Binary and Decimal Number System

Binary Number System: a base-2 number system represented by two symbols, usually zero (0) and on (1). The number system is commonly used in digital electronic circuitry and internally by all modern computers. A sequence of binary digits are used to represent a binary number.

Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000
Decimal	0	1	2	3	4	5	6	7	8

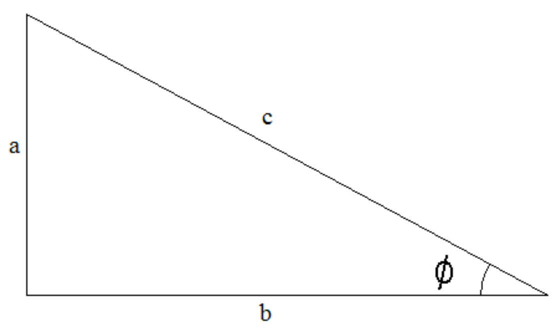
Binary	1001	1010	1011	1100	1101	1110	1111	10000
Decimal	9	10	11	12	13	14	15	16

Decimal Number System: a base-10 number system. The ten digits, zero (0) to nine (9), can be used in combination to represent any number.

Converting Binary and Decimal Numbers: binary numbers are expressed as a series of powers of two and the terms in the series are added (Bushberg page 62).

$$11011101 = +(1\times 2^7)+(1\times 2^6)+(0\times 2^5)+(1\times 2^4)+(1\times 2^3)+(1\times 2^2)+(0\times 2^1)+(1\times 2^0)=221$$

Basic Geometric and Trigonometric Principles



a = opposite side to the defined angle
b = adjacent side to the defined angle
c = hypotenuse

Pythagorean Theorem: for right triangles (triangles with a 90 degree angle), the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$a^2 + b^2 = c^2$$

Common Trigonometry Functions:

- <u>*Sine*</u> – generally written as sin(φ)

$$\sin(\varphi) = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

- <u>*Cosine*</u> – generally written as cos(φ)

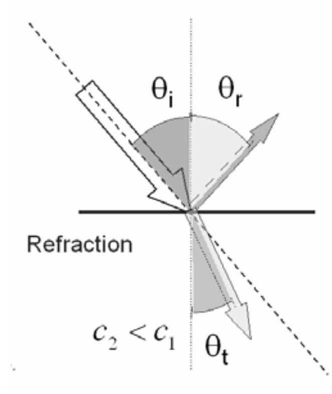
$$\cos(\varphi) = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

- <u>*Tangent*</u> – generally written as tan(φ)

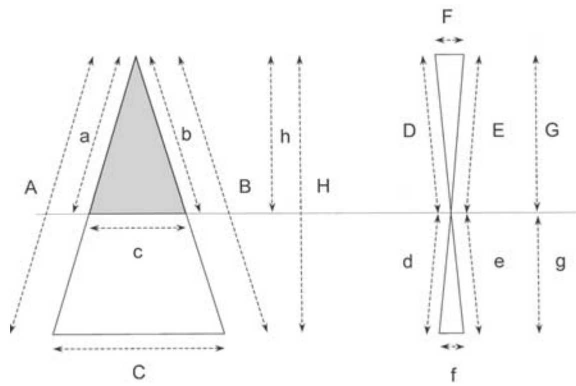
$$\tan(\varphi) = \frac{opposite}{adjacent} = \frac{a}{b}$$

- <u>*Snell's Law (law of refraction)*</u> – this formula describes the relationship between the angles of incident and refraction as waves pass through an interface between two different materials. The waves will propagate through material at a given speed, c, that is characterized by the bulk modulus and density of the medium. This formula is commonly used in ultrasound.

$$\frac{\sin(\theta_t)}{\sin(\theta_i)} = \frac{c_2}{c_1}$$



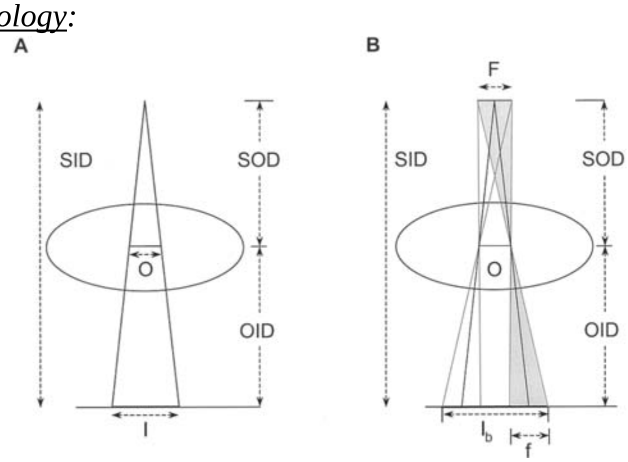
Similar Triangles: two triangles that share the same angles but are different sizes are similar triangles. The area of one triangle is proportional to the area of the second triangle.



