

Z-tables

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This tutorial covers how to find areas under normal distributions using a z-table.

The standard normal distribution

Thanks to the central limit theorem distributions of means often fall into a normal 'bell-shaped' distribution. Since we'll be dealing with means as dependent measures a lot this quarter and in our research, we'll need to be familiar with the properties of the normal distribution.

All normal distributions have the same shape. They only differ by their means and standard deviations. The general equation for the normal probability distribution is:

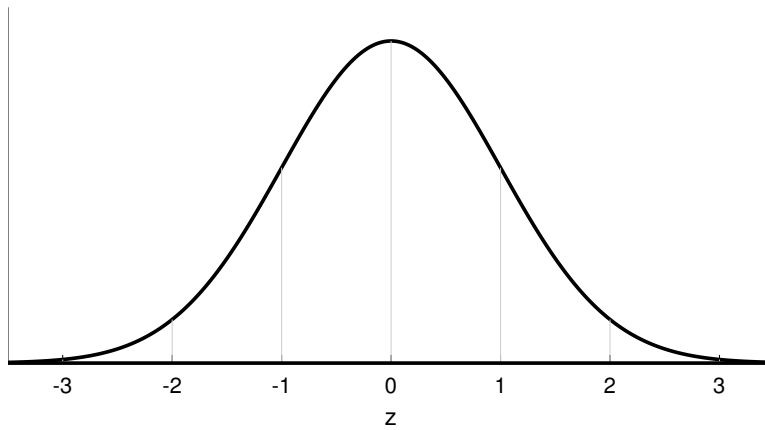
$$\frac{e^{-(x-\mu)^2}}{\sqrt{2\pi}\sigma}$$

Where μ is the mean and σ is the standard deviation of the distribution. (It's kind of remarkable that this ubiquitous function has two famous transcendental numbers in it, e , and π , plus the irrational number $\sqrt{2}$).

We choose one particular normal distribution, the standard normal, as a reference for tables. The standard normal distribution, or 'z-distribution' has a mean of zero and a standard deviation of 1. The standard normal's probability distribution function simplifies to:

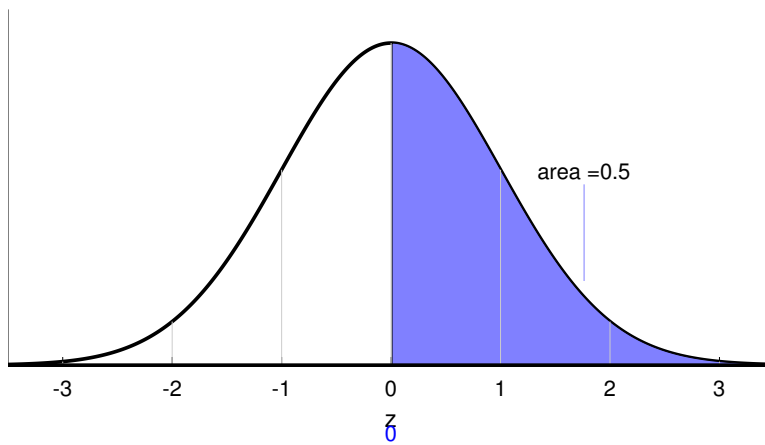
$$\frac{e^{-x^2}}{\sqrt{2\pi}}$$

It looks like this:



Here are some exercises on using the z-table to find areas under this standard normal distribution (either in the book, Excel spreadsheet, handout, R, or one of many websites or statistics programs). We'll start with an easy one:

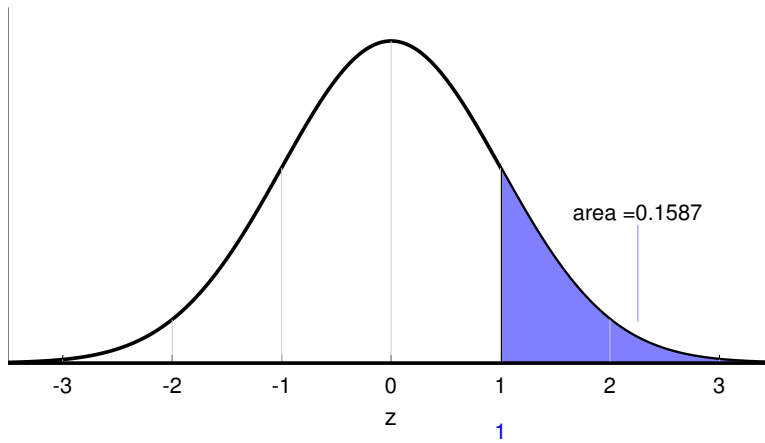
What is the area under the standard normal distribution above $z=0$? The area is shaded in the figure below:



The answer is 0.5 because the normal distribution has a total area of 1 and is symmetric about the mean of 0.

