( \frac{6.7453+11.1963}{2} = 8.9708 \)


\[ df_C = 4 - 1 = 3 \]

\[ MS_C = \frac{393.243}{3} = 131.0811 \]

\[ F_C(3,144) = \frac{131.0811}{75.2718} = 1.7414 \]

Interaction:
\[ SS_{RXC} = 12559.9 - (10839.143 + 1151.591 + 393.2433) = 175.922 \]

\[ df_{RXC} = (2-1)(4-1) = 3 \]

\[ MS_{RXC} = \frac{175.9216}{3} = 58.6405 \]

\[ F_{RXC}(3,144) = \frac{58.6405}{75.2718} = 0.7791 \]

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There is a significant main effect for psych 315 students (rows) on the recognition of exams, \(F(1,144) = 15.300, p = 0.000\).
There is not a significant main effect for facial expressions (columns), \(F(3,144) = 1.740, p = 0.162\).
There is not a significant interaction between psych 315 students and facial expressions, \(F(3,144) = 0.780, p = 0.507\).
3) Within cell:

\[ SS_{wc} = 483.0966 + 245.3403 + 464.487 + 498.5705 + 506.1125 + 647.5841 + 488.0813 + 443.5555 = 3776.8278 \]

\[ df_{wc} = 160 - (2)(4) = 152 \]

\[ MS_{wc} = \frac{3776.83}{152} = 24.8476 \]

Rows: poo

Mean row 1 = \[ \frac{87.42+84.563+88.4095+84.9765}{4} = 86.3422 \]

Mean row 2 = \[ \frac{83.5465+82.9855+87.3415+85.115}{4} = 84.7471 \]

\[ SS_R = (4)(20)(86.3422 - 85.5447)^2 + (4)(20)(84.7471 - 85.5447)^2 = 101.777 \]

\[ df_R = 2 - 1 = 1 \]

\[ MS_R = \frac{101.777}{1} = 101.777 \]

\[ F_R(1, 152) = \frac{101.777}{24.8476} = 4.1 \]

Columns: monkeys

Mean col 1 : \[ \frac{87.42+83.5465}{2} = 85.4833 \]

Mean col 2 : \[ \frac{84.563+82.9855}{2} = 83.7743 \]

Mean col 3 : \[ \frac{88.4095+87.3415}{2} = 87.8755 \]

Mean col 4 : \[ \frac{84.9765+85.115}{2} = 85.0458 \]


\[ df_C = 4 - 1 = 3 \]

\[ MS_C = \frac{352.7938}{3} = 117.5979 \]

\[ F_C(3, 152) = \frac{117.5979}{24.8476} = 4.7328 \]

Interaction:
\[ SS_{RXC} = 4316.14 - (3776.8277 + 101.777 + 352.7938) = 84.7463 \]
\[ df_{RXC} = (2 - 1)(4 - 1) = 3 \]
\[ MS_{RXC} = \frac{84.7463}{3} = 28.2488 \]
\[ F_{RXC}(3,152) = \frac{28.2488}{24.8476} = 1.1369 \]

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There is a significant main effect for poo (rows) on the smell of potatoes, \(F(1,152) = 4.100, p = 0.045\).
There is a significant main effect for monkeys (columns), \(F(3,152) = 4.730, p = 0.004\).
There is not a significant interaction between poo and monkeys, \(F(3,152) = 1.140, p = 0.335\).
4)

Within cell:

\[ SS_{wc} = 104.082 + 155.6483 + 83.1262 + 32.8122 + 65.4205 + 215.3636 + 91.9814 + 32.8705 = 781.3047 \]

\[ df_{wc} = 64 - (2)(4) = 56 \]

\[ MS_{wc} = \frac{781.305}{56} = 13.9519 \]

