

## STAT 512: MATH/PROB DIAGNOSTIC

This quiz is intended solely to help assess your mathematical preparation for 512. It will be self-graded and the grade will *not* be recorded. If questions 1-5 deal with material unfamiliar to you, you should consider taking preparatory courses in mathematics and/or probability before taking 512, such as MATH/STAT 394-5. If questions 6-7 are very easy for you, you might consider taking a more advanced probability course, such as MATH/STAT 491.

1. Evaluate  $\int_0^1 e^{-2x} dx$ .

2. Evaluate  $\frac{d}{dx} \frac{1}{(1-x)}$ .

3. Evaluate  $\sum_{k=0}^{\infty} x^k$  for  $|x| < 1$ .

4. Evaluate  $\sum_{k=0}^{\infty} kx^{k-1}$  for  $|x| < 1$ .

5. Let  $X$  be a random variable uniformly distributed on the interval  $(a, b)$ . Find  $E(X)$  and  $P[X \leq E(X)]$ .

6. Define  $f(x, y) = \begin{cases} c, & \text{if } 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$

Show that  $c = 2$  makes this a probability density function.

7. Let  $(X, Y)$  be a pair of random variables with joint probability density function  $f(x, y)$  as given in 6.

(a) Find the marginal density  $f(y)$  of  $Y$  and find  $E(Y)$ .

(b) For  $0 < y < 1$ , find the conditional distribution of  $X$  given  $Y = y$ .

(c) Find the conditional expectation  $E(X|Y = y)$  and find  $E(X)$ .

(d) Are  $X$  and  $Y$  independent?

(e) Are  $X/Y$  and  $Y$  independent?

(f) Evaluate  $E(XY)$ .