Areas above z

Example: find the area above $z=1$. The area is the shaded region below:



The area can be found by using table A in the book. Find the value in the first column for $z=1$. The third column gives the area under the standard normal above z . The relevant part of the table should look something like this:

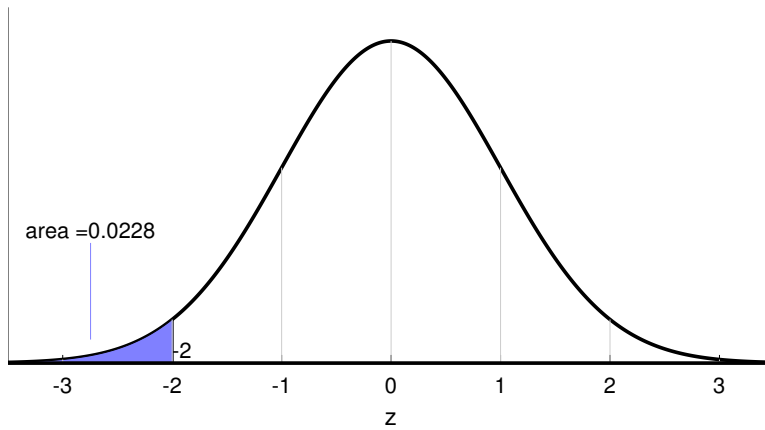
z	Area between mean and z	Area beyond z
\vdots	\vdots	\vdots
0.98	0.3365	0.1635
0.99	0.3389	0.1611
1.00	0.3413	0.1587
1.01	0.3438	0.1562
1.02	0.3461	0.1539
\vdots	\vdots	\vdots

On the row where the first column as $z = 1$, the third column shows that the area under the curve above z is 0.1587.

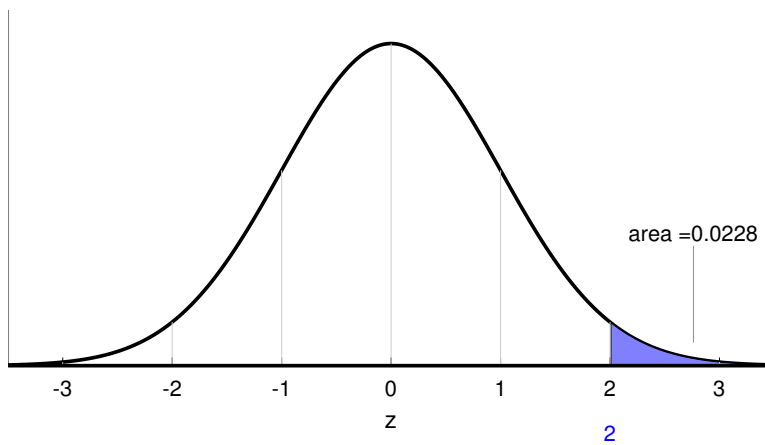
The middle column is the area between zero and z . Since right-half of the area is 0.5, you can see that columns 2 and 3 add up to 0.5 (for $z=1$, $0.3413 + 0.1587 = 0.5$).

Areas below the mean

Example: What is the area under the standard normal distribution below $z = -2$?



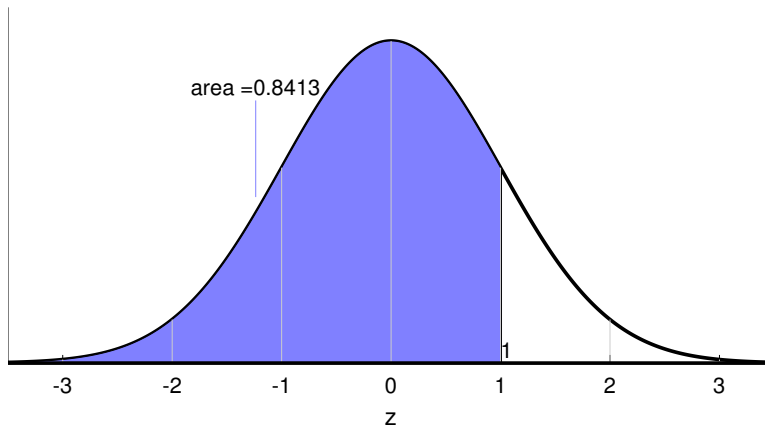
Notice that the z-table doesn't show areas for negative values of z . That's because the z -distribution is symmetrical, so for our example, the area below $z = -2$ is the same as the area above $z = 2$:



