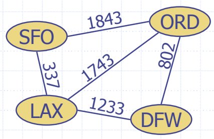
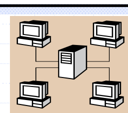
Problems on Graphs

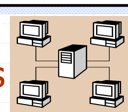


Simple Problems



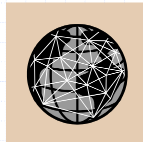
- Given an undirected graph G:
  - Determine if G connected.
  - Compute the connected components of G
  - Find and report a path between two given vertices
  - Find a cycle in the graph
- Use Graph Traversal algorithm to solve above
  - Depth First Search (DFS)
  - Breadth First Search (BFS)

Directed Graph Problems



- Given an directed graph G:
  - Determine if G strongly connected.
  - Compute the strongly connected components of G
  - Find and report a path between two given vertices
  - Find a cycle in the graph

Graph Traversal Algs

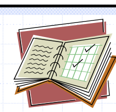


- Depth-first search (§6.3.1)
- Breadth-first search (§6.3.3)
- Directed DFS (§6.4)
- Applications of DFS (§6.5)
  - Path finding
  - Cycle finding

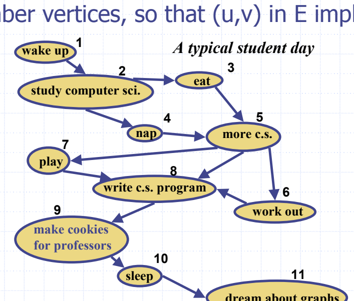
Topological Sorting

- Given a Directed Acyclic Graph (DAG)
- Find a topological ordering of the graph
  - topological ordering = ordering of vertices that obey "constraints" defined by directed edges.

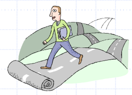
Topological Sorting



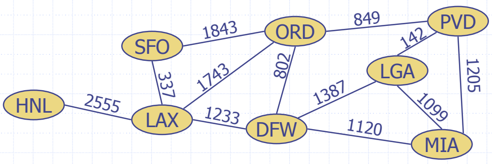
- Number vertices, so that (u,v) in E implies u < v



Weighted Graphs

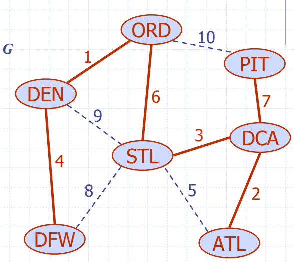


- In a weighted graph, each edge has a weight (an associated numerical value)
- Edge weights may represent, distances, costs, etc.
- Example:
  - In a flight route graph, the weight of an edge represents the distance in miles between the endpoint airports

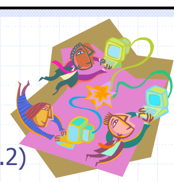


Minimum Spanning Tree

- Spanning subgraph
  - Subgraph of a graph ***G*** containing all the vertices of ***G***
- Spanning tree
  - Spanning subgraph that is itself a (free) tree
- Minimum spanning tree (MST)
  - Spanning tree of a weighted graph with minimum total edge weight
- Applications
  - Communications networks
  - Transportation networks



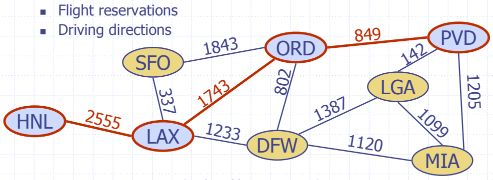
MST Algorithms



- The Prim-Jarnik Algorithm (§7.3.2)
- Kruskal's Algorithm (§7.3.1)
- Baruvka's Algorithm (§7.3.3)

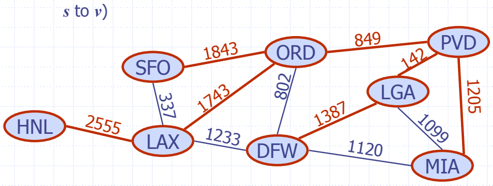
Shortest Path Problem

- Given a weighted graph and two vertices u and v, we want to find a path of minimum total weight of a path between u and v.
  - Length (or weight) of a path is the sum of the weights of its edges.
- Distance of u from v is the length of a shortest path from u to v.
- Example: Shortest path between Providence and Honolulu
- Applications
  - Internet packet routing
  - Flight reservations
  - Driving directions



Single-Source Shortest Paths Problem

- Given a weighted graph and one source vertex s, find the shortest path tree T.
- T is a tree rooted at s representing shortest path from s to every other vertex v in the graph.
  - (The simple path from s to v in tree T is a shortest path from s to v)



All-Pairs Shortest Paths

- Given a weighted directed graph G
- Find the (length of the) shortest path between every pair of vertices in G.

Shortest Paths algorithms

- For single-source shortest paths
  - Dijkstra's algorithm (§7.1.1)
  - The Bellman-Ford algorithm (§7.1.2)
- For single-source shortest paths in DAGs
  - Unnamed algorithm (§7.1.3) (called "DAGShortestPaths")
- For all-pairs shortest paths
  - Floyd-Warshall algorithm (§7.2.1)