Contrast masking was studied psychophysically. A two-alternative forced-choice procedure was used to measure contrast thresholds for 2.0 cpd sine-wave gratings in the presence of masking sine-wave gratings. Thresholds were measured for 11 masker contrasts spanning three log units, and seven masker frequencies ranging \pm one octave from the signal frequency. Corresponding measurements were made for gratings with horizontal widths of 0.75\textdegree (narrow fields) and 6.0\textdegree (wide fields). For high contrast maskers at all frequencies, signal thresholds were related to masking contrast by power functions with exponents near 0.6. For a range of low masking contrasts, signal thresholds were reduced by the masker. For the wide fields, high contrast masking tuning functions peaked at the signal frequency, were slightly asymmetric, and had approximately invariant half-maximum frequencies that lie 3/4 octave below and 1 octave above the signal frequency. The corresponding low contrast tuning functions exhibited peak threshold reduction at the signal frequency, with half-minimum frequencies at roughly \pm 0.25 octaves. For the narrow fields, the masking tuning functions were much broader at both low and high masking contrasts. A masking model is presented that encompasses contrast detection, discrimination, and masking phenomena. Central constructs of the model include a linear spatial frequency filter, a nonlinear transducer, and a process of spatial pooling that acts at low contrasts only.

INTRODUCTION

The term masking is used commonly to refer to any destructive interaction or interference among transient stimuli that are closely coupled in space or time.\textsuperscript{1} Adaptation is often distinguished from masking, but the distinction is not very precise. An adapting stimulus onset always precedes the test stimulus and an adapting stimulus usually has a relatively long duration. The effect of masking may be a decrease in brightness, errors in recognition, or a failure to detect. This paper is concerned only with the effect of one stimulus on the detectability of another where the stimuli are coincident in space and simultaneous in time.

The effect of one stimulus on the detectability of another need not be to decrease detectability. Indeed, several studies have shown that a low contrast masker increases the detectability of a signal.\textsuperscript{2-4} This effect is variously called negative masking, facilitation, or the pedestal effect. This paper is concerned with this effect as well as the more common masking effect.

In the experiments, an observer was required to discriminate the superposition $a + b$ of two sine-wave grating stimuli $a$ and $b$ from grating $b$ presented alone. Grating $b$ is termed the masker. Its contrast remained fixed throughout a measurement. Grating $a$ is termed the signal. Its contrast was varied to find its threshold. This masking procedure includes, as special cases, contrast detection and contrast discrimination. In contrast detection, the masking contrast is 0. In contrast discrimination, masker and signal gratings have the same frequency and phase, and differ only in contrast.\textsuperscript{1}
Contrast discrimination is sometimes referred to as contrast increment detection. Our use of the term masking and our paradigm for studying it are very similar to those commonly used in the study of auditory frequency analysis.5

Contrast masking has been used to study the spatial frequency selectivity of pattern vision by measuring signal thresholds at one spatial frequency in the presence of masking patterns of other spatial frequencies.3,4,6,9,10 For high contrast maskers, and signals at medium and high spatial frequencies, these studies have found that signal threshold elevation is maximal when the masker and signal have the same frequency. Threshold elevation decreases regularly as the masking frequency departs from the signal frequency. The functions relating signal threshold elevation to masker frequency may be termed spread of masking functions or masking tuning functions.

It has been suggested that the bandwidths of these tuning functions provide estimates for spatial frequency channel bandwidths.7 These estimates are in rough agreement with tuning functions obtained in measurements of the spatial frequency adaptation effect.11,12 However, there is considerable variability in the form of tuning functions, both within and between methods, even at similar spatial frequencies. Widths at half-maximum range from 2 octaves10 in a masking experiment, to 0.62 octaves13 in an adaptation experiment. Estimates of the spatial frequency channel bandwidth from measurements of the detectability of complex gratings consisting of two sinusoidal components have yielded bandwidths as narrow as 0.4 octaves14 or less than 1.0 cycle per degree (cpd) in the range 5 to 20 cpd.15 However, it has been pointed out that the channel bandwidths derived from the detection of complex gratings depend very much on one’s model of detection.16-18 The same is true of masking and adaptation. It is not possible to relate the frequency selectivity of the masking or adaptation tuning functions to properties of underlying spatial frequency channels without invoking models of contrast masking or adaptation. To date, there exists no model of contrast masking. Moreover, there is evidence suggesting that adaptation may be determined in part by the inhibition of one channel by another,19,20 although the need for this additional complication has been challenged.21 Consequently, at this point, the spatial frequency channel bandwidth underlying frequency selectivity in masking or adaptation is still undetermined.

In the present study, masked thresholds were measured for a 2.0 cpd signal over a 2 octave range of masker frequencies and a 3 log unit range of masker contrasts. If the masking tuning function is to be taken as indicative of the relative spatial frequency sensitivity of the detecting channel, then, at a minimum, this function ought to have the same form independent of masker contrast. One objective of the study was to determine whether contrast masking possesses this property. It does not. However, the results may be interpreted in the context of a model that includes a spatial frequency channel whose sensitivity function is independent of contrast and peaks at the signal frequency. This model is an extension of the nonlinear transducer model of Foley and Legge for contrast detection and discrimination.4

The second purpose of the study was to investigate properties of contrast masking under conditions of restricted field size. It has been observed that masking tuning functions become broader (on a log frequency scale) for low signal frequencies.5,9 The broadening of the masking tuning functions bears some similarity to peak shift and/or broadening of spatial frequency adaptation tuning functions at low spatial frequencies.12,22 Comparison of these studies suggests a loose covariation between bandwidth estimates and field size. This covariation raises the possibility that the broadening of the tuning functions at low spatial frequencies may be related, not so much to inherent properties of low spatial frequency channels, but to the restricted number of cycles in the stimulus. Hence, corresponding masking measurements were made with stimuli that subtended either 6° or 0.75° horizontally. This stimulus width variation was found to have a large effect, but the model developed to account for the effects of varying contrast accounts for this result as well.

