QSci 291 • answers • Hmwk #10

class handout on differentials

$dy = (y')\,dx$ or $dy = \left(\frac{dy}{dx}\right)\cdot dx$

1. $y = x^2 + 7x + 1$; $\underline{dy = (2x+7)\cdot dx}$

3. $y = t\cdot \ln t$; $\underline{dy = (\ln t + 1)\cdot dt}$

4. $y = u e^{-u}$; $\underline{dy = (1-u)e^{-u}\cdot du}$

5. $y = \ln(z^2+1)$; $\underline{dy = \left(\frac{2z}{z^2+1}\right)\cdot dz}$

11. $y = x^3$; $dy = (3x^2)\cdot dx$; $x = 2$; $dx = 0.01$

$\therefore dy = (3\cdot 2^2)(0.01)$ or $\underline{dy = 0.12}$

13. $x = y\cdot \ln y$; $dx = (\ln y + 1)dy$; $y = 1$; $dy = 0.003$

$\therefore dx = (\ln(1)+1)(0.003)$ or $\underline{dx = 0.003}$

17. $y = \ln u$; $dy = \frac{1}{u}\cdot du$; $u = 3$; $du = 0.06$

$dy = \left(\frac{1}{3}\right)(0.06)$ or $\underline{dy = 0.02}$

20.) $\sqrt[4]{17} = x.xxx(?)$; Let $y = x^{1/4}$; $dy = \frac{x^{-3/4}}{4}\,dx$

Notice: $\Delta y \approx dy = \frac{1}{4}\frac{1}{(\sqrt[4]{x^3})}\,dx \approx \sqrt[4]{17} - \sqrt[4]{16}$

Thus, $\sqrt[4]{17} = \sqrt[4]{16} + \frac{1}{4}\frac{1}{(\sqrt[4]{16})^3}\cdot(1)$ with $x = 16$ & $dx = 1$

$\sqrt[4]{17} \approx 2 + 0.03125$ or $\underline{\sqrt[4]{17} \approx 2.03125}$

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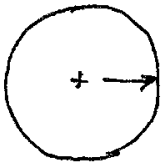
23.) $V = \frac{4}{3}\pi r^3$; $dV = (4\pi r^2)dr$



$r = 8\,cm$

$dr = \pm 0.002\,cm$

Smallest Volume : $V_1 = V_0 - dV$

Largest Volume : $V_2 = V_0 + dV$

Maximum Error in Volume : $\Delta V = V_2 - V_1 = 2dV$

$\Delta V = 2(4\pi 8^2)(0.002)$ or $\underline{\Delta V = 3.2\ cm^3}$

Note : Relative Maximum Error would be

$$\frac{2\Delta V}{V_0} \times 100 \approx 1.5\%$$

25.) Relative Volume Error : $100\frac{dV}{V} = 2\%$

$V = \frac{4\pi}{3} r^3$; $dV = (4\pi r^2)\,dr$

$$\therefore \quad 100\,\frac{dV}{V} = 100\,\frac{4\pi r^2}{(\frac{4\pi}{3})r^3} \cdot dr = 3\left(\frac{dr}{r}100\right) = 2$$

Allowable maximum percentage error

$\underline{\frac{dr}{r} \times 100 = \frac{2}{3}\%}$ or $\frac{dr}{r} \times 100 = 0.667\%$