

**UNIVERSITY OF WASHINGTON**  
**ED TEP 522: TEACHING AND LEARNING NUMERACY**  
**Autumn 2005**

<http://courses.washington.edu/tep522>

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**COURSE OVERVIEW**

This quarter we will focus on planning units and designing lessons. You will study a curriculum unit, learning more about the mathematics it entails, how students will approach tasks and how their understandings might develop. In teams, you will work on outlining the unit and designing benchmark lessons. At the end of the quarter, we will have the opportunity to teach and analyze several of those benchmark lessons. The work we undertake this year will prepare you in your first years of teaching as you make sense of the curriculum units you are expected to teach.

**COURSE GOALS**

- Our course activities will help you continue to build your content knowledge of the mathematics you will be expected to teach.
- Our work will build your ability to adapt and design curriculum units to support children's learning, differentiating instructional design where necessary to meet the language, cultural, and learning needs of your students.
- Our work will build your ability to design and facilitate lessons that involve eliciting and building student thinking and assessing students' understanding.

If you would like to request academic accommodations due to a disability, please contact Disabled Student Services, 448 Schmitz, (206)543-8924 (V/TTY). If you have a letter from Disabled Student Services indicating you have a disability that requires academic accommodations, please present the letter to me so we can discuss the accommodations you might need for the class.

## EXPECTATIONS

- This class will run on a “workshop” model, which means that teams will have tasks to complete each day during class. It is important that you contribute to your team and stay focused on the task. Bring materials to class. You may also need to bring a laptop to class if you have one and prefer to work on your own laptop rather than a university one.
- Participating actively in classroom work is vital to this class. Participation does not mean talking a lot. You need to be mindful that you are not dominating the discussions. Also be mindful of the ways your comments are connecting to the flow of the conversation and what has already been discussed.
- Assignments must be completed on time. Late papers will not be accepted without penalty, excepting extraordinary circumstances.
- All of your written work will be held to high standards and should conform to proper rules of grammar, usage, punctuation, and spelling. Proofread your paper before you turn it in. Please use Times 12 pt font with 1 inch margins, and use APA guidelines for citations. Please do not put report covers on your work. Always include student work when relevant.
- We expect you to be on time for class. If you cannot attend class, please let us know ahead of time. You will be responsible for the material that you miss.
- We welcome you to talk to us about our discussions, assignments, readings, and your observations in the field.

## EVALUATION

Assignments will be weighted according to the following scheme.

Participation*	20 points	Grading Scale	
Unit goals/assessment	20 points	95-100	4.0
Unit tasks and student responses	30 points	90-94	3.7
Unit overview	15 points	85-89	3.4
Lesson plan & analysis	15 points	80-84	3.2
		75-79	3.0
		70-74	2.7
		< 70	

### \*Participation

This part of your final grade includes: active engagement with small groups, professional demeanor, and participation in class activities, and preparedness for class (doing assigned readings, tasks, etc.)

NOTE: READINGS AND ASSIGNMENTS SHOULD BE COMPLETED ON THE DAY THEY ARE LISTED.  
THIS IS A WORKING DOCUMENT. WE MAY MAKE CHANGES TO THE PLAN AS NEEDED.

W	DAY	CLASS	ASSIGNMENTS
Week 1	Thursday 9/29	<b>Introduction to class</b> This quarter's focus: Planning units and teaching lessons	
	Tuesday 10/4	<b>Cognitive demand</b> How does the cognitive demand of the task influence student learning?	
Week 2	Thursday 10/6	<b>Identifying curricular goals &amp; developing rationale</b> What are the big mathematical ideas of the unit you are planning? Where do those ideas fit in relation to the GLEs? Why is it important that students learn these ideas?	<i>Read GLEs for your unit. Read parts of the unit that describe mathematical goals. Check Seattle curriculum guide if using TERC: <a href="http://www.seattleschools.org/are/math/curriculumguides/curriculum_guides_home.xml">http://www.seattleschools.org/are/math/curriculumguides/curriculum_guides_home.xml</a></i>  <b>TURN IN: Unit Goals. Rationale.</b>
	Tuesday 10/11	<b>Refining goals, linking to culminating assessment</b> What are the culminating assessments that you will use? How do your goals align with culminating assessment? What kind of student performance will be evidence of learning the big ideas and why?	<b>TURN IN: Culminating assessments with example of proficient student performance and explanation of why it is evidence of learning unit goals.</b>
Week 3	Thursday 10/13	<b>Taking into account student understanding</b> What initial ideas will students bring to the unit? What knowledge & skills will students need to develop? What aspects of the unit might need to be adapted to make it culturally relevant to your students? How do articles on student understanding help you create a hypothetical learning trajectory?	<i>Read selected articles and watch video on student understanding related to the topic of your unit.</i>
	Tuesday 10/18	<b>Identifying benchmark lessons and assessment signposts</b> What lessons will serve as benchmark lessons? What assessments or lessons will you use as assessment signposts?	<i>Finish reading articles and watching video on student understanding</i>
Week 4	Thursday 10/21	<b>Creating unit overview maps</b> How will your unit progress? Create a visual to show overview of where big ideas are addressed and pacing of the unit	<b>TURN IN: Complete unit tasks from benchmark lessons showing student thinking issues</b>

