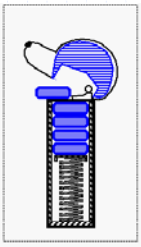
Elementary Data Structures

Stacks, Queues, Vectors, Lists & Sequences

Trees



The Stack ADT (§2.1.1)

- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine or C++ runtime environment
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Array-based Stack (§2.1.1)

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable t keeps track of the index of the top element (size is t+1)

Algorithm *pop*():
  if *isEmpty*() **then**
    throw *EmptyStackException*
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$

Algorithm *push*(*o*)
  if $t = S.length - 1$ **then**
    throw *FullStackException*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$

The Queue ADT (§2.1.2)

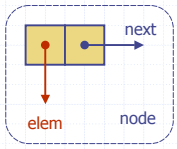
- The Queue ADT stores arbitrary objects
- Insertions and deletions follow the first-in first-out scheme
- Insertions are at the rear of the queue and removals are at the front of the queue
- Main queue operations:
  - enqueue(object): inserts an element at the end of the queue
  - object dequeue(): removes and returns the element at the front of the queue
- Auxiliary queue operations:
  - object front(): returns the element at the front without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored
- Exceptions
  - Attempting the execution of dequeue or front on an empty queue throws an EmptyQueueException

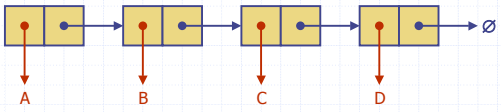
Applications of Queues

- Direct applications
  - Waiting lines
  - Access to shared resources (e.g., printer)
  - Multiprogramming
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Singly Linked List

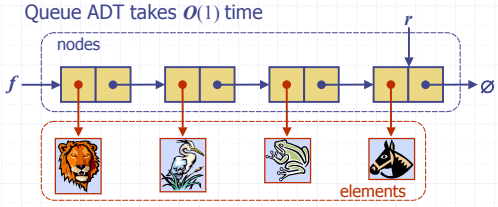
- A singly linked list is a concrete data structure consisting of a sequence of nodes
- Each node stores
  - element
  - link to the next node





Queue with a Singly Linked List

- We can implement a queue with a singly linked list
  - The front element is stored at the first node
  - The rear element is stored at the last node
- The space used is $O(n)$ and each operation of the Queue ADT takes $O(1)$ time



The Vector ADT

- The Vector ADT extends the notion of array by storing a sequence of arbitrary objects
- An element can be accessed, inserted or removed by specifying its rank (number of elements preceding it)
- An exception is thrown if an incorrect rank is specified (e.g., a negative rank)
- Main vector operations:
  - object elemAtRank(integer r): returns the element at rank r without removing it
  - object replaceAtRank(integer r, object o): replace the element at rank with o and return the old element
  - insertAtRank(integer r, object o): insert a new element o to have rank r
  - object removeAtRank(integer r): removes and returns the element at rank r
- Additional operations size() and isEmpty()

Applications of Vectors

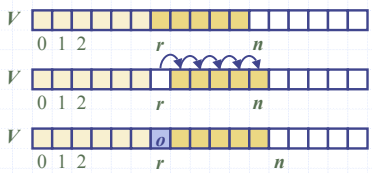
- Direct applications
  - Sorted collection of objects (elementary database)
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Array-based Vector

- Use an array V of size N
- A variable n keeps track of the size of the vector (number of elements stored)
- Operation $elemAtRank(r)$ is implemented in $O(1)$ time by returning $V[r]$

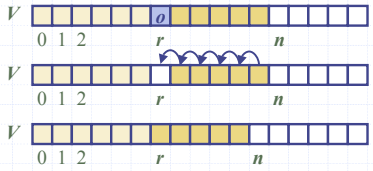
Insertion

- In operation $insertAtRank(r, o)$, we need to make room for the new element by shifting forward the $n - r$ elements $V[r], \ldots, V[n-1]$
- In the worst case ($r = 0$), this takes $O(n)$ time



Deletion

- In operation *removeAtRank*(r), we need to fill the hole left by the removed element by shifting backward the $n - r - 1$ elements $V[r+1], \ldots, V[n-1]$
- In the worst case ($r = 0$), this takes $O(n)$ time