METHOD

Apparatus

Vertical sine-wave gratings were presented on a CRT display by Z-axis modulation.23 The display, designed and constructed at the Physiological Laboratory, Cambridge, had a P31 phosphor. The constant mean luminance was 200 cd/m². The raster was derived from a 100 Hz horizontal sweep and 100 kHz vertical triangle wave.

The relation between grating contrast and Z-axis voltage and frequency was measured with a narrow slit and UDT 80X Opto-meter. As the grating pattern drifted behind the stationary slit, the Opto-meter obtained 256 luminance samples per cycle. The resulting sequence of values was Fourier analyzed. By this means the harmonic composition of the modulation was obtained for a given Z-axis voltage and frequency. During the experiments, all contrasts were kept within the range for which the amplitudes of harmonic distortion products were less than 3% of the fundamental. Luminance modulation was horizontally restricted to a region symmetric about the center of the screen. The remainder of the screen was maintained at the constant mean luminance level without modulation. This mode of presentation was produced by passing the Z-axis signal through an electronic switch. The onset and period of switch closure were controlled by logic pulses from the CRT circuitry and corresponded to that portion of the display sweep for which luminance modulation was desired.

Voltage waveforms, corresponding to masker and signal gratings, were derived from two function generators. Their outputs were electronically added and applied to the switch prior to Z-axis input. As a result, CRT luminance modulation consisted of the superposition of masker and signal gratings. When the masker and signal had the same spatial frequency, both were derived from the same function generator.

A DEC PDP-8 computer controlled stimulus durations, and contrast levels with D/A converters, DB attenuators, and analog multipliers. The computer sequenced stimulus presentations and collected data.

Procedure

Observers viewed the display binocularly, with natural pupils, at a distance of 114 cm. The screen subtended 10°
horizontally and 6° vertically, with a white cardboard surround. For the wide field masking, modulation was restricted horizontally to 6°, symmetric about a central fixation mark. For narrow field masking, modulation was restricted horizontally to 0.75°, symmetric about the same fixation mark. When signals were presented, their onsets and offsets were simultaneous with those of the masker.

Psychophysical threshold estimates were obtained with a version of the two-alternative forced-choice staircase procedure.24 The observer began by adjusting signal grating contrast to a value just above threshold by turning a hand-held logarithmic attenuator. The observer was then given a block of self-initiated trials. Each trial consisted of two 200 ms exposures, marked by auditory tones, and separated by 750 ms. Only one exposure contained the signal grating, but both exposures contained the masking grating. The observer identified the signal interval by pressing one of two keys. A correct choice was followed by a tone. Three correct choices at one contrast level were followed by a constant decrement in contrast, and one incorrect choice was followed by an increment. The mean of the first six contrast peaks and valleys in the resulting sequence was taken as an estimate of the 0.79 proportion correct contrast level.24 Typically, a block consisted of about 35 trials. For each condition, the threshold measure was the geometric mean of the threshold estimates from four to six blocks. The error bars in Figs. 2 and 4 correspond to ± one standard error of the mean.

Contrast thresholds were obtained in two masking experiments:

(1) Wide Field Masking: Signals and maskers subtended 6° by 6°. Contrast thresholds for sine-wave grating signals at 2.0 cpd were measured as a function of masking contrast for seven masking frequencies. The masking frequencies, arranged to lie 0, approximately ±1/4, ±1/2, and ±1 octave from the signal frequency were: 1.0, 1.4, 1.7, 2.0, 2.4, 2.8, and 4.0 cpd. For each masking frequency, the 11 masking contrasts (percent contrasts) were: 0.05, 0.10, 0.20, 0.40, 0.80, 1.6, 3.2, 6.4, 12.8, 25.6, and 51.2%.

An experimental run, lasting about an hour, was devoted to a single masking frequency. Twelve signal threshold estimates were obtained, one in the absence of masking (detection threshold), and one for each of the 11 masking contrasts. Masking contrasts were presented in either ascending or descending order. No consistent differences could be observed in the resulting threshold estimates, suggesting that neither observer fatigue nor cumulative pattern adaptation played a role.

Three observers participated in the experiments, although time constraints prevented one observer from completing all conditions. An observer’s participation in a given condition involved two runs through the set of contrasts, one in ascending and one in descending order. Except in Fig. 1, the data points are always the geometric means of either four or six threshold estimates, obtained from two or three observers. For a given observer, the order of masking frequencies was randomized across runs.

(2) Narrow Field Masking: Signals and maskers subtended 0.75° horizontally by 6.0° vertically, symmetrically arranged about the center of the screen. The remainder of the screen was maintained at the same mean luminance level. Signal gratings at 2.0 cpd were truncated to 1.5 periods, consisting of a central bright half-cycle, flanked on either side by dark half-cycles.

The same set of masking frequencies and contrasts was used in the narrow field case, except that the lowest masking contrast of 0.05% was omitted. Procedural details were like those for wide field masking. Runs of narrow and wide field masking were interleaved.

Observers
There were three observers, all practiced with the methods and stimuli. WWL is a female, and SH a male, both in their mid twenties. JMF is a male in his late thirties. Throughout the experiments, observers were optically corrected.

RESULTS AND DISCUSSION

Individual data
At the signal frequency of 2.0 cpd, the three observers’ threshold contrasts (percent contrasts) for the 6.0° and 0.75° gratings were, respectively: JMF, 0.28 and 0.51; WWL, 0.31 and 0.49; SH, 0.28 and 0.50. These values are geometric means of 7 to 16 separate detection threshold estimates, collected on different days in different runs.

