QSci 291 • answers • Hmwk #3

Neu 3rd ed, p. 143, sec. 4.1.4
Neu 2nd ed, p. 177, sec. 4.1.4

⇦

#6) $f(x) = (x+2)^2 = x^2+4x+4$; $f'(x) = 2(x+2)$; $\boxed{f'(1)=6}$

note: Neu 2nd ed → $f(x) = (x-1)^2$; $f'(x) = 2(x-1)$; $f'(1) = 0$ ans

#22) $f(x) = -2x^2$; $f(x) = -4x$

note: #14 in Neu 2nd

a) formal 4 step def^n

step 1: $f(x+h) \to -2(x+h)^2 = -2(x^2+2xh+h^2)$

2: $-f(x) \to -2h(2x+h)$

3: $\div h \to -2(2x+h)$

4: $h \to 0 \to -2(2x) = -4x$

thus, $\boxed{f'(1) = -4}$ ans

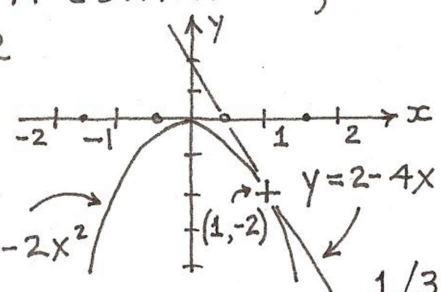
b) let $y = -2x^2$, then at $x = 1$, $y = -2(1)^2 = -2$ hence the point $(x, y) = (1, -2)$ is on the graph of $y = -2x^2$

c.) the equation of a tangent line is: $y = a + bx$ where b is the slope; but $b = -4$ since $f'(1) = -4$, $\therefore y = a - 4x$

now both graphs: $y = -2x^2$ and $y = a - 4x$ have the point $(1, -2)$ in common, so $(-2) = a - 4(1)$ or $a = 2$

clearly, the tangent line is:

$\boxed{y = 2 - 4x}$ ans



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#26) (#18 2nd ed) formal definition

- $f(x+h) = \dfrac{1}{(x+h)+1}$

- $f(x+h) - f(x) = \dfrac{1}{x+h+1} - \dfrac{1}{x+1}$

- $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x+1) - (x+h+1)}{h(x+h+1)(x+1)}$

- $\lim\limits_{h \to 0}\left(\dfrac{f(x+h)-f(x)}{h}\right) = \lim\limits_{h \to 0}\left(\dfrac{-h}{h(x+h+1)(x+1)}\right) = \boxed{\dfrac{-1}{(x+1)^2}}$ ans

or using the chain rule $\rightarrow$ let $z = x+1$ and $y = z^{-1}$ then $\dfrac{dz}{dx} = 1$ & $\dfrac{dy}{dz} = \dfrac{-1}{z^2}$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \left(-\frac{1}{z^2}\right) \cdot 1 = \boxed{\frac{-1}{(x+1)^2}}$$

Neu 3rd ed., p. 149, sec. 4.2
(Neu 2nd ed., p. 183, sec. 4.2.1)

#7) $g(s) = 5s^7 + 2s^3 - 5s$, $g'(s) = \boxed{35s^6 + 6s^2 - 5}$ ans

#19) $f(x) = 20x^3 - 4x^6 + 9x^8$, $f'(x) = \boxed{60x^2 - 24x + 72x^7}$ ans

#33) $g(t) = a^3t - at^3$, $g'(t) = \boxed{a^3 - 3at^2}$ ans

#41) $R(T) = AT^4$ where : $A = \dfrac{2\pi^5}{15} \cdot \dfrac{k^2}{c^2h^3}$ (a constant)

$R'(T) = 4AT^3$

or $R'(T) = \boxed{\left(\dfrac{8\pi^5}{15}\right) \cdot \left(\dfrac{k^2}{c^2h^3}\right) T^3}$ ans

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Neu 3rd ed, p 158, sec. 4.3
Neu 2nd ed, p 192, sec. 4.3

#13) $g(s) = (2s^2 - 5s)^2 = 4s^4 - 20s^3 + 25s^2$

$$g'(s) = \boxed{16s^3 - 60s^2 + 50s} \text{ ans}$$

E.C.E.

note: C.R. $z = 2s^2 - 5s$, $y = g(s) = z^2$

$$\frac{dy}{ds} \equiv g'(s) = \frac{dy}{dz} \cdot \frac{dz}{ds} = 2z(4s-5) = \boxed{2(4s-5)(2s^2-5s)}$$

ans

#34) $h(t) = \sqrt{a}\,(t - a) + a$ (a is a constant)

$$\boxed{h'(t) = \sqrt{a}}$$

#63) $f(x) = x^3 - \frac{1}{x^3} = x^3 - x^{-3}$

$$f'(x) = 3x^2 - (-3)x^{-4} = \boxed{3\left(x^2 + x^{-4}\right)}$$

ans

or $f'(x) = \boxed{3\left(x^2 + \frac{1}{x^4}\right)}$ E.C.E.