The area above $z = 2$ can be found in the table:

z	Area between mean and z	Area beyond z
\vdots	\vdots	\vdots
1.99	0.4767	0.0233
2.00	0.4772	0.0228
2.01	0.4778	0.0222
\vdots	\vdots	\vdots

Example: Find the area under the standard normal distribution below $z = 1$:

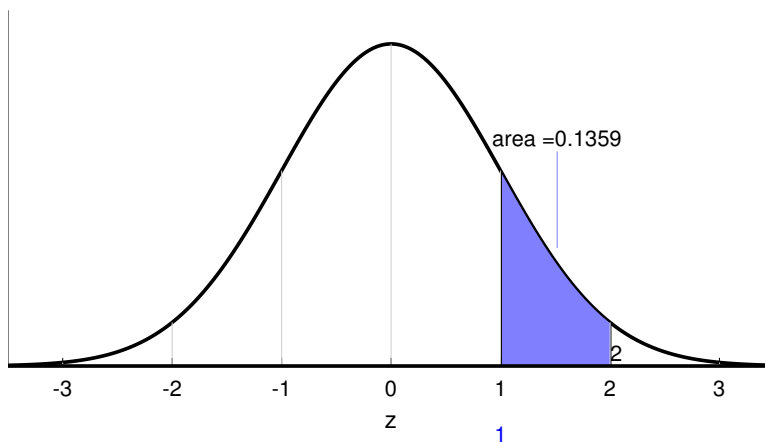


There are a couple of ways to do this one. One way is to realize that since the total area is 1, the area below $z = 1$ is equal to 1 minus the area above $z = 1$ which we know from before is 0.1587. So the area below 1 is $1 - 0.1587 = 0.8413$.

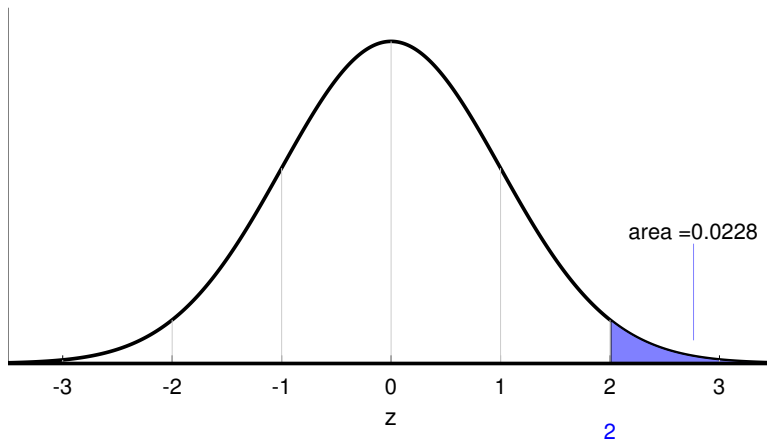
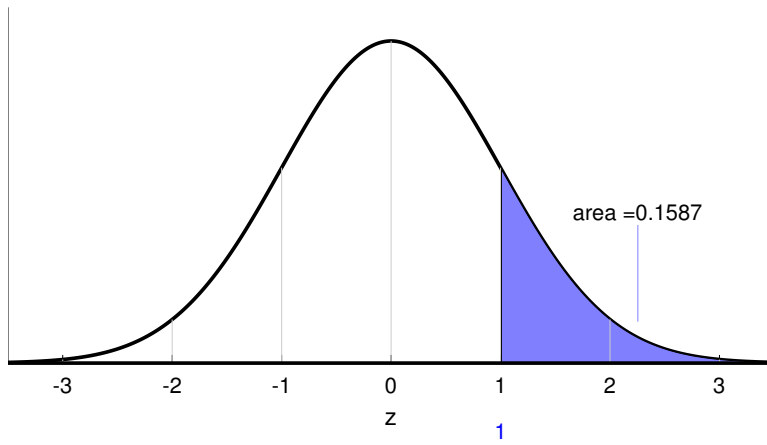
Another way to do this is to see that the area below 1 is the sum of the area between zero and 1 and the area below zero which is 0.5. From second column in the table, the area between zero and 1 is 0.6915. So the total area is $0.5 + 0.3413 = 0.8413$

Areas between two values of z

Example: What is the area under the standard normal distribution between 1 and 2?