Figure 1 shows data separately for the three observers when the masker frequency was 2.8 cpd and the gratings were 0.75° wide. Each point is the geometric mean of two threshold estimates. The important properties of the masking data will be discussed in the next two subsections. Note here, however, that the scatter of points across individuals is typical of data collected in all masking conditions. No systematic individual differences in sensitivity or shape of the masking curves were apparent across conditions. Accordingly, data were combined across observers. The remaining figures show combined data.
There is a range of high masking contrasts for which the signal thresholds at all masking frequencies appear to lie along straight lines in these double logarithmic coordinates. Moreover, because the slopes of these lines are similar, they appear to be roughly parallel. Best-fitting straight lines (method of least squares) have been computed over the contrast range from 3.2% to 51.2%. They are drawn as the straight line portions of the solid curves through the data in Fig. 2. The slopes of these straight lines are given in Table I. For wide field masking, the slopes range from 0.525 to 0.711 with a mean value of 0.620. There appear to be no systematic variations of slope with frequency. To a good approximation, the slopes at the seven masking frequencies may be taken as equal, and the high contrast portions of the masking functions in Fig. 2 as parallel. Parallel plots in double logarithmic coordinates mean that the corresponding functions are scaled versions of one another. The straight lines through the high contrast portions of the masking functions in Fig. 2 are roughly parallel, but they are not coincident. For masking frequencies more and more remote from the signal frequency of 2.0 cpd, corresponding straight lines are increasingly shifted to the right, representing a decrease in the effectiveness of masking. Denoting signal contrast threshold by $C_s$, masking contrast by $C_m$, and taking the power function exponent to be 0.62, the high contrast masking functions take the form

$$C_t = [k(f)C]^0.62,$$

(1)

where $k(f)$ is a frequency-dependent scaling factor. The values of $k(f)$ were computed using a least-square criterion to find the best-fitting straight lines with slopes 0.62 through the high contrast data in Fig. 2. Values of $k(f)$ are presented in Table I.

Values of $k(f)$, normalized by the peak value at 2.0 cpd, are plotted as the filled circles in Fig. 8(a) (see below). Best-fitting straight lines have been drawn through the points to the right and left of the peak for purposes of interpolation and extrapolation later. $k(f)$ may be termed a sensitivity function. It represents the sensitivity of signal detection at 2.0 cpd to maskers of different frequencies. The values plotted in Fig. 8(a) are related reciprocally to masker contrasts required to produce a criterion signal threshold contrast. In this respect, the sensitivity function of Fig. 8(a) is analogous to an action spectrum. It corresponds to the "equivalent contrast

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**Table I. Exponents of the power functions relating signal thresholds to masking contrast.**

<table>
<thead>
<tr>
<th>Masking frequency</th>
<th>Wide field Exponent</th>
<th>Narrow field Exponent</th>
<th>$k(f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.711</td>
<td>0.788</td>
<td>0.0025</td>
</tr>
<tr>
<td>1.4</td>
<td>0.610</td>
<td>0.567</td>
<td>0.0059</td>
</tr>
<tr>
<td>1.7</td>
<td>0.525</td>
<td>0.526</td>
<td>0.0072</td>
</tr>
<tr>
<td>2.0</td>
<td>0.558</td>
<td>0.672</td>
<td>0.0152</td>
</tr>
<tr>
<td>2.4</td>
<td>0.651</td>
<td>0.561</td>
<td>0.0083</td>
</tr>
<tr>
<td>2.8</td>
<td>0.673</td>
<td>0.577</td>
<td>0.0089</td>
</tr>
<tr>
<td>4.0</td>
<td>0.615</td>
<td>0.769</td>
<td>0.0050</td>
</tr>
<tr>
<td>Mean</td>
<td>0.620</td>
<td>0.627</td>
<td></td>
</tr>
</tbody>
</table>

a Based on slope of best-fitting straight line in log-log coordinates in the range 3.2%-51.2% masking contrast.

b Based on slope of best-fitting straight line in log-log coordinates in the range 6.4%-51.2% masking contrast.
transformation” introduced by Blakemore and Nachmias. In the masking model to be presented below, this sensitivity function represents the frequency selectivity of a linear, spatial frequency filter (channel). Notice that the sensitivity function is broader to the right of its peak than to the left. The half-maximum frequencies lie near 1/4 octave below the peak and slightly more than 1/2 octave above the peak. The overall bandwidth between half-maximum frequencies is therefore about 0.75 octaves. Blakemore et al. found a contrast-invariant channel bandwidth of 0.75 octaves from grating adaptation studies using the equivalent contrast transformation.

Pantle used binocular viewing and forced-choice methods to measure thresholds for sine-wave gratings presented for 1.7 s upon a continuously present, slowly drifting background grating. Frequencies of the background gratings varied from those of the signal gratings by factors of 1, 2, 3, and 5. Over several conditions of masker and signal frequencies, and contrast ranges, there was evidence for power-law relations between signal threshold and background contrast. The log-log slopes obtained by Pantle are not inconsistent with the conclusion reached here, that masking functions in which maskers differ from signals by as much as an octave in frequency are scaled versions of the same power law, for a range of high masking contrasts.

When the masker and signal both have spatial frequency 2.0 cpd, the observer’s task is one of contrast discrimination. Table I indicates that the best-fitting straight line through the contrast discrimination data of Fig. 2 has a slope of 0.558, well below the value of 1.0 predicted by Weber’s law. Legge, using the same forced-choice procedure, luminance and monocular viewing, has measured contrast discrimination functions of this sort at several spatial frequencies. At 1.0, 4.0, and 16.0 cpd, the corresponding slopes were found to be 0.62, 0.67, and 0.70, respectively. (At the very low spatial frequency of 0.25 cpd, the slope was only 0.28.) Several other investigators have measured the contrast discrimination function using a variety of stimulus conditions and procedures. Some have found Weber’s law behavior. Others find that contrast discrimination thresholds increase more slowly than Weber’s law predicts. Still others find Weber’s law behavior under some conditions and departures from it under others. For a discussion of possible reasons for these discrepant findings, see Legge.