Performance

- In the array based implementation of a Vector
  - The space used by the data structure is $O(N)$
  - *size*, *isEmpty*, *elemAtRank* and *replaceAtRank* run in $O(1)$ time
  - *insertAtRank* and *removeAtRank* run in $O(n)$ time
- If we use the array in a circular fashion, *insertAtRank*(0) and *removeAtRank*(0) run in $O(1)$ time
- In an *insertAtRank* operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

Position ADT

- The Position ADT models the notion of place within a data structure where a single object is stored
- It gives a unified view of diverse ways of storing data, such as
  - a cell of an array
  - a node of a linked list
- Just one method:
  - object element(): returns the element stored at the position

List ADT (§2.2.2)

- The List ADT models a sequence of positions storing arbitrary objects
- It establishes a before/after relation between positions
- Generic methods:
  - size(), isEmpty()
- Query methods:
  - isFirst(p), isLast(p)

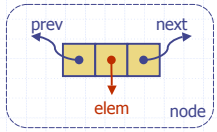
Accessor methods:

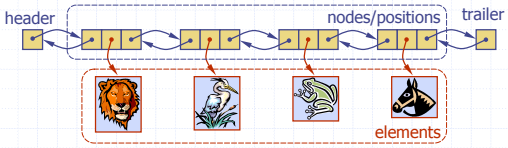
- first(), last()
- before(p), after(p)

- Update methods:
  - replaceElement(p, o), swapElements(p, q)
  - insertBefore(p, o), insertAfter(p, o),
  - insertFirst(o), insertLast(o)
  - remove(p)

Doubly Linked List

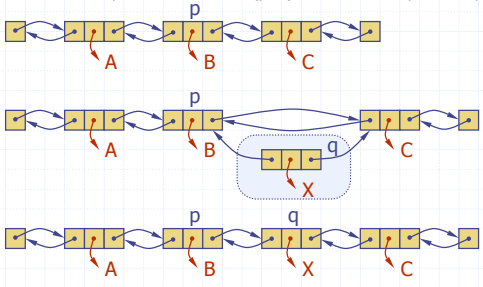
- A doubly linked list provides a natural implementation of the List ADT
- Nodes implement Position and store:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header nodes





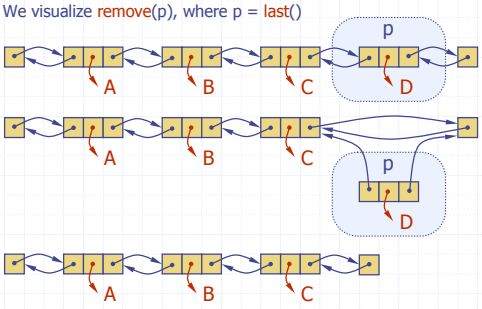
Insertion

- We visualize operation insertAfter(p, X), which returns position q



Deletion

- We visualize remove(p), where p = last()



Performance

- In the implementation of the List ADT by means of a doubly linked list
  - The space used by a list with n elements is $O(n)$
  - The space used by each position of the list is $O(1)$
  - All the operations of the List ADT run in $O(1)$ time
  - Operation element() of the Position ADT runs in $O(1)$ time

Sequence ADT

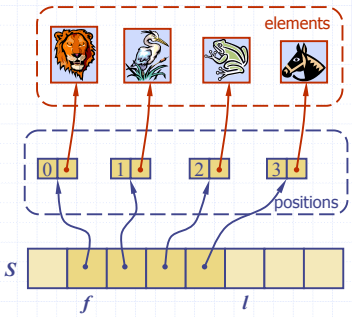
- The Sequence ADT is the union of the Vector and List ADTs
- Elements accessed by
  - Rank, or
  - Position
- Generic methods:
  - size(), isEmpty()
- Vector-based methods:
  - elemAtRank(r), replaceAtRank(r, o), insertAtRank(r, o), removeAtRank(r)
- List-based methods:
  - first(), last(), before(p), after(p), replaceElement(p, o), swapElements(p, q), insertBefore(p, o), insertAfter(p, o), insertFirst(o), insertLast(o), remove(p)
- Bridge methods:
  - atRank(r), rankOf(p)