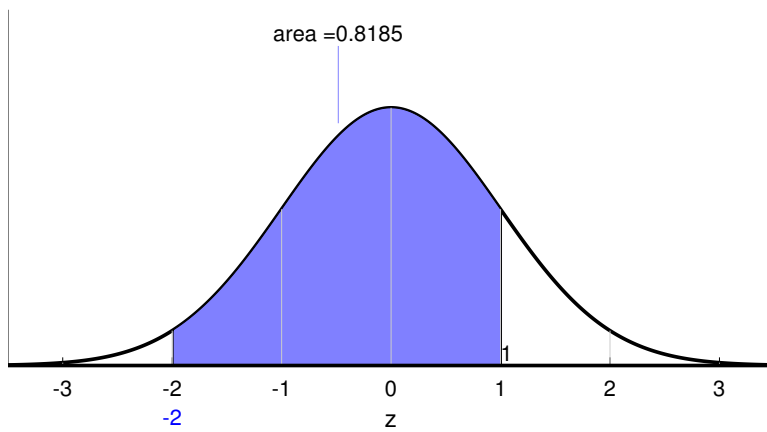


The trick is to understand that the area can be computed by subtracting the area above $z = 2$ from the area above $z = 1$:

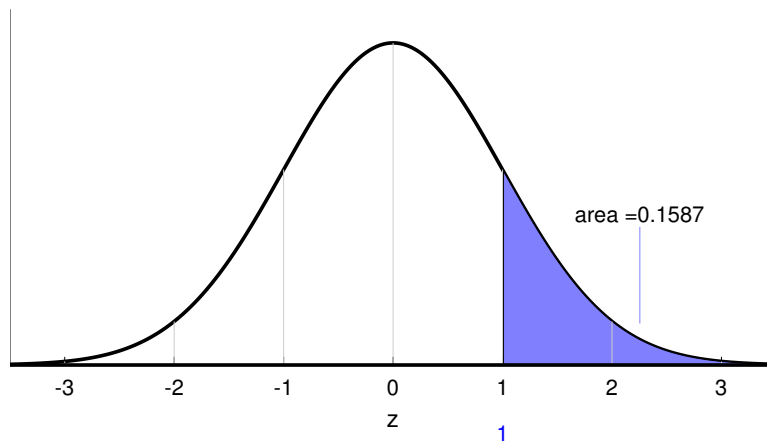
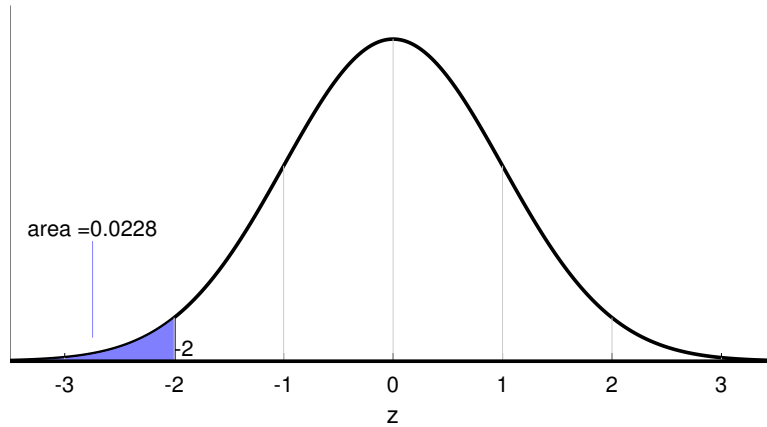


The difference is $0.1587 - 0.0228 = 0.1359$

Example: What is the area under the standard normal between $z = -2$ and 1 ?