Now consider the nonmonotonicity of the masking functions in Fig. 2. For masking contrasts between about 0.05% and 0.8%, signal thresholds are lower than thresholds measured in the absence of masking. For instance, for a 2.0 cpd masker having contrast 0.4%, the signal threshold is about two-and-one-half times lower than the detection threshold. In such cases, the masker may be said to facilitate signal detection. This phenomenon has been termed the pedestal effect. The pedestal effect also occurs for the discrimination of luminance increments. It has been observed elsewhere for gratings contrast discrimination, and has been discussed at some length by Foley and Legge. The pedestal effect of Fig. 2 is frequency selective. It is readily apparent when the masker and signal have the same frequency, but is greatly diminished when the masker and signal differ by ±0.5 octaves. Unlike high contrast portions of the masking functions of Fig. 2, the frequency-dependent shape of the pedestal effect means that the low contrast portions of the masking functions are not parallel. Hence, the low contrast portions of the masking functions are not simply scaled versions of one another.

The facilitation effect should be distinguished from “subthreshold summation.” In the subthreshold summation paradigm, the observer adjusts the contrast C1 of a superimposed pair of stimuli C1 + C2 until the combination reaches threshold. In the forced-choice discrimination paradigm of this paper, the masker C1 is presented in both intervals of a trial, and the signal C2 is presented in one interval only. The observer must discriminate between stimuli C1 and C1 + C2. The two procedures would not, in general, be expected to yield the same result. In any case, a simple “summation to threshold” model cannot account for the facilitation effect, because signal thresholds are reduced by masking contrasts well above the unmasked threshold, and because the sum of masker contrast and threshold contrast is not constant when the masker’s contrast is below the detection threshold.

The frequency selectivity of masking can be examined more carefully in Fig. 3. Data points, representing signal thresholds, have been replotted from Fig. 2 as a function of masking frequency. The masking contrasts parametrize these masking tuning functions. The ordinate is relative threshold elevation, the ratio of masked to unmasked signal threshold. The ten symbols represent masked thresholds obtained with ten masking contrasts. Values for 0.8% masking contrast have been omitted for the sake of clarity. An ordinate value of 1.0 means that there is no effect of masking. For very low contrast, there is no effect of masking. For masking contrasts in the range from about 0.1% to 0.8%, there is a facilitation or pedestal effect with very narrow frequency selectivity. As the masking contrasts increase beyond 0.8%, the tuning functions turn inside out and become the more familiar threshold elevation functions. These have medium band-
The high contrast tuning functions are asymmetric, being slightly broader above their peaks than below. This asymmetry in masking tuning functions has been noted elsewhere.\textsuperscript{7,8,38,39}

Some studies have found that masking contrasts that increase signal threshold when they are close in frequency to the signal act to reduce signal threshold when masking frequency is about \( \frac{1}{3} \) the signal frequency.\textsuperscript{3,29,38} This phenomenon has been called "remote facilitation." In our experiments, no masker frequency is one-third the signal frequency. However, facilitation is apparent at and near the signal frequency when the masker has a contrast of 0.8% or less. These observations suggest that facilitation is proximal at low contrasts, but becomes remote at high contrasts.

Can a single empirical measure be used to characterize the bandwidth of spatial frequency masking? To the extent that the high contrast portions of the masking functions in Fig. 2 are scaled versions of one another, estimates of the half-maximum bandwidth frequencies in Fig. 3 will be invariant over a range of high masking contrasts. The half-maximum frequencies so obtained lie roughly one octave (or slightly less) above the signal frequency, and about \( \frac{3}{4} \) of an octave below it.

However, such an empirical bandwidth estimate is certainly inadequate for description of the frequency selectivity of the pedestal effect. Its half-minimum frequencies lie roughly \( \pm \frac{1}{4} \) octave from the signal frequency. This very narrow tuning of the pedestal effect in contrast masking has been noted earlier.\textsuperscript{36}

Certainly no single empirical measure can characterize the frequency selectivity of masking over the full range of contrasts used in this study. Nevertheless, the masking model, to be presented below, postulates the existence of a linear filter whose (channel) bandwidth is invariant with contrast.

Narrow field masking

In the narrow field masking experiment, sinusoidal luminance modulation was confined to a horizontal extent of 0.75°. The remainder of the screen was maintained at a mean luminance level of 200 cd/m\(^2\).

Figure 4 presents 2.0 cpd signal thresholds as a function of masker contrast, for seven masking frequencies. The conventions for plotting the data are like those for Fig. 2. The signals and maskers were always truncated sine-wave gratings in cosine phase with a central fixation mark.

Once again, there is a range of high masking contrasts for which the sets of data are well approximated by straight lines in the double logarithmic coordinates. The lines are approximately parallel and have slopes well below 1.0. (See Table 1.) The mean slope is 0.627, very close to the value of 0.620 for wide field masking.

For a range of low masking contrasts, the pedestal effect is evident at all masking frequencies.

In Fig. 5, data points from Fig. 4 are replotted as signal thresholds versus masking frequency. The conventions for plotting the data are like those in Fig. 3. The nine symbols represent masked thresholds obtained with nine masking contrasts. Values for 1.6% masking contrast have been omitted for the sake of clarity. Smooth curves have been drawn through the data at each masking frequency. For masking contrasts below about 2 to 3%, there is a general facilitation effect that appears to extend across the range of masking frequencies used. This "broad tuning" of the pedestal effect is very different from the extremely narrow tuning of the pedestal effect in wide field masking (see Fig. 3). At higher masking contrasts, the tuning functions turn inside out so that the presence of masking elevates signal thresholds. At high masking contrasts, these tuning functions become more broadly tuned than those in Fig. 3 for wide field masking. There appears to be a tendency for the peak threshold elevation to occur for masking frequencies above the signal frequency. Analogous effects have been observed under rather
different spatial frequency masking conditions. Legge observed that masking tuning functions for signals at 0.375 and 0.75 cpd were broader than those at higher signal frequencies, and tended to have their peaks shifted to masking frequencies above the signal frequency. The tuning functions were obtained with gratings subtending 10°. Legge observed similar effects for monocular masking tuning functions at 0.125 and 0.25 cpd, using a 13° field. Legge hypothesized that characteristics of masking tuning functions at low signal frequencies might reflect properties of a transient mechanism, having low-pass spatial frequency sensitivity, coexisting with a set of band-pass sustained mechanisms. The results of the present study, however, suggest that the broadening of masking tuning functions may not be confined to low signal frequencies below 1.0 cpd. Instead, it may be that broadening is related to a reduced number of cycles in the stimulus gratings. The number of cycles and the spatial frequency of signals covary for stimuli of fixed angular subtense. Further research will be required to unravel the separate dependences of the spread of masking upon signal frequency and field subtense.