Notice that the angles of both triangle 'abc' and 'ABC' are equal. Similarly, the triangles 'def' and 'DEF' have equal angles. (Bushberg page 147, figure 6-2)

- <u>*Common uses of Similar Triangles in Radiology*</u>*:*
  - <u>*Magnification factors*</u> – the mag factor (M) is related to the image size (I) and object size (O). Additionally, the mag factor can also be calculated from the source-to-image distance (SID) and the source-to-object distance (SOD). See figure A at right from Bushberg page 147, figure 6-3.

    $$M = \frac{I}{O} = \frac{SID}{SOD}$$

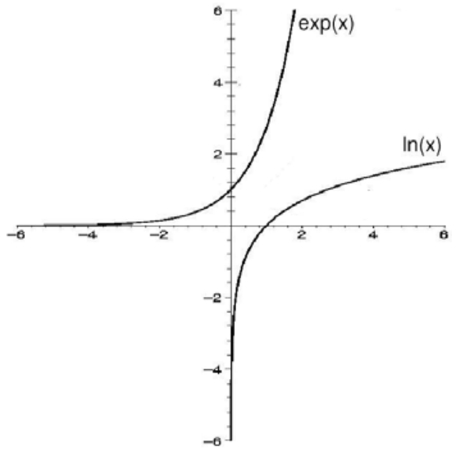


  - <u>*Geometric Blur*</u> – the penumbra (f) is related to the focal spot size (F), the object-to-image distance (OID), and the SOD. See figure B at right from Bushberg page 147, figure 6-3.

    $$\frac{f}{F} = \frac{OID}{SOD} = \frac{SID - SOD}{SOD} = M - 1$$

Logarithmic (also know as Log) Functions:

Definition: the log of a number 'x' to the base 'b' is written as $\log_b(x)$. The log of the number 'x' to a given base 'b' is the power or exponent to which the base must be raised in order to produce the number 'x'. The advantage to logarithmic functions is that small values of the function are expanded and large values of the function are compressed, making the range of the function more manageable.



Functions are normally written in the form:

$$y = \log_b(x)$$

However, another way to write a log function is

$$b^y = x$$

Basic examples:

- $y = \log_{10}(1000) = 3$ because $10^3 = 1000$.
- $y = \log_2(32) = 5$ because $2^5 = 32$.

Two commonly used log functions:

- *Natural logarithm (base = e = 2.71828)*: the function is commonly written as $y = \log_e(x) = \ln(x)$; this is the inverse of exponential functions.
    - Example: recall the attenuation equation for x-rays.

      $$I = I_o e^{-\mu x}$$

      30% of a x-ray beam was transmitted through a material with a linear attenuation coefficient of 0.05 cm^{-1}. Determine the thickness of the material (i.e. solve for the variable 'x') by using the inverse of the exponential function (i.e. the log function).

      Because 30% of the beam is transmitted, $\frac{I}{I_o} = 0.3$.

      $e^{-0.05\text{cm}^{-1}(x)} = 0.3$ **Define the equation.

      $\ln\left(e^{-0.05\text{cm}^{-1}(x)}\right) = \ln(0.3)$ **Take the natural log of both sides.

      $-0.05\text{cm}^{-1}(x) = \ln(0.3)$ **The $\ln(e^a) = a$.

      $x = \frac{\ln(0.3)}{-0.05\text{cm}^{-1}}$ **Solve for x.