<b>Week 5-7</b>	<b>FIELD</b> <b>TURN IN DRAFT OF UNIT OUTLINE DURING FIRST WEEK OUT</b>		
<b>Week 8</b>	Tuesday 11/15	<b>Lesson Design I</b> How do you assess and maintain cognitive demand of the lesson?	<i>Read cognitive demand readings (chapters 2 and 7)</i>
	Thursday 11/17	<b>Lesson Design II</b> How do you involve students in discussions? What role do manipulatives play in supporting learning?	<i>Read Magical Hopes: Ball, 1992</i>
<b>Week 9</b>	Tuesday 11/22	<b>Structuring Observations.</b>	
	Thursday 11/24	<b><i>Thanksgiving Break. No class.</i></b>	
<b>Week 10</b>	Tuesday 11/29	<b>Rehearsing Lesson you will teach</b>	
	Thursday 12/1	<b>Practice Teaching I</b>	
<b>Week 11</b>	Tuesday 12/6	<b>Practice Teaching II</b>	
	Thursday 12/8	<b>Culminating our Work in Math Methods</b>	<b><i>TURN IN: Lesson &amp; unit plan with analysis of your learning Monday 12/12/05</i></b>

## Planning for a Unit of Study in Mathematics

“Use curriculum as a means to an end. . . . many teachers *begin* with textbooks, favored lessons, and time-honored activities rather than deriving those tools from targeted goals or standards. . . . We advocate the reverse: One starts with the end—the desired results (goals or standards)—and then derives the curriculum from the evidence of learning (performances) called for by the standard and the teaching needed to equip students to perform” (Wiggins & McTighe, 8, 1998).



### Phase I: Making explicit to yourself what is important enough to teach.

#### a. IDENTIFYING A TOPIC

Determine the concept, topic, or theme—based on the appropriate Grade Level Expectations (GLEs), Essential Academic Learning Requirements (EALRs) and other documents like the NCTM Principles and Standards. If your ideas do not appear in these documents, it is likely not a key idea agreed upon by the broader community.

#### b. UNCOVERING THE BIG IDEAS

In sentence form, write two to four big ideas or big ideas for the mathematics in the unit. These are essential understandings of concepts and process that make up the unit. Big ideas should come from reading articles that describe challenges students face in learning the mathematics of this topic, solving problems yourself and with colleagues, and reading curriculum materials that describe students' ideas about the topic. For example in a unit on categorical and numerical data for primary students the big ideas might be:

- Defining clear and precise questions are essential for data collection.
- Designing methods for data collection that are accurate and consider the population.
- Creating representations that afford accurate insights on the data and use appropriate mathematical elements, e.g., scale, range, independent/dependent relationship, etc..

#### c. RATIONALE FOR THE UNIT

Write a one-paragraph rationale for the unit. The paragraph should answer two questions: **“Why is it important that students understand this**

**topic?” and “What kind of topics (encompassing particular types of knowledge and skills) would logically precede and follow this unit in a course plan?”** In answering these questions, think about what you might say to a school board member, principal, or concerned parent asked you about your teaching. The following stems may help you write the rationale:

Students need to understand \_\_\_\_\_. They might approach it by \_\_\_\_\_. They need to develop skills in \_\_\_\_\_ so that they \_\_\_\_\_. It might be hard for them to \_\_\_\_\_. The point of doing these activities is to learn \_\_\_\_\_.