Implicit in the data of Figs. 2 and 4 is an important difference between wide field and narrow field masking. In Fig. 6, contrast discrimination thresholds (both signal and masker at 2.0 cpd) have been replotted as a function of masker contrast for the wide field condition (●) and for the narrow field condition (+). The two theoretical curves are discussed below. For masking contrasts above about 6.4%, there is very little difference between wide field and narrow field thresholds. For low masking contrasts, below about 0.8%, wide field thresholds fall well below narrow field thresholds. Similarly, contrast detection thresholds (see Individual data) are lower in the wide field condition than in the narrow field condition. A notable peculiarity is that for a narrow range of intermediate contrasts, the narrow field thresholds actually drop below the wide field thresholds.

The decrease in contrast detection threshold with an increase in the number of grating cycles has been well documented. The new observation here is that a similar improvement in sensitivity occurs for contrast discrimination at very low contrasts as field size is increased, but that contrast discrimination is relatively insensitive to field size at high contrasts.

CONTRAST MASKING MODEL

There exists no quantitative model of spatial frequency masking. The model proposed here is an extension of the nonlinear transducer model of contrast detection and discrimination of Foley and Legge. The masking model is comprehensive insofar as it treats processes of detection, discrimination, and masking within a single theoretical framework. The development owes much to the earlier work of Nachmias and Sansbury, and Stromeyer and Klein. The model bears similarities to detection and discrimination models in audition (see, e.g., McGill and Goldberg and Hall and Sundh). The model is presented in the spirit of a first effort to account for a broad and complex body of masking data. It accounts for most of the diverse results of this paper.

The masking model postulates that detection of a 2.0 cpd sine-wave grating signal is accomplished by an ensemble of spatially localized detectors, differing only in their positions. Each detector's response is determined by three processes—a linear filter characterized by a spatial frequency sensitivity function $k'(f)$, a nonlinear transducer $F$, and additive, zero-mean, constant-variance Gaussian noise. The model will be fully characterized by specifying: (i) the function $k'(f)$; (ii) the functional $F$; and (iii) the “decision rules” by which detector outputs are combined to determine a response in the forced-choice procedure. Figure 7 is a schematic representation of the model. In the following subsections, $k'(f)$ is identified with the sensitivity function plotted in Fig. 8(a), and the nonlinear transducer $F$ is shown to have the form given in Fig. 8(b). The model's decision rules incorporate a form of spatial pooling at low contrasts, but not at high contrasts.

**Linear filter**

Recent theoretical treatments have attributed phenomena of spatial frequency selectivity in vision to hypothetical spatial weighting functions or "receptive fields" associated with spatially localized detectors. Legge has modeled grating detection by an ensemble of such detectors, distributed across the visual field. The ensemble, all members of which have the same spatial weighting function, was termed a spatial frequency channel. The spatial frequency selectivity associated with the channel is characterized by the Fourier transform of the spatial weighting function. For a detector centered at position $x_0$ in visual coordinates, having spatial weighting function $S(x)$, the output $r_0$, associated with stimulus waveform $L(x)$, is

$$r_0[L(x)] = \int_{-\infty}^{\infty} L(x)S(x - x_0)dx.$$

This convolution is linear, and constitutes the linear filter of the masking model. For a sine-wave grating of frequency $f$ in cosine phase with the origin of coordinates, the stimulus

![Graph](image_url)
waveform $L(x)$ is

$$L(x) = L_0(1 + C \cos 2\pi fx),$$  

(3)

where $L_0$ is mean luminance and $C$ is the grating contrast. If the detector is assumed to be insensitive to mean luminance, its dependence on the grating frequency $f$ is found from Eqs. (2) and (3) to be

$$r_0(f) = C \int_{-\infty}^{\infty} \cos 2\pi fx S(x - x_0) dx.$$  

(4)

Assuming that $S(x)$ is center symmetric and of finite extent, we obtain

$$r_0(f) = C \cos 2\pi fx_0 \left[ \int_{-\infty}^{\infty} \cos 2\pi fx S(x) dx \right]$$

$$= C \cos 2\pi fx_0 k'(f)$$

where the bracketed quantity has been rewritten as $k'(f)$ and is the Fourier transform of the symmetric function $S(x)$. Equation (5) indicates that the output of the linear filter, resulting from a stimulus grating of frequency $f$, is proportional to the product of three factors—grating contrast $C$, the value of the sensitivity function $k'(f)$ at $f$, and a phase-sensitive factor $\cos 2\pi fx_0$ that depends on the position $x_0$ of the detector.

The filter output for a detector located at the fixation point, $x_0 = 0$, is simply $Ck'(f)$. The contrast discrimination data in Fig. 6 suggest that the effects of spatial pooling are largely absent at high contrasts. Frequency selectivity of masking at high contrast may, therefore, predominantly reflect the filtering characteristics of detectors located at or very near the fixation point. It has already been shown that, for a range of high contrasts, the masking functions of Fig. 2 may be approximated by scaled versions of one another. This frequency-dependent scaling is characterized in a contrast-invariant manner by the sensitivity function of Fig. 8(a). In the context of the masking model, this spatial frequency sensitivity function can be identified as the function $k'(f)$ associated with the linear filter. The filter attenuates signal inputs to the detectors in accordance with Eq. (5).

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**FIG. 7.** Schematic representation of the masking model.