Rows: teenagers

mean row 1 = \( \frac{64.7863+61.2563+64.0325+62.9838}{4} = 63.2647 \)

mean row 2 = \( \frac{65.8513+61.4375+60.34+63.0813}{4} = 62.6775 \)

\[ SS_R = (4)(8)(63.2647 - 62.9711)^2 + (4)(8)(62.6775 - 62.9711)^2 = 5.5166 \]

\[ df_R = 2 - 1 = 1 \]

\[ MS_R = \frac{5.5166}{1} = 5.5166 \]

\[ F_R(1, 56) = \frac{5.5166}{13.9519} = 0.4 \]

Columns: examples

mean col 1 : \( \frac{64.7863+65.8513}{2} = 65.3188 \)

mean col 2 : \( \frac{61.2563+61.4375}{2} = 61.3469 \)

mean col 3 : \( \frac{64.0325+60.34}{2} = 62.1863 \)

mean col 4 : \( \frac{62.9838+63.0813}{2} = 63.0326 \)


\[ df_C = 4 - 1 = 3 \]

\[ MS_C = \frac{140.3092}{3} = 46.7697 \]

\[ F_C(3, 56) = \frac{46.7697}{13.9519} = 3.3522 \]

Interaction:
\[ SS_{RXC} = 980.858 - (781.3046 + 5.5166 + 140.3092) = 53.728 \]

\[ df_{RXC} = (2 - 1)(4 - 1) = 3 \]

\[ MS_{RXC} = \frac{53.728}{3} = 17.9093 \]

\[ F_{RXC}(3,56) = \frac{17.9093}{13.9519} = 1.2836 \]

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There is not a significant main effect for teenagers (rows) on the farts of statistics problems, \( F(1,56) = 0.400 \), \( p = 0.530 \).
There is a significant main effect for examples (columns), \( F(3,56) = 3.350 \), \( p = 0.025 \).
There is not a significant interaction between teenagers and examples, \( F(3,56) = 1.280 \), \( p = 0.290 \).
5)

Within cell:

\[ SS_{wc} = 413.7444 + 296.404 + 385.2423 + 234.1379 + 410.3429 + 379.7682 = 2119.6397 \]
\[ df_{wc} = 132 - (2)(3) = 126 \]
\[ MS_{wc} = \frac{2119.64}{126} = 16.8225 \]

Rows: fingers

mean row 1 = \( \frac{64.375+70.7568+65.7986}{3} = 66.9768 \)
mean row 2 = \( \frac{62.8423+63.5882+62.4109}{3} = 62.9471 \)
\[ SSR = (3)(22)(66.9768 - 64.962)^2 + (3)(22)(62.9471 - 64.962)^2 = 535.8691 \]
\[ df_R = 2 - 1 = 1 \]
\[ MS_R = \frac{535.8691}{1} = 535.8691 \]
\[ F_R(1,126) = \frac{535.8691}{16.8225} = 31.85 \]

Columns: photoreceptors

mean col 1 : \( \frac{64.375+62.8423}{2} = 63.6087 \)
mean col 2 : \( \frac{70.7568+63.5882}{2} = 67.1725 \)
mean col 3 : \( \frac{65.7986+62.4109}{2} = 64.1048 \)
\[ SSC = (2)(22)(63.6087 - 64.962)^2 + (2)(22)(67.1725 - 64.962)^2 + (2)(22)(64.1048 - 64.962)^2 = 327.921 \]
\[ df_C = 3 - 1 = 2 \]
\[ MSC = \frac{327.9207}{2} = 163.9604 \]
\[ F_C(2,126) = \frac{163.9604}{16.8225} = 9.7465 \]

Interaction:

\[ SS_{RXC} = 3164.93 - (2119.6396 + 535.8691 + 327.9207) = 181.499 \]
$df_{RXC} = (2 - 1)(3 - 1) = 2$

$MS_{RXC} = \frac{181.4991}{2} = 90.7496$

$F_{RXC}(2, 126) = \frac{90.7496}{16.8225} = 5.3945$

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There is a significant main effect for fingers (rows) on the happiness of personalities, $F(1,126) = 31.850$, $p = 0.000$.
There is a significant main effect for photoreceptors (columns), $F(2,126) = 9.750$, $p = 0.000$.
There is a significant interaction between fingers and photoreceptors, $F(2,126) = 5.390$, $p = 0.006$. 