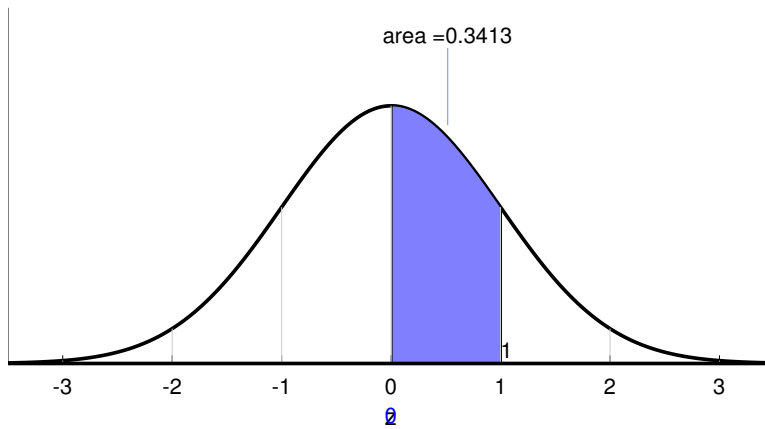
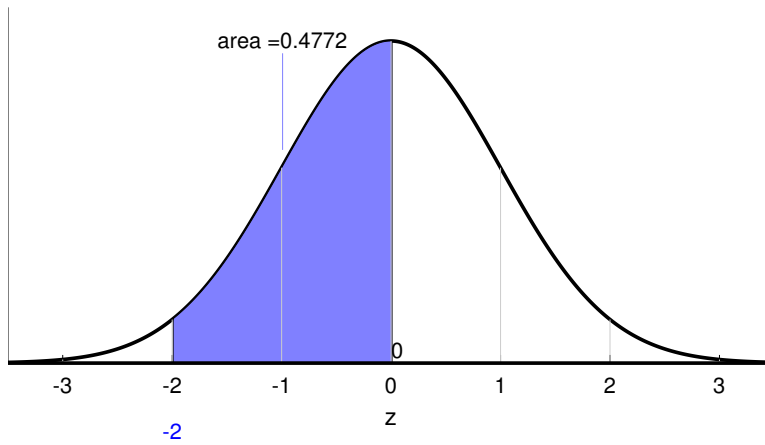


Again, there are a couple of ways to solve this one. One way is to use the fact that the total area is 1, so the area between -2 and 1 is equal to 1 minus the areas in the tails. The area below $z = -2$ is 0.0228 and the area above 1 is 0.1587:



So the total area is equal to $1 - 0.0228 - 0.1587 = 0.8185$

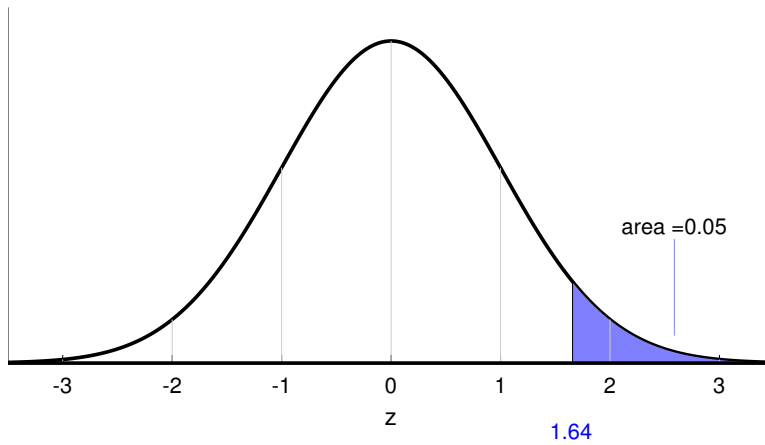
Another way to solve this one is to use the second column in table, which is the area between the mean and z . The area between $z = -2$ and $z = 0$ is the same as the area between $z = 0$ and $z = 2$, which according to the table is 0.4772. The table also tells us that the area between 0 and 1 is 0.3413:



So the total area is $0.4772 + 0.3413 = 0.8185$

Finding z-scores from areas

Example: Find the z score for which 5% of the area under the standard normal distribution lies above.