**FIG. 8.** (a) Spatial frequency sensitivity function used in the masking model. The scale factors $k(f)$ (Table I) obtained from high contrast masking are plotted in normalized form as a function of frequency. Straight lines have been fit through the points to the left and right of 2.0 cpd for purposes of interpolation and extrapolation. (b) Nonlinear transducer used in the masking model. $F$ is the nonlinear transducer of the masking model: $F(r) = (a_1 |r|^2)^4 \sqrt{(|r|^2 + a_2^2)}$ where $a_1 = 45$ and $a_2 = 0.0075$. $F$ is plotted for the case in which $r = C$—detector located at $x_0$ responding to a 2.0 cpd sine-wave grating.
More generally, the spectral representations of stimuli of finite width will be continuous functions of frequency. The output of the linear filter for a detector centered at \( x_0 \) generalizes from Eq. (5) to

\[
r_0 \propto \int_{-\infty}^{\infty} C(f)k'(f) \cos2\pi fx_0 df, \tag{6}
\]

where \( C(f) \) is the contrast spectrum of the stimulus. The spectral spread of the stimulus becomes important when modeling the narrow field results for which gratings were truncated to widths of 0.75°. The fitted straight lines to the points of Fig. 8(a) approximate \( k'(f) \). This approximation is used in the numerical evaluation of the integral in Eq. (6).

It is known that contrast sensitivity decreases with increasing retinal eccentricity. Effects of retinal inhomogeneity were probably slight in the current experiments. The points of Fig. 8(a) approximate truncated to widths of 0.75°. The fitted straight lines to the spectral spread of the stimulus becomes important when

\[
\frac{r}{a_2} \propto \int_{-\infty}^{\infty} C(f)k'(f) \cos2\pi fx_0 df, \tag{6}
\]

where \( C(f) \) is the contrast spectrum of the stimulus. The spectral spread of the stimulus becomes important when modeling the narrow field results for which gratings were truncated to widths of 0.75°. The fitted straight lines to the points of Fig. 8(a) approximate \( k'(f) \). This approximation is used in the numerical evaluation of the integral in Eq. (6).

It is known that contrast sensitivity decreases with increasing retinal eccentricity. Effects of retinal inhomogeneity were probably slight in the current experiments. The wide field gratings extended only 3° on either side of the fixation point. Nevertheless, in the masking model, the filter output of each detector was multiplied by the factor \( \exp(-0.116|x_0|) \). This function, taken from Wilson and Giese, describes the decline in sensitivity with retinal eccentricity for 2.0 cpd gratings. Inclusion of this factor in the masking model had only slight effects upon its performance.

\[ F = aC^n, \tag{7}\]

where \( a \) and \( n \) are constants, and \( C \) is the grating contrast. They found values of the exponent \( n \) that ranged from 2.11 to 3.04, depending on observer and spatial frequency. A positively accelerating nonlinear transducer of this type accounts for contrast detection and discrimination data over a range of low contrasts. With increasing contrast \( C \), fixed increments in transducer output are associated with ever decreasing increments in transducer inputs. This property of the nonlinear transducer can be used to model the facilitation that occurs with low contrast discrimination. However, the nonlinear transducer of Eq. (7) will not work at high contrasts, because it predicts that threshold will continue to decrease with increasing masking contrast. The choice of nonlinear transducer to use in the masking model is

\[
F = a_1|r|^2.4/\left(|r|^2 + a_2^2\right), \tag{8}\]

where \( r \) is the input to the nonlinear transducer, and \( a_1 \) and \( a_2 \) are constants. This equation is plotted in Fig. 8(b) in log-log coordinates, with values of the constants computed as discussed below. For small inputs \( |r| < a_2 \), the transducer function reduces to \( F = (a_1/a_2^2)|r|^{2.4} \). This form of the transducer is compatible with the transducer inferred by Foley and Legge, Eq. (7). For large inputs \( |r| > a_2 \), the transducer has the form \( F = |r|^{0.4} \). At high contrasts, the transducer is compressive, but nonsaturating. In the compressive region, greater and greater input increments are required to achieve equal output increments. In the case of contrast discrimination, the empirical consequence is that signal thresholds increase with increasing contrast.

Notice that the constant \( a_2 \) in Eq. (8) determines the ranges of inputs \( r \) that lie in the positively accelerating and compressive regions of the nonlinearity. If \( a_2 = 0 \), there would be no positively accelerated portion of the nonlinearity and, consequently, no dip in the masking functions of Figs. 2 and 4. \( a_1 \) is simply a constant of proportionality.

\[ F = aC^n \]

\[ F = a_1|r|^2.4/\left(|r|^2 + a_2^2\right) \]

\[
F = a_1|r|^{0.4} \]

\[ E(r) = F(r) + e. \tag{9}\]

\[ F \] is the nonlinear transducer; \( e \) is the additive, zero-mean, unit-variance Gaussian noise; the mean of the random variable \( E(r) = F(r) \), and its constant standard deviation is 1.0.

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\[ F \] is the nonlinear transducer; \( e \) is the additive, zero-mean, unit-variance Gaussian noise; the mean of the random variable \( E(r) = F(r) \), and its constant standard deviation is 1.0.
There have been several investigations of spatial pooling models associated with contrast detection. The most popular model has been termed probability summation. Probability summation has been well developed for contrast detection. However, it is not applicable, without considerable refinement, to contrast discrimination or masking.

As an alternative model of spatial pooling that is applicable to contrast detection, discrimination, and masking, the decision rule for one detector may be extended to many detectors, as follows:

**DECISION RULE:** When the decision in a forced-choice trial is based upon the outputs of many detectors, identify the detector whose outputs in the two intervals have the greatest difference. Choose the interval in which this detector’s output is greater.

This rule implies that the observer computes the difference $E(r_s + r_{m1}) - E(r_{m1})$ for each detector, and then bases his decision upon the detector for which this difference has the largest magnitude.

Many decision rules are possible that include versions of spatial pooling. Several decision rules, including linear and quadratic summation across detector outputs, were studied in attempts to model the masking data of this paper. None were found to be as adequate as the decision rule given here, although the differences were often small.