#### d. **CULMINATING ASSESSMENTS**

Determine what students can do as a culminating assessment. This should be a project or product to demonstrate their knowledge and/or skills acquisition around your big ideas. You may have two or three smaller products as opposed to a single comprehensive one. These products or projects do not have to include all the big ideas, but they should include at least two or them.

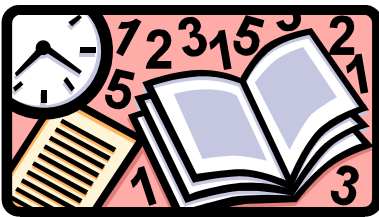
- Note: evidence of understanding includes ongoing formal and informal assessment as well as the final, end-of-unit product or task.

#### Phase II: Modified Backward Planning

Start with the final product developed in Phase I and work backward.

- Ask “What skills or knowledge would each student need to have in order to complete the final project?” Then consider what knowledge or skills students would need for these prerequisites...and so on. There is no formula to the process of working backwards. Eventually, at the bottom of your “backwards list”, you will have some basic skills or “knowledge nuggets” that are necessary to launch a unit. It is messy.

#### Phase II: Planning the Assessment Sign Posts



- Once you have worked backward to the beginning, the most basic information - then begin working forward to plan the assessments (remember assessment should serve as instruction and vice versa) which will build toward the final project. These can be the lessons or problem tasks that comprise the unit plan but should focus on the evidence you will have of students' understanding of concepts and skill.
- Also, one of the first things you will want to do in a unit is elicit student ideas.** Their existing knowledge will be a key part of how the unit unfolds and may influence the development of your plan!

### **Phase III: Creating A Unit Overview**

*Construct the pacing for the unit with a visual that will help you see when and how the big ideas are emphasized as the unit progresses.*

*Do not write full lesson plans. Instead, jot brief notes about:*



*1) Describe what the students will be doing*

*2) What you might be looking for as students engage in the task. Over the course of the unit, these brief jottings should help you see how you are going to progress towards achieving the big ideas.*

*It is possible that you would “lump” two or three days together as students engage in the same activity over an extended period.*

Please remember, unit design is a complex and iterative process.  
It is not neat and straightforward.

## REQUIRED READINGS

Ball, D.L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator*, 16 (2), 28-33.

## STUDENT THINKING READINGS

### Geometry

Battista, M. (1999). The importance of spatial structuring in geometric reasoning. *Teaching Children Mathematics*, 6(3), 170-7.

Battista, M. & Clements, D. (1998). Finding the number of cubes in rectangular cube buildings. *Teaching Children Mathematics*, 4(5), 258-264.

Battista, M., Clements, D., Sarama, J. (1998). Development of geometric and measurement ideas. In R. Lehrer & D. Chazan (Eds). *Designing learning environments for developing understanding of geometry and space* (pp. 201-225). Mahwah, NJ: Erlbaum.

Burns, Barbara A., Clements, D. (2000). Students' development of strategies for turn and angle measure. *Educational Studies in Mathematics* 41, 31-45.

Lehrer, R., Jacobson, C., Thoyre, G., Kemeny, V., Strom, D., Horvath, J., Gance, S., & Koehler, M. (1998). Developing understanding of geometry and space in the primary grades. In R. Lehrer & D. Chazan (Eds). *Designing learning environments for developing understanding of geometry and space* (pp. 169-200). Mahwah, NJ: Erlbaum.

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's reasoning about space and geometry. In R. Lehrer & D. Chazan (Eds). *Designing learning environments for developing understanding of geometry and space* (pp. 137-167). Mahwah, NJ: Erlbaum.

Schifter, D. (1999). Learning geometry: Some insights drawn from teachers' writing. *Teaching Children Mathematics*, 5, 360-366.

### Measurement

Hiebert, J. (1984). Why do some children have trouble learning measurement concepts? *Arithmetic Teacher*, 31(7), 19-24.

Outhred, L.N. & Mitchelmore, M.C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education*, 31, 144-167.

Thompson, C. S. & Van de Walle, J. (1985). Learning about rulers and measuring. *Arithmetic Teacher*, 32(8), 8-12.

Wilson, P.S., & Rowland, R. Teaching measurement.