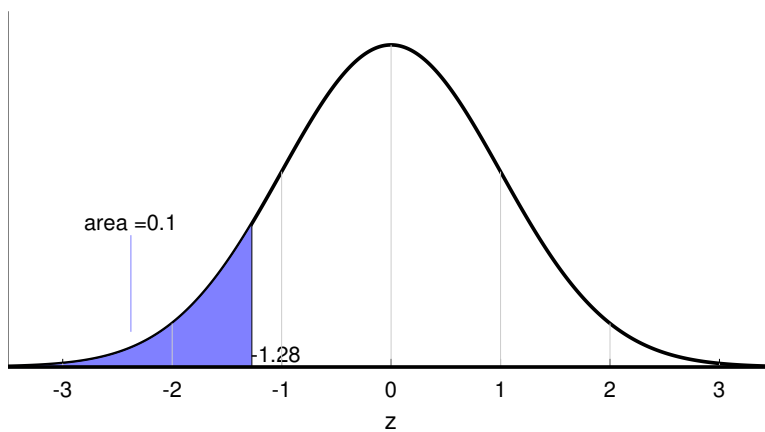


To solve this one we need to find the row in the table for which the third column, the area beyond z , is nearest to 0.05:

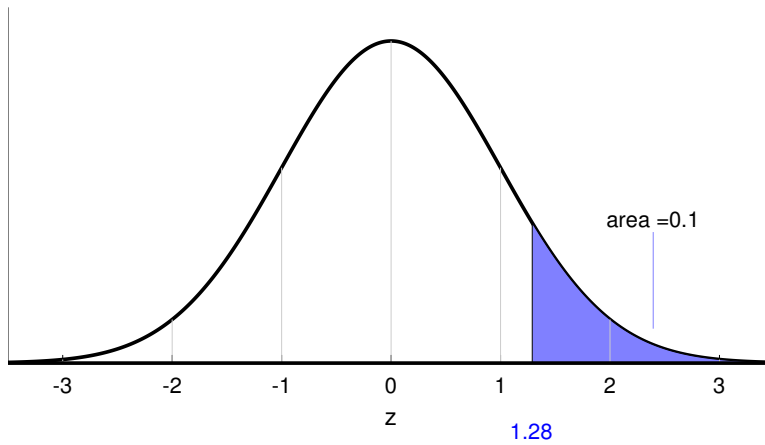
z	Area between mean and z	Area beyond z
\vdots	\vdots	\vdots
1.63	0.4484	0.0516
1.64	0.4495	0.0505
1.65	0.4505	0.0495
\vdots	\vdots	\vdots

So the answer is $z = 1.64$

Example: Find the value of z for which 10% of the area under the standard normal distribution lies below:



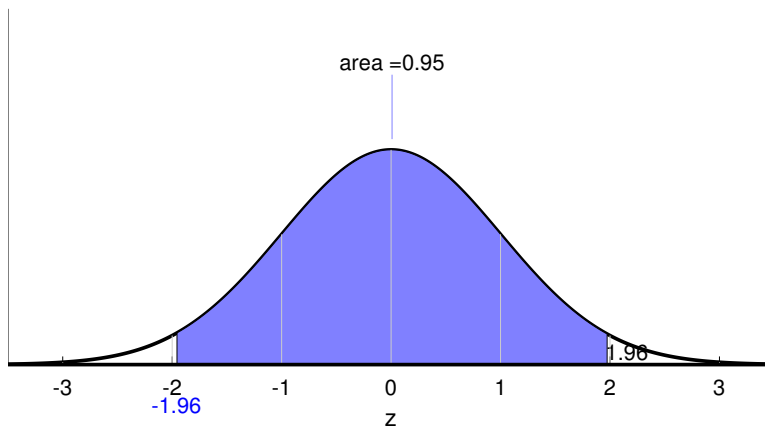
We'll use the fact that the normal distribution is symmetrical, and find the z -value for which 10% lies above:



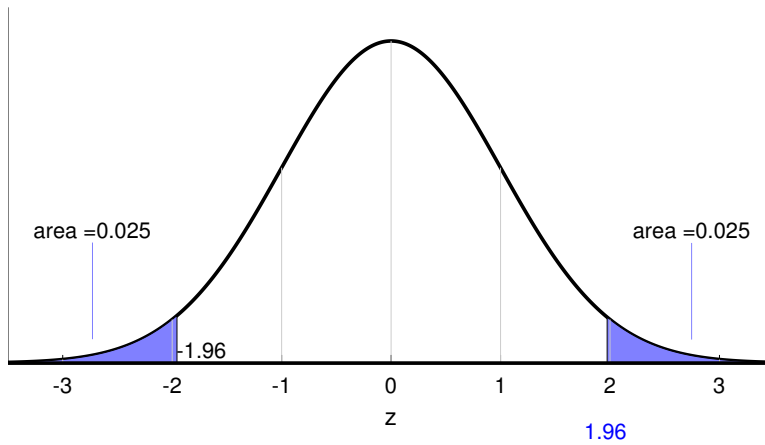
z	Area between mean and z	Area beyond z
⋮	⋮	⋮
1.27	0.3980	0.1020
1.28	0.3997	0.1003
1.29	0.4015	0.0985
⋮	⋮	⋮

The closest value of z is 1.28. Using symmetry, we know that 10% of the area under the standard normal distribution lies below $z = -1.28$.