![Graph showing the relationship between photoreceptors and happiness of personalities for two fingers](image-url)
6)

Within cell:

\[ SS_{wc} = 1841.4889 + 1644.2684 + 1335.6235 + 1962.2844 + 1114.4215 + 2270.9023 = 10168.989 \]

\[ df_{wc} = 114 - (2)(3) = 108 \]

\[ MS_{wc} = \frac{10169}{108} = 94.1573 \]

Rows: sponges

mean row 1 = \( \frac{74.3616 + 79.6484 + 71.9221}{3} = 75.3107 \)

mean row 2 = \( \frac{77.76 + 75.2526 + 82.4047}{3} = 78.4724 \)

\[ SS_R = (3)(19)(75.3107 - 76.8916)^2 + (3)(19)(78.4724 - 76.8916)^2 = 284.9056 \]

\[ df_R = 2 - 1 = 1 \]

\[ MS_R = \frac{284.9056}{1} = 284.9056 \]

\[ F_R(1,108) = \frac{284.9056}{94.1573} = 3.03 \]

Columns: elbows

mean col 1 : \( \frac{74.3616 + 77.76}{2} = 76.0608 \)

mean col 2 : \( \frac{79.6484 + 75.2526}{2} = 77.4505 \)

mean col 3 : \( \frac{71.9221 + 82.4047}{2} = 77.1634 \)

\[ SS_C = (2)(19)(76.0608 - 76.8916)^2 + (2)(19)(77.4505 - 76.8916)^2 + (2)(19)(77.1634 - 76.8916)^2 = 40.9082 \]

\[ df_C = 3 - 1 = 2 \]

\[ MS_C = \frac{40.9082}{2} = 20.4541 \]

\[ F_C(2,108) = \frac{20.4541}{94.1573} = 0.2172 \]

Interaction:

\[ SS_{RC} = 11547.1 - (10168.9889 + 284.9056 + 40.9082) = 1052.29 \]
\[ df_{RXC} = (2 - 1)(3 - 1) = 2 \]

\[ MS_{RXC} = \frac{1052.2934}{2} = 526.1467 \]

\[ F_{RXC}(2, 108) = \frac{526.1467}{94.1573} = 5.588 \]

<table>
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There is not a significant main effect for sponges (rows) on the education of Americans, \( F(1, 108) = 3.030, p = 0.085 \).
There is not a significant main effect for elbows (columns), \( F(2, 108) = 0.220, p = 0.803 \).
There is a significant interaction between sponges and elbows, \( F(2, 108) = 5.590, p = 0.005 \).
7)

Within cell:

\[ SS_{wc} = 1245.9752 + 842.0207 + 2229.8991 + 1800.0023 = 6117.8973 \]

\[ df_{wc} = 92 - (2)(2) = 88 \]

\[ MS_{wc} = \frac{6117.9}{88} = 69.5216 \]

Rows: eggs

mean row 1 = \[ \frac{57.2491 + 56.4335}{2} = 56.8413 \]

mean row 2 = \[ \frac{56.8817 + 53.3413}{2} = 55.1115 \]

\[ SSR = (2)(23)(56.8413 - 55.9764)^2 + (2)(23)(55.1115 - 55.9764)^2 = 68.8194 \]

\[ df_R = 2 - 1 = 1 \]

\[ MS_R = \frac{68.8194}{1} = 68.8194 \]

\[ F_R(1, 88) = \frac{68.8194}{69.5216} = 0.99 \]

Columns: monkeys

mean col 1 : \[ \frac{57.2491 + 56.8817}{2} = 57.0654 \]

mean col 2 : \[ \frac{56.4335 + 53.3413}{2} = 54.8874 \]

\[ SSC = (2)(23)(57.0654 - 55.9764)^2 + (2)(23)(54.8874 - 55.9764)^2 = 109.109 \]

\[ df_C = 2 - 1 = 1 \]

\[ MS_C = \frac{109.1091}{1} = 109.1091 \]

\[ F_C(1, 88) = \frac{109.1091}{69.5216} = 1.5694 \]