### Fractions

Ball, D. (1993). Halves, pieces, and twos: Constructing and using representational contexts in teaching fractions. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 157-196). Hillsdale, NJ: Erlbaum.

DMI Cases (see electronic reserves): Empson, S. (2000 Oct.). Organizing diversity in early fraction thinking. *Presented at the first annual conference: Cognitively Guided Instruction and Beyond*. Phoenix, AZ.

Empson, S. (1995). Using sharing situations to help children learn fractions. *Teaching Children Mathematics*, 2, 110-114.

Ma, L. (1999). Generating representations: Division by fractions. In *Knowing and teaching mathematics: Teachers' understanding of fundamental mathematics in China and the United States* (pp. 55-83). Mahwah, NJ: Erlbaum.

Mack, N. (1998). Building a foundation for understanding the multiplication of fractions. *Teaching Children Mathematics*, 5, 34-38.

Watanabe, T. (2002). Representations for teaching and learning fractions. *Teaching Children Mathematics*, April, 457-463.

## Data

*Developing Mathematical Ideas* cases.

Bright, G.W., & Friel, S.N. (1998). Graphical representations: Helping students interpret data. In S. Lajoie (Ed.). *Reflections on statistics: Agenda for learning, teaching, and assessment in K-12* (pp.63-89). Mahwah, NJ: Erlbaum.

Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning*, 1, 5-43.

Curcio, F.R., & Folkson, S. (1996). Exploring data: Kindergarten children do it their way. *Teaching Children Mathematics*, 2, 382-385.

Folkson, S. (1996). Meaningful communication among children: Data collection. In P.C. Elliott (Ed.). *NCTM Yearbook: Communication in Mathematics, K-12 and Beyond* (pp. 29-34). Reston, VA: NCTM.

Friel, S.N. (1998, October). Comparing data sets: How do students interpret information displayed using box plots. *Proceedings of the Psychology of Mathematics Education – North American Chapter*, Raleigh, NC.

Friel, S.N., Curcio, F.R., Bright, G.W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education*, 32, 124-158.

Hancock, C., Kaput, J. J., & Goldsmith, L. T. (1992). Authentic inquiry with data: Critical barriers to classroom implementation. *Educational Psychologist*, 27, 337-364.

Lehrer, R., Giles, N.D., & Schauble, L. (2002). Children's work with data. In R. Lehrer & L. Schauble (Eds). *Investigating real data in the classroom: Expanding children's understanding of math and science* (pp. 1-26). New York: Teachers College.

McClain, K., McGatha, M., & Hodge, L.L. (2000). Improving data analysis. *Mathematics Teaching in the Middle School*, 5, 548-553.

Mokros, J., & Russell, S. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education*, 26, 20-39.

Russell, S.J., & Friel, S.N. (1989). Collecting and analyzing real data in the elementary school classroom. In P.R. Trafton & A.B. Shulte (Eds.) *National Council of Teachers of Mathematics yearbook: New directions for elementary school mathematics* (pp. 134-148). Reston, VA: National Council of Teachers of Mathematics.

Russell, S.J., & Mokros, J. (1996). What do children understand about average? *Teaching Children Mathematics*, 2, 360-364.

## Number

Ambrose, R., Baek, J., Carpenter, T. (2003). Children's invention of multidigit multiplication and division algorithms. In A.J. Baroody & A. Dowker (Eds) *The development of arithmetic concepts and skills* (pp. 305-336). Mahwah, NJ: Erlbaum.

- Baroody, A. (1984). Children's difficulties in subtraction: Some causes and questions. *Journal for Research in Mathematics Education*, 15(3), 203-213.
- Carpenter, T. P., Franke, M. L., Jacobs, V., & Fennema, E. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 29, 3-20.
- Carpenter, T., Levi, L. (1999 Apr). Developing conceptions of algebraic reasoning in the primary grades. *Presented at the annual meeting of the American Educational Research Association*. Montreal, Canada.
- Freckman, L., Huinker, D., Steinmeyer, M. (2003). Subtraction strategies from children's thinking: Moving toward fluency with greater numbers. *Teaching Children Mathematics*, 9(6), 347-353.
- Fuson, K. (1984). More complexities in subtraction. *Journal for Research in Mathematics Education*, 15, 214-225.
- Falkner, K., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics*, 6, 232-236.