Problems on Graphs

Simple Problems
- Given an undirected graph G:
  - Determine if G connected.
  - Compute the connected components of G
  - Find and report a path between two given vertices
  - Find a cycle in the graph
- Use Graph Traversal algorithm to solve above
  - Depth First Search (DFS)
  - Breadth First Search (BFS)

Directed Graph Problems
- Given a directed graph G:
  - Determine if G strongly connected.
  - Compute the strongly connected components of G
  - Find and report a path between two given vertices
  - Find a cycle in the graph
Graph Traversal Algs

- Depth-first search (§6.3.1)
- Breadth-first search (§6.3.3)
- Directed DFS (§6.4)
- Applications of DFS (§6.5)
  - Path finding
  - Cycle finding

Topological Sorting

- Given a Directed Acyclic Graph (DAG)
- Find a topological ordering of the graph
  - topological ordering = ordering of vertices that obey “constraints” defined by directed edges.

A typical student day

1. Wake up
2. Study computer sci.
3. Eat
4. More c.s.
5. Work out
6. Nap
7. Study computer sci.
8. Eat
9. Make cookies for professors
10. More c.s.
11. Dream about graphs
Weighted Graphs

- In a weighted graph, each edge has a weight (an associated numerical value)
- Edge weights may represent distances, costs, etc.
- Example: In a flight route graph, the weight of an edge represents the distance in miles between the endpoint airports

Minimum Spanning Tree

- Spanning subgraph: Subgraph of a graph $G$ containing all the vertices of $G$
- Spanning tree: Spanning subgraph that is itself a (free) tree
- Minimum spanning tree (MST): Spanning tree of a weighted graph with minimum total edge weight
- Applications: Communications networks, Transportation networks

MST Algorithms

- The Prim-Jarnik Algorithm (§7.3.2)
- Kruskal’s Algorithm (§7.3.1)
- Baruva’s Algorithm (§7.3.3)
Shortest Path Problem

- Given a weighted graph and two vertices \( u \) and \( v \), we want to find a path of minimum total weight of a path between \( u \) and \( v \).
- Length (or weight) of a path is the sum of the weights of its edges.
- Distance of \( u \) from \( v \) is the length of a shortest path from \( u \) to \( v \).
- Example: Shortest path between Providence and Honolulu
- Applications
  - Internet packet routing
  - Flight reservations
  - Driving directions

Single-Source Shortest Paths Problem

- Given a weighted graph and one source vertex \( s \), find the shortest path tree \( T \).
- \( T \) is a tree rooted at \( s \) representing shortest path from \( s \) to every other vertex \( v \) in the graph.
- (The simple path from \( s \) to \( v \) in tree \( T \) is a shortest path from \( s \) to \( v \))

All-Pairs Shortest Paths

- Given a weighted directed graph \( G \)
- Find the (length of the) shortest path between every pair of vertices in \( G \).
Shortest Paths algorithms

- For single-source shortest paths
  - Dijkstra’s algorithm (§7.1.1)
  - The Bellman-Ford algorithm (§7.1.2)
- For single-source shortest paths in DAGs
  - Unnamed algorithm (§7.1.3) (called “DAGShortestPaths”)
- For all-pairs shortest paths
  - Floyd-Warshall algorithm (§7.2.1)