Interaction:

\[ SS_{RXC} = 6338.52 - (6117.8973 + 68.8194 + 109.1091) = 42.6905 \]

\[ df_{RXC} = (2 - 1)(2 - 1) = 1 \]

\[ MS_{RXC} = \frac{42.6905}{1} = 42.6905 \]
There is not a significant main effect for eggs (rows) on the health of teams, $F(1,88) = 0.990$, $p = 0.323$. 
There is not a significant main effect for monkeys (columns), $F(1,88) = 1.570$, $p = 0.213$. 
There is not a significant interaction between eggs and monkeys, $F(1,88) = 0.610$, $p = 0.437$. 

$$F_{RXC}(1, 88) = \frac{42.6905}{69.5216} = 0.6141$$

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Within cell:

\[ SS_{wc} = 5.1546 + 2.9919 + 3.5921 + 7.0072 = 18.7458 \]

\[ df_{wc} = 68 - (2)(2) = 64 \]

\[ MS_{wc} = \frac{18.7458}{64} = 0.2929 \]

Rows: video games

mean row 1 = \( \frac{78.8247 + 82.0406}{2} = 80.4327 \)

mean row 2 = \( \frac{80.1094 + 80.9159}{2} = 80.5126 \)

\[ SSR = (2)(17)(80.4327 - 80.4726)^2 + (2)(17)(80.5126 - 80.4726)^2 = 0.1088 \]

\[ df_R = 2 - 1 = 1 \]

\[ MS_R = \frac{0.1088}{1} = 0.1088 \]

\[ F_R(1, 64) = \frac{0.1088}{0.2929} = 0.37 \]

Columns: otter pops

mean col 1 : \( \frac{78.8247 + 80.1094}{2} = 79.4671 \)

mean col 2 : \( \frac{82.0406 + 80.9159}{2} = 81.4783 \)

\[ SSC = (2)(17)(79.4671 - 80.4726)^2 + (2)(17)(81.4783 - 80.4726)^2 = 68.7621 \]

\[ df_C = 2 - 1 = 1 \]

\[ MS_C = \frac{68.7621}{1} = 68.7621 \]

\[ F_C(1, 64) = \frac{68.7621}{0.2929} = 234.7631 \]

Interaction:

\[ SSRXC = 112.289 - (18.7458 + 0.1088 + 68.7621) = 24.6724 \]

\[ df_{RXC} = (2 - 1)(2 - 1) = 1 \]

\[ MS_{RXC} = \frac{24.6724}{1} = 24.6724 \]
\[ F_{RXC}(1, 64) = \frac{24.6724}{0.2929} = 84.2349 \]

<table>
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<th>F</th>
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<th>p-value</th>
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There is not a significant main effect for video games (rows) on the taste of fathers, \( F(1, 64) = 0.370, p = 0.545 \).
There is a significant main effect for otter pops (columns), \( F(1, 64) = 234.760, p = 0.000 \).
There is a significant interaction between video games and otter pops, \( F(1, 64) = 84.230, p = 0.000 \).
9)

Within cell:

\[ SS_{wc} = 415.0559 + 834.9543 + 1049.8392 + 89.5251 + 322.9629 + 517.8356 + 815.3837 + 284.7882 = 4330.3449 \]

\[ df_{wc} = 56 - (2)(4) = 48 \]

\[ MS_{wc} = \frac{4330.3449}{48} = 90.2155 \]

Rows: spaghetti

mean row 1 = \[ \frac{52.6229+56.8143+64.4486+55.9614}{4} = 57.4618 \]

mean row 2 = \[ \frac{53.3371+59.9571+52.26+57.5457}{4} = 55.775 \]

\[ SS_R = (4)(7)(57.4618 - 56.6184)^2 + (4)(7)(55.775 - 56.6184)^2 = 39.8335 \]

\[ df_R = 2 - 1 = 1 \]

\[ MS_R = \frac{39.8335}{1} = 39.8335 \]

\[ F_R(1,48) = \frac{39.8335}{90.2155} = 0.44 \]