      $x = 24.1\,\text{cm}$

- *Common logarithm (base = 10)*: $y = \log_{10}(x)$
    - Example: in ultrasound, the attenuation of sound beams is determined by the relative intensity on a decibel (dB) scale.

$$\text{relative intensity (dB)} = 10 \cdot \log_{10}\left(\frac{I_2}{I_1}\right)$$

Calculate the remaining intensity of a 100-mW ultrasound pulse that loses 50 dB while traveling through tissue (Bushberg pg. 476).

$$10 \cdot \log_{10} \frac{I_2}{100\,mW} = -50dB \qquad \text{**Define the function.}$$

$$\log_{10} \frac{I_2}{100\,mW} = -5 \qquad \text{**Divide by 10.}$$

$$\frac{I_2}{100\,mW} = 10^{-5} \qquad \text{**Recall: } b^y = x \text{ for } \log_b(x) = y$$

$$I_2 = 0.001\,mW = 1\,\mu W \qquad \text{**Solve for } I_2.$$

Properties of Log Functions:

- $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
- $c \cdot \log_b(x) = \log_b(x^c)$

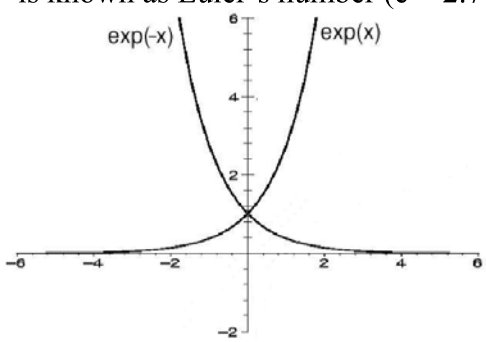
<u>Exponential Functions:</u>

Definition: functions in which negative values of the variable 'x' increase at a slower rate and as the value of 'x' increases into positive values, the function growth increases more quickly. This is known as "exponential growth." An exponential function approaches a value of zero, but will never equal zero.

Functions are normally written in the form:

$$y = \exp(x) = e^x$$

The mathematical constant 'e' is known as Euler's number (e = 2.71828).



Note that the function $y = \exp(-x) = e^{-x}$ decreases quickly for negative values of 'x' and decrease more slowly for positive values of 'x'. These types of functions are seen regularly in radiology (see examples below).

Basic examples:

- $y = \exp(0.5) = e^{0.5} = 2.71828^{0.5} = 1.65$
- $y = \exp(3) = e^{3} = 2.71828^{3} = 20.1$

The exponential function models many scenarios in radiology, including radioactive decay and x-ray attenuation.

- ***<u>Radioactive decay</u>*** – the fundamental decay equation used in nuclear medicine relates the number of radioactive atoms, N, remaining in a sample after a given time period. The formula can also be written in terms of activity remaining since A = λN.

  $N = N_o e^{-\lambda t}$ **<u>OR</u>** $A = A_o e^{-\lambda t}$

  N = remaining # of radioactive atoms after a time interval, t.
  N_o = initial number of atoms.
  A = remaining activity after a time interval, t.
  A_o = initial activity.
  λ = decay constant and $\tau_{1/2}$ = half-life of the radioactive sample.

  $$\lambda = \frac{\ln(2)}{\tau_{1/2}}$$

  <u>Example</u>*:* A technologist assays a syringe of Tc-99m sulfur colloid and measures an activity of 4 mCi. The half-life of Tc-99m is 6.02 hours. If the technologist waits 3.5 hours and assays the syringe again, how much activity will be present in the syringe?

  $$A = A_o e^{-\lambda t} = (4\text{ mCi})\cdot e^{-\left(\ln(2)/6.02hr\right)\cdot 3.5hr}$$
  $$A = (4\text{ mCi})\cdot e^{-0.403} = (4\text{ mCi})\cdot(0.668)$$
  $$A = 2.67\text{ mCi}$$

- ***<u>X-ray attenuation</u>*** – the removal of monoenergetic, incident photons as they pass through matter is a result of absorption and scattering of primary photons. The number of transmitted photons can be calculated using the following exponential function, where.

  $N = N_o e^{-\mu x}$

  N = # of transmitted photons.
  N_o = # of incident photons.
  μ = linear attenuation coefficient (function of the photon energy and absorption material).
  x = thickness of the absorption material.

  <u>Example</u>*:* If a 2-mm thickness of material transmits 25% of a monoenergetic beam of photons, calculate the HVL of the beam.

  The number of photons transmitted is 25%, therefore $\frac{N}{N_o} = 0.25$.

  $\frac{N}{N_o} = 0.25 = e^{-\mu(0.2\text{cm})}$ **Define the problem.

  $\ln(0.25) = -\mu(0.2\text{ cm})$ **Take the natural log of both sides of the equation.

  $\mu = 6.93\text{ cm}^{-1}$ **Solve for μ.

  Plug the value of μ into the HVL equation.

  $$\text{HVL} = \frac{\ln(2)}{\mu} = \frac{\ln(2)}{0.693\text{ cm}^{-1}} = 0.1\text{ cm}$$