Example: Find the values of z that bracket the middle 95% of the area under the standard normal distribution.



The middle 95% of the area leaves $(100-95)/2 = 2.5\%$ in each of the two tails;



So we need to find the z -value for which the area above is 0.025.

z	Area between mean and z	Area beyond z
\vdots	\vdots	\vdots
1.95	0.4744	0.0256
1.96	0.4750	0.0250
1.97	0.4756	0.0244
\vdots	\vdots	\vdots

From the table, $z = 1.96$. Therefore 95% of the area under the standard normal distribution lies between $z = -1.96$ and $z = 1.96$.

Z tables in R:

R has two functions that give you values in the z-table. 'pnorm' converts z-scores to areas, and 'qnorm' converts areas to z-scores.

The R commands shown below can be found here: [zTable.R](#)

```
# The z-distribution
#
# R's function 'pnorm' calculates the area under the normal distribution
# below a z-score. By default, it uses a standard normal distribution
# (mean 0, s.d. 1).
#
# For example, to find the area under the standard normal below z=1, we use:
pnorm(1)
[1] 0.8413447

# Note that the tables in the book and the Excel spreadsheet give you
# the areas ABOVE z. To get the values in these tables we need to subtract
# the result from 'pnorm' from 1:
1-pnorm(1)
[1] 0.1586553

# You should see that this value matches the value in the third column in
# the standard normal table for z=1.

# The area under the standard normal below z = -2 is:
pnorm(-2)
[1] 0.02275013

# The area under the standard normal between z=1 and z=2 can be found
# by finding the area below z=2 and subtracting the area below z=1:
pnorm(2) - pnorm(1)
[1] 0.1359051

# The area between z = -2 and z=1 can be calculated similarly:
pnorm(1) - pnorm(-2)
[1] 0.8185946

# To go the other way and find z-scores from areas we use
# the function 'qnorm'.
#
# For example, the z-score for which 95% of the area of the
# standard normal falls below is:
qnorm(.95)
[1] 1.644854

# This is, of course the same as the z-score for which 5% falls above.
```

```

# To find the z-score for which 10% of the area lies below we use:
qnorm(.1)
[1] -1.281552

# To find the values of z that bracket the middle 95% is found
# by first finding the lower z-score. This is the z-score
# for which only 2.5% falls below:
qnorm(.025)
[1] -1.959964

# Since the z-distribution is symmetric, the upper value of z
# is just the positive sign of this:
-qnorm(.025)
[1] 1.959964

# In general, if you want to find the z scores that bracket
# the middle p% we can first set a variable 'p' to some value:
p <- .5
# And then use 'c' to concatenate the two z-scores into a single
# vector:
c(qnorm((1-p)/2), -qnorm((1-p)/2))
[1] -0.6744898  0.6744898

```

Questions

Now it's your turn. Here are 30 random z-distribution problems and answer (including R commands). Draw pictures if it helps.

1) Find the area under the standard normal distribution below $z = 1.20$

```

pnorm(1.2)
[1] 0.8849303

```

Answer: 0.8849

2) Find the value of z for which 93 percent of the area under standard normal distribution lies above.

```

qnorm(1-0.93)
[1] -1.475791

```

Answer: $z = -1.48$

3) Find the area under the standard normal distribution between $z = 0.10$ and $z = 0.60$

```
pnorm(0.6) - pnorm(0.1)
[1] 0.185919
```

Answer: 0.1859

4) Find the value of z for which 5 percent of the area under standard normal distribution lies below.

```
qnorm(0.05)
[1] -1.644854
```

Answer: $z = -1.64$

5) Find the value of z for which 1 percent of the area under standard normal distribution lies above.

```
qnorm(1-0.01)
[1] 2.326348
```

Answer: $z = 2.33$

6) Find the area under the standard normal distribution below $z = 0.40$

```
pnorm(0.4)
[1] 0.6554217
```

Answer: 0.6554

7) Find the area under the standard normal distribution above $z = -0.70$

```
1-pnorm(-0.7)
[1] 0.7580363
```