Columns: dinosaurs

mean col 1 : \[ \frac{52.6229+53.3371}{2} = 52.98 \]

mean col 2 : \[ \frac{56.8143+59.9571}{2} = 58.3857 \]

mean col 3 : \[ \frac{64.4486+52.26}{2} = 58.3543 \]

mean col 4 : \[ \frac{55.9614+57.5457}{2} = 56.7536 \]


\[ df_C = 4 - 1 = 3 \]

\[ MS_C = \frac{271.501}{3} = 90.5003 \]

\[ F_C(3,48) = \frac{90.5003}{90.2155} = 1.0032 \]

Interaction:
\[ SS_{RXC} = 5166.95 - (4330.3449 + 39.8335 + 271.501) = 525.273 \]

\[ df_{RXC} = (2 - 1)(4 - 1) = 3 \]

\[ MS_{RXC} = \frac{525.273}{3} = 175.091 \]

\[ F_{RXC}(3, 48) = \frac{175.091}{90.2155} = 1.9408 \]

<table>
<thead>
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There is not a significant main effect for spaghetti (rows) on the warmth of grandmothers, \( F(1,48) = 0.440, p = 0.510 \).

There is not a significant main effect for dinosaurs (columns), \( F(3,48) = 1.000, p = 0.401 \).

There is not a significant interaction between spaghetti and dinosaurs, \( F(3,48) = 1.940, p = 0.136 \).
10

Within cell:

\[ SS_{wc} = 22.209 + 89.7686 + 39.4826 + 17.856 + 77.0077 + 107.3566 = 353.6805 \]
\[ df_{wc} = 60 - (2)(3) = 54 \]
\[ MS_{wc} = \frac{353.681}{54} = 6.5496 \]

Rows: brains

mean row 1 = \( \frac{35.114 + 34.326 + 34.669}{3} = 34.703 \)
mean row 2 = \( \frac{33.507 + 37.875 + 34.402}{3} = 35.2613 \)
\[ SS_R = (3)(10)(34.703 - 34.9822)^2 + (3)(10)(35.2613 - 34.9822)^2 = 4.6761 \]
\[ df_R = 2 - 1 = 1 \]
\[ MS_R = \frac{4.6761}{1} = 4.6761 \]
\[ F_R(1, 54) = \frac{4.6761}{0.5496} = 0.71 \]

Columns: cartoon characters

mean col 1 : \( \frac{35.114 + 33.507}{2} = 34.3105 \)
mean col 2 : \( \frac{34.326 + 37.875}{2} = 36.1005 \)
mean col 3 : \( \frac{34.669 + 34.402}{2} = 34.5355 \)
\[ SS_C = (2)(10)(34.3105 - 34.9822)^2 + (2)(10)(36.1005 - 34.9822)^2 + (2)(10)(34.5355 - 34.9822)^2 = 38.0263 \]
\[ df_C = 3 - 1 = 2 \]
\[ MS_C = \frac{38.0263}{2} = 19.0132 \]
\[ F_C(2, 54) = \frac{19.0132}{0.5496} = 2.903 \]

Interaction:

\[ SS_{RXC} = 467.953 - (353.6806 + 4.6761 + 38.0263) = 71.5696 \]
\[ df_{RXC} = (2 - 1)(3 - 1) = 2 \]

\[ MS_{RXC} = \frac{71.5696}{2} = 35.7848 \]

\[ F_{RXC}(2, 54) = \frac{35.7848}{6.5496} = 5.4637 \]

<table>
<thead>
<tr>
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</table>

There is not a significant main effect for brains (rows) on the clothing of beers, \( F(1,54) = 0.710, p = 0.403 \).

There is not a significant main effect for cartoon characters (columns), \( F(2,54) = 2.900, p = 0.064 \).

There is a significant interaction between brains and cartoon characters, \( F(2,54) = 5.460, p = 0.007 \).