Applications of Sequences

- The Sequence ADT is a basic, general-purpose, data structure for storing an ordered collection of elements
- Direct applications:
  - Generic replacement for stack, queue, vector, or list
  - small database (e.g., address book)
- Indirect applications:
  - Building block of more complex data structures

Array-based Implementation

- We use a circular array storing positions
- A position object stores:
  - Element
  - Rank
- Indices f and l keep track of first and last positions



Sequence Implementations

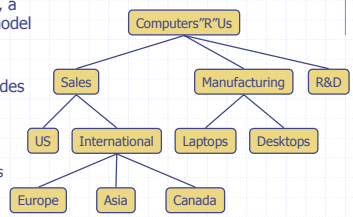
Operation	Array	List
size, isEmpty	1	1
atRank, rankOf, elemAtRank	1	n
first, last, before, after	1	1
replaceElement, swapElements	1	1
replaceAtRank	1	n
insertAtRank, removeAtRank	n	n
insertFirst, insertLast	1	1
insertAfter, insertBefore	n	1
remove	n	1

Iterators

- An iterator abstracts the process of scanning through a collection of elements
- Methods of the ObjectIterator ADT:
  - object object()
  - boolean hasNext()
  - object nextObject()
  - reset()
- Extends the concept of Position by adding a traversal capability
- Implementation with an array or singly linked list
- An iterator is typically associated with an another data structure
- We can augment the Stack, Queue, Vector, List and Sequence ADTs with method:
  - ObjectIterator elements()
- Two notions of iterator:
  - snapshot: freezes the contents of the data structure at a given time
  - dynamic: follows changes to the data structure

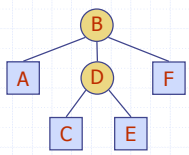
Trees (§2.3)

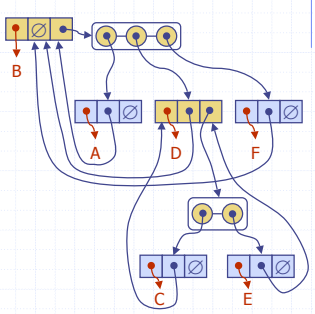
- In computer science, a tree is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Applications:
  - Organization charts
  - File systems
  - Programming environments



Linked Data Structure for Representing Trees (§2.3.4)

- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes
- Node objects implement the Position ADT





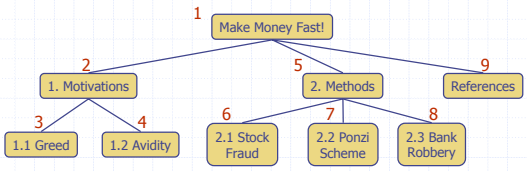
Tree ADT (§2.3.1)

- We use positions to abstract nodes
- Generic methods:
  - integer size()
  - boolean isEmpty()
  - objectIterator elements()
  - positionIterator positions()
- Accessor methods:
  - position root()
  - position parent(p)
  - positionIterator children(p)
- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Update methods:
  - swapElements(p, q)
  - object replaceElement(p, o)
- Additional update methods may be defined by data structures implementing the Tree ADT

Preorder Traversal (§2.3.2)

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document

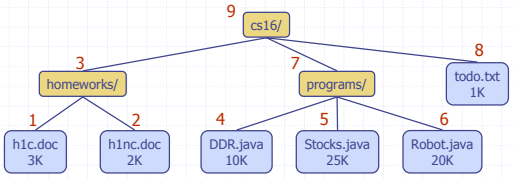
Algorithm *preOrder*(*v*)
  visit(*v*)
  for each child *w* of *v*
    preorder (*w*)



Postorder Traversal (§2.3.2)

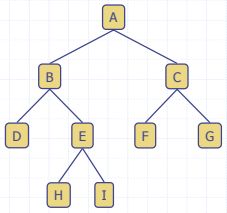
- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories

Algorithm *postOrder*(*v*)
  for each child *w* of *v*
    postOrder (*w*)
  visit(*v*)