Answer: 0.7580

8) Find the value of z for which 85 percent of the area under standard normal distribution lies above.

```
qnorm(1-0.85)
[1] -1.036433
```

Answer: $z = -1.04$

9) Find the value of z for which 65 percent of the area under standard normal distribution lies below.

```
qnorm(0.65)
[1] 0.3853205
```

Answer: $z = 0.39$

10) Find the area under the standard normal distribution above $z = 1.70$

```
1-pnorm(1.7)
[1] 0.04456546
```

Answer: 0.0446

11) Find the value of z for which 26 percent of the area under standard normal distribution lies above.

```
qnorm(1-0.26)
[1] 0.6433454
```

Answer: $z = 0.64$

12) Find the area under the standard normal distribution above $z = 0.10$

```
1-pnorm(0.1)
[1] 0.4601722
```

Answer: 0.4602

13) Find the area under the standard normal distribution between $z = 0.30$ and $z = 1.80$

```
pnorm(1.8) - pnorm(0.3)
[1] 0.3461583
```

Answer: 0.3462

14) Find the value of z for which 37 percent of the area under standard normal distribution lies above.

```
qnorm(1-0.37)
[1] 0.3318533
```

Answer: $z = 0.33$

15) Find the area under the standard normal distribution above $z = 0.90$

```
1-pnorm(0.9)
[1] 0.1840601
```

Answer: 0.1841

16) Find the value of z for which 65 percent of the area under standard normal distribution lies above.

```
qnorm(1-0.65)  
[1] -0.3853205
```

Answer: $z = -0.39$

17) Find the area under the standard normal distribution between $z = -1.40$ and $z = -0.70$

```
pnorm(-0.7) - pnorm(-1.4)  
[1] 0.161207
```

Answer: 0.1612

18) Find the value of z for which 44 percent of the area under standard normal distribution lies below.

```
qnorm(0.44)  
[1] -0.1509692
```

Answer: $z = -0.15$

19) Find the range of z values which covers the middle 24 percent of the area under the standard normal distribution.

```
c(qnorm(1-(1-0.24)/2), -qnorm(1-(1-0.24)/2))  
[1] 0.3054808 -0.3054808
```

Answer: Between $z = -0.31$ and $z = 0.31$

20) Find the area under the standard normal distribution between $z = -0.60$ and $z = 1.20$

```
pnorm(1.2) - pnorm(-0.6)  
[1] 0.6106772
```

Answer: 0.6107

21) Find the area under the standard normal distribution between $z = -0.40$ and $z = 0.60$

```
pnorm(0.6) - pnorm(-0.4)  
[1] 0.3811686
```

Answer: 0.3812

22) Find the value of z for which 6 percent of the area under standard normal distribution lies above.


```
qnorm(1-0.06)
[1] 1.554774
```

Answer: $z = 1.55$

23) Find the range of z values which covers the middle 52 percent of the area under the standard normal distribution.

```
c(qnorm(1-(1-0.52)/2), -qnorm(1-(1-0.52)/2))
[1] 0.7063026 -0.7063026
```

Answer: Between $z = -0.71$ and $z = 0.71$

24) Find the area under the standard normal distribution above $z = -0.10$

```
1-pnorm(-0.1)
[1] 0.5398278
```

Answer: 0.5398

25) Find the area under the standard normal distribution below $z = -1.30$

```
pnorm(-1.3)
[1] 0.09680048
```

Answer: 0.0968

26) Find the area under the standard normal distribution above $z = -1.30$

```
1-pnorm(-1.3)
[1] 0.9031995
```

Answer: 0.9032

27) Find the area under the standard normal distribution between $z = -0.00$ and $z = 1.00$

```
pnorm(1) - pnorm(-0)
[1] 0.3413447
```

Answer: 0.3413

28) Find the area under the standard normal distribution below $z = -0.80$

```
pnorm(-0.8)
[1] 0.2118554
```

Answer: 0.2119

29) Find the range of z values which covers the middle 46 percent of the area under the standard normal distribution.

```
c(qnorm(1-(1-0.46)/2), -qnorm(1-(1-0.46)/2))  
[1] 0.612813 -0.612813
```

Answer: Between $z = -0.61$ and $z = 0.61$

30) Find the value of z for which 76 percent of the area under standard normal distribution lies below.

```
qnorm(0.76)  
[1] 0.7063026
```

Answer: $z = 0.71$