The contrast discrimination data of Fig. 6 suggest that the effects of spatial pooling are primarily confined to low contrasts. For purposes of modeling, the following decision rules were adopted: (i) At low masking contrasts, the forced-choice decision is based upon the output of many detectors, according to the decision rule in Eq. (11); (ii) at high contrasts, the forced-choice decision is based upon the output of a single detector, located at the fixation point. The switch in decision rules occurs at a contrast of about 1%.

In the case of spatial pooling, the following simplification was made for purposes of calculation. Only detectors centered on peaks and valleys of the 2.0 cpd signal gratings were included in the decision process.

The properties of the masking model were studied with a computer program simulation. For a given signal and masker, a forced-choice trial was simulated by the following steps: (i) A set of detectors was chosen by specifying their center positions $x_i$ in the visual field; (ii) the linear filter response of each detector was calculated for the masker and signal-plus-masker from Eq. (6), and weighted by the retinal inhomogeneity factor; (iii) for each detector, the linear filter outputs were subjected to the nonlinear transformation of Eq. (8); (iv) for each detector, the difference in signal-plus-masker and masker outputs was computed, and the forced-choice was determined by the decision rule given in either (10) or (11). For a given masker and signal, 250 trials were simulated, and the percent correct calculated. The staircase procedure used in the experiments of this paper estimated the signal contrast that yielded 79% correct. Accordingly, the computer simulation repeated its computations for signal contrasts until the corresponding values of percent correct ranged from 60% to 90%. From these simulated “frequency-of-seeing” curves, the 79% signal contrast was obtained. Although the Gaussian noise component of the simulation means that the model’s predictions have some variability, repetitions indicated that its threshold estimates are good to better than 5%.

The masking model has now been fully specified, except for evaluation of the constants $a_1$ and $a_2$ in Eq. (8). As noted earlier, at high contrasts, the nonlinear transducer reduces to $a_1 [r]^0.4$ and is independent of $a_2$. The value of $a_1$ was chosen by simulating contrast discrimination at 51.2% contrast. A value of $a_1 = 45$ was chosen because it yielded a threshold of about 4.5% in agreement with observation. Similarly, $a_2 = 0.0075$ was chosen so that the model’s predictions for contrast detection thresholds for wide fields would approximate closely the observed threshold of 0.3%.

The model may now be used to predict signal thresholds for contrast detection, discrimination, and masking. The model is characterized by a linear spatial frequency filter [Fig. 8(a)], a nonlinear transducer [Fig. 8(b)], separate decision rules for low and high contrast masking, and the constants $a_1$ and $a_2$.

**PREDICTIONS OF THE MASKING MODEL**

**Contrast discrimination**

Two theoretical curves accompany the contrast discrimination data in Fig. 6. The solid curve, labeled NO SPATIAL POOLING, was derived from the masking model by applying the single detector decision rule, Eq. (10), across the full range of contrasts. The broken line, labeled SPATIAL POOLING, is the masking model’s prediction based upon the decision rule, Eq. (11), that incorporates spatial pooling over the 6° field. A third theoretical curve, not shown, incorporates effects of spatial pooling in the narrow field case. It lies very close to the NO SPATIAL POOLING curve at low contrasts, and drops well below it at high contrasts. The two theoretical curves in Fig. 6 indicate that the NO SPATIAL POOLING predictions fit the narrow field data at all contrasts, and the wide field data at high contrasts. On the other hand, spatial pooling leads to predictions that provide a reasonably good fit to the wide field data at low contrasts. Apparently, low contrast discrimination at 2.0 cpd involves some form of spatial pooling, but high contrast discrimination does not.

When the spatial pooling decision rule is used, the masking model predicts contrast detection thresholds of 0.31% and 0.55% for wide and narrow field gratings, respectively, compared with measured values (means across three observers) of 0.29% and 0.50%. The predicted psychometric functions for detection have the shape found experimentally by Foley and Legge.

**Masking tuning functions at high contrast**

Figure 9 presents the masking model’s tuning functions (solid curves) for masking contrasts of 51.2% and 25.6%. Panel (a) presents wide field results and panel (b) narrow field results. Data points are replotted from Figs. 3 and 5.

The model’s predictions fit the wide field data quite well, reflecting the broad and asymmetric form of the tuning functions. The model’s predictions for all masking contrasts in the range 3.2% to 51.2% are similar in shape, and give
is a reflection of a form of phase sensitivity inherent in the tuning of the facilitation effect. In part, the narrow tuning of 0.2% and 0.4%. Data points have been replotted from Figs. 3.

Panel (b) presents narrow field results for masking contrasts with the channel sensitivity function than does the spectrum cpd masking stimulus actually exhibits slightly greater overlap of this sort, arising because the phase selective narrowing due to spatial pooling has been removed and because stimulus spectra have been broadened. Note that both the data and the model's predictions are so broadly tuned that frequency sensitivity is hardly apparent. Comparison of the upper and lower panels of Fig. 10 shows the very large influence that field size has upon the frequency selectivity of the facilitation effect. For wide fields, upper panel, the frequency tuning is extremely narrow. For narrow fields, lower panel, the tuning is extremely broad. The model makes predictions about the presence or absence of the 2.0 cpd signal always result from decision rules that are based on the outputs of detectors that are optimally sensitive at 2.0 cpd. This is an assumption of parsimony. Instead, it may be that observers refer their decisions to different sets of detectors for different masking conditions. Limitations of the model in describing the data may result from a violation of this assumption.

For narrow field masking at low contrasts, the model predicts very broad tuning of the facilitation effect. The broadening occurs because the phase selective narrowing due to spatial pooling has been removed and because stimulus spectra have been broadened. Note that both the data and the model's predictions are so broadly tuned that frequency sensitivity is hardly apparent. Comparison of the upper and lower panels of Fig. 10 shows the very large influence that field size has upon the frequency selectivity of the facilitation effect. For wide fields, upper panel, the frequency tuning is extremely narrow. For narrow fields, lower panel, the tuning is extremely broad. The model also possesses these properties.