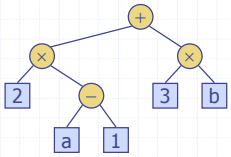
Binary Trees (§2.3.3)

- A binary tree is a tree where:
  - Each internal node has at most two children
- A **proper** binary tree is a binary tree where:
  - each internal node has exactly two children
  - The children are an ordered pair, denoted left child and right child.
- Alternative recursive definition: a (proper) binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a (proper) binary tree
- Applications:
  - arithmetic expressions
  - decision processes
  - searching



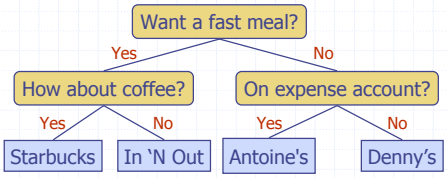
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $((2 \times (a - 1)) + (3 \times b))$



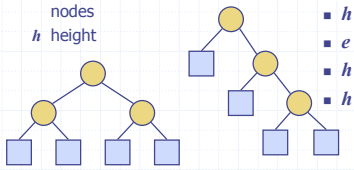
Decision Tree

- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision



Properties of (Proper) Binary Trees

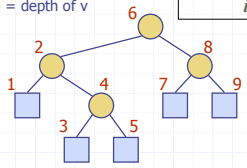
- Notation
  - n number of nodes
  - e number of external nodes
  - i number of internal nodes
  - h height
- Properties:
  - $e = i + 1$
  - $n = 2e - 1$
  - $h \leq i$
  - $h \leq (n - 1)/2$
  - $e \leq 2^h$
  - $h \geq \log_2 e$
  - $h \geq \log_2 (n + 1) - 1$



Inorder Traversal

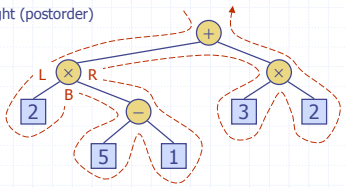
- In an inorder traversal a node is visited after its left subtree and before its right subtree
- Application: draw a binary tree
  - x(v) = inorder rank of v
  - y(v) = depth of v

Algorithm *inOrder*(*v*)
  if *isInternal* (*v*)
    inOrder (*leftChild* (*v*))
  visit(*v*)
  if *isInternal* (*v*)
    inOrder (*rightChild* (*v*))



Euler Tour Traversal

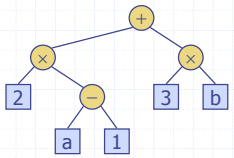
- Generic traversal of a binary tree
- Includes a special cases the preorder, postorder and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



Printing Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print "(" before traversing left subtree
  - print ")" after traversing right subtree

Algorithm *printExpr*(*v*)
  if *isInternal* (*v*)
    print("(")
    printExpr (*leftChild*(*v*))
  print(*v.element* ())
  if *isInternal* (*v*)
    printExpr(*rightChild*(*v*))
    print (")")



((2 × (a − 1)) + (3 × b))

Linked Data Structure for Binary Trees (§2.3.4)

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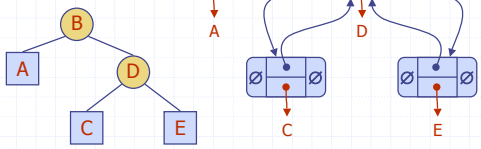
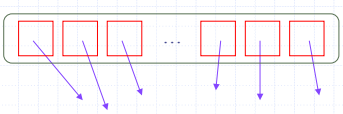
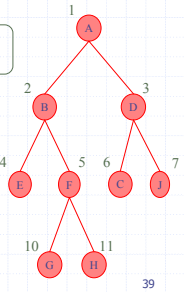
  - Right child node
- Node objects implement the Position ADT



Array-Based Representation of Binary Trees

- nodes are stored in an array





- let rank(node) be defined as follows:
  - rank(root) = 1
  - if node is the left child of parent(node), rank(node) = 2*rank(parent(node))
  - if node is the right child of parent(node), rank(node) = 2*rank(parent(node))+1