A rather strong assumption of the masking model is that decisions concerning the presence or absence of the 2.0 cpd signal always result from decision rules that are based on the outputs of detectors that are optimally sensitive at 2.0 cpd. This is an assumption of parsimony. Instead, it may be that observers refer their decisions to different sets of detectors for different masking conditions. Limitations of the model in describing the data may result from a violation of this assumption.

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FIG. 10. Masking tuning functions at low contrast: data and model. Solid curves are predictions of the masking model for 2.0 cpd signal threshold reduction plotted as a function of the spatial frequency of masking gratings.

(a) Wide field: 6° by 6°, data replotted from Fig. 3. (b) Narrow field: 0.75° by 6°, data replotted from Fig. 5.
acterized by the same linear filter, followed by a compressive nonlinearity, and the relative absence of spatial pooling. Empirically, high contrast masking elevates signal threshold, exhibits medium to broad tuning, and is relatively insensitive to field size. The striking differences between masking at low and high contrasts may reflect the properties of separate, underlying mechanisms. Accelerating and compressive portions of the nonlinear transducer may be descriptive of separate mechanisms. Foley and Legge\(^4\) have suggested that the accelerating portion of the transducer may be the result of a threshold process followed by the addition of Gaussian noise. By comparison, the compressive portion of the nonlinear transducer may reflect the operation of signal compression in the transmission of high amplitude signals along visual pathways.

Why is spatial pooling less prominent at high contrasts? One possible reason involves the properties of the Gaussian noise that is added to the transducer output of each detector. If, as assumed in the masking model, the additive noise is uncorrelated across detectors, the decision rule in (11) results in spatial pooling. If, however, the Gaussian noise were perfectly correlated across detectors, no spatial pooling would be present. This account has some phenomenological plausibility. Observers sometimes report that for very low contrasts, grating detection involves "seeing something" at different places in the stimulus field on different trials, as if local response fluctuations were occurring. At high contrasts, however, observers do not seem to report seeing patches of a grating fluctuating in contrast. But why should the detector noise become correlated at high contrast? Although we have assumed constant-variance additive noise in the current model, further study may indicate that the noise is signal dependent, in which case some correlation might not be surprising.

An important feature of the masking model is the ordering of its elements. The linear filter precedes the nonlinear transducer. Burton\(^6\) attributed visual "beats" to a frequency filter that followed a nonlinearity. The possibility exists that spatial frequency filtering occurs both before and after the nonlinearity. However, there are two compelling reasons for placing a stage of linear filtering before the nonlinearity. First, it would be difficult to account for the fact that the high contrast portions of the masking functions in Figs. 2 and 4 are approximately parallel without assuming that the scaling operation (linear filtering) occurs prior to the nonlinearity. Second, evidence that pattern detection thresholds can be predicted from a knowledge of pattern Fourier spectra\(^67-69\) requires that a stage of linear filtering precedes the nonlinearity.

**ACKNOWLEDGMENTS**

The masking experiments were performed in the laboratory of Dr. F. W. Campbell, Cambridge University, while G. E. L. held a Postdoctoral Research Fellowship from the Medical Research Council of Canada, and J. M. F. held a grant from the James McKeen Cattell Fund. We wish to thank Dr. Campbell for his hospitality, encouragement, and for his many instructive comments and suggestions. We thank A. H. Harcourt, C. Hood, W. W. Legge, and D. Pelli for their help, and N. Graham, S. Klein, and C. F. Stromeyer III for their helpful comments on a draft of this paper. Some of the results of this paper were reported at the annual meeting of the Association for Research in Vision and Ophthalmology, Sarasota, 1979. The research was supported in part by Public Health Service Grant EO02857.


\(^{26}\) C. Blakemore, J. P. J. Muncey, and R. Ridley, "Stimulus specificity


55The only restriction upon Eq. (6) is that the spatial sensitivity function $S(x)$ be center-symmetric and finite. All "phase effects" due to the position of the detector are accounted for by the factor $\cos 2\pi x_0$.


58Fechner (see Boring, Chap. 14) derived a sensory magnitude function from the discrimination data. If the effects of spatial masking are ignored, Fechner's technique may be used to derive the transducer function $F$ from the discrimination functions in Figs. 2 or 4. The discrimination function is proportional to the reciprocal of the derivative of $F$.


60If the constant variance assumption is relaxed, the transducer function cannot be inferred directly from forced-choice data. Information concerning the dependence of noise variance on contrast could be obtained from other psychophysical procedures, such as the "bootstrap" procedure of Nachmias and Kocher. With this information in hand, a corrected form of the transducer function could be computed. Changes in variance with contrast would not be expected to affect the shape of the channel sensitivity function, as shown in Fig. 8(a). There exists little evidence concerning the dependence of variance on stimulus contrast. Paired comparison data of Foley and Legge\(^1\) indicate small changes in variance over a range of low contrasts. Further discussion of the effects of relaxing the constant variance assumption is given by Nachmias and Sansbury.


65H. R. Wilson, "Quantitative characterization of two types of line-spread function near the fovea," Vision Res. 18, 971–981 (1978).


68The spatial frequency sensitivity function of Fig. 8(a) will be identified with the linear filter. Extrapolation of the fitted straight line toward 0 cphd (uniform field) suggests little sensitivity to mean luminance.

69In Eq. (4), let $x' = x - x_0$. Then:

$$r_0(f) = C \int_{-\infty}^{\infty} \cos 2\pi f(x' + x_0)S(x')dx'$$

$$= C \int_{-\infty}^{\infty} \cos 2\pi f x' \cos 2\pi f x_0 - \sin 2\pi f x_0 \sin 2\pi f x_0 S(x')dx'$$

$$= C \cos 2\pi f x_0 \int_{-\infty}^{\infty} \cos 2\pi f x S(x')dx'$$

$$- C \sin 2\pi f x_0 \int_{-\infty}^{\infty} \sin 2\pi f x S(x')dx'.$$

Assuming $S$ is even-symmetric and finite, the second term is 0, and we are left with Eq. (5).

