QSci 291 • answers • Hmwk #4

Neu 3^{rd} ed, p. 159, sec. 4.3.2
Neu 2^{nd} ed, p. 194, sec. 4.3.2

#58) $y = f(s) = u(s)/v(s)$, where:

$u = 2s^3 - 4s^2 + 5s - 7$; $u' = 6s^2 - 8s + 5$

$v = (s^2-3)^2$; $v' = 2(s^2-3)\cdot(2s)$ C.R.

• use Q.R. $y' = (v\cdot u' - u\cdot v')/v^2$

$v\cdot u' = (s^2-3)^2\cdot(6s^2 - 8s + 5)$

$u\cdot v' = (s^2-3)\cdot(4s)\cdot(2s^3 - 4s + 5s - 7)$

$v\cdot u' - u\cdot v' = (s^2-3)\left[(s^2-3)(6s^2-8s+5) - (8s^4 - 16s^3 + 20s^2 - 28s)\right]$

$$\frac{dy}{ds} = \frac{(s^2-3)\left[-2s^4 + 8s^3 - 33s^2 + 52s - 15\right]}{(s^2-3)^4}$$

$$\therefore \boxed{f'(s) = (-2s^4 + 8s^3 - 33s^2 + 52s - 15)/(s^2-3)^3} \text{ ans.}$$

#68) $y = g(s) = u(s)/v(s)$, where:

$u = s^{1/7} - s^{2/7}$; $u' = \frac{1}{7}s^{-6/7} - \frac{2}{7}s^{-5/7}$

$v = s^{3/7} + s^{4/7}$; $v' = \frac{3}{7}s^{-4/7} + \frac{4}{7}s^{-3/7}$

• use Q.R.

$v\cdot u' - u\cdot v' = \frac{1}{7}\left[(s^{3/7}+s^{4/7})(s^{-6/7} - 2s^{-5/7}) - (s^{1/7} - s^{2/7})(3s^{-4/7} + 4s^{-3/7})\right]$

$$\frac{dy}{ds} = \frac{1}{7}\left[\frac{(-2s^{-3/7} - 2s^{-2/7} + 2s^{-1/7})}{(s^{3/7}+s^{4/7})^2}\right]$$

$$\therefore \boxed{g'(s) = -\frac{2}{7}s^{-9/7}\left(\frac{1+s^{1/7} - s^{2/7}}{(1+s^{1/7})^2}\right)} \text{ ans}$$

<u>note</u>: Let $z = s^{1/7}$, then $y = (1-z)/z^2(1+z)$

• use C.R. and Q.R. to get $\frac{dy}{ds} = -\frac{2}{7}z^{-9}\left[(1+z-z^2)/(1+z)^2\right]$

QSci 291 • answers • Hmwk #4

#72) $y = f(x) = 3x^{-1} - 4x^{-1/2} + 2x^{-2}$

$$\frac{dy}{dx} \equiv y' = f'(x) = -3x^{-2} + 4\left(\tfrac{1}{2}\right)x^{-3/2} - 2(2)x^{-3}$$

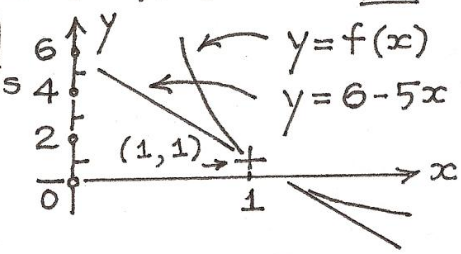
or $\boxed{y' = -\dfrac{3}{x^2} + \dfrac{2}{x\sqrt{x}} - \dfrac{4}{x^3}}$ ans

at $x = x_0 = 1$, $y_0 = f(1) = 3 - 4 + 2 = \underline{\underline{1}}$; $f'(1) = -3 + 2 - 4 = \underline{\underline{-5}}$

tangent line: $y = a + bx$

since $b = f'(1) = \underline{\underline{-5}}$, then $\overset{y_0}{1} = a + \overset{b}{(-5)}\overset{x_0}{(1)}$ $\therefore a = \underline{\underline{6}}$

tang. line formula: $\boxed{y = 6 - 5x}$ ans



Neu 3rd, p. 172, sec 4.4.1
Neu 2nd, p. 208, sec 4.4.1

#5) $y = f(x) = \sqrt{x^2+3}$; C.R. Let $u = x^2 + 3$; $\dfrac{du}{dx} = 2x$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \left(\frac{1}{2\sqrt{u}}\right)\cdot(2x) \qquad y = u^{1/2};\ \frac{dy}{du} = \frac{1}{2\sqrt{u}}$$

$\boxed{f'(x) = \dfrac{x}{\sqrt{x^2+3}}}$ ans

#21) $y = h(x) = \sqrt[5]{3 - x^4} = (3 - x^4)^{1/5}$

C.R. Let $z = 3 - x^4$ and then $y = z^{1/5}$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \left(\frac{1}{5}z^{-4/5}\right)\cdot\left(-4x^3\right) \quad \text{or}$$

$$y' = -\frac{4}{5}(3 - x^4)^{-4/5}\cdot x^3$$

$\boxed{h'(x) = -\dfrac{4}{5}\left[\dfrac{x^3}{(3 - x^4)^{4/5}}\right]}$ ans

QSci 291 • answers • Hmwk #4

#27) $y = h(t) = \left(3t + \frac{3}{t}\right)^{2/5}$ or $y = 3^{2/5}\left(t + t^{-1}\right)^{2/5}$

C.R. Let $p = t + t^{-1}$, then $y = 3^{2/5} \cdot p^{2/5}$

$$\frac{dy}{dt} = \frac{dy}{dp} \cdot \frac{dp}{dt} = 3^{2/5}\left(\frac{2}{5} p^{-3/5}\right) \cdot \left(1 - t^{-2}\right)$$

or

$$\boxed{h'(t) = \left(\frac{2 \cdot 3^{2/5}}{5}\right) \cdot \left[\frac{1 - \frac{1}{t^2}}{\left(t + \frac{1}{t}\right)^{3/5}}\right]}$$ ans

#33) $y = g(T) = a(T_0 - T)^3 - b$ (a, b, T_0 are constants)

C.R. Let $u = T_0 - T$, then $y = a u^3 + b$

$$\frac{dy}{dT} = \frac{dy}{du} \cdot \frac{du}{dT} = (3au^2) \cdot (-1) \quad \therefore \boxed{g'(T) = -3a(T_0 - T)^2}$$ ans

Neu 3rd, p. 181, sec 4.6

Neu 2nd, p. 221, sec 4.6

#2) $y = f(x) = e^{-2x}$; C.R. $w = -2x$, $y = e^{w}$

$$\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{dx} = (e^{w}) \cdot (-2) \quad \therefore \boxed{f'(x) = -2e^{-2x}}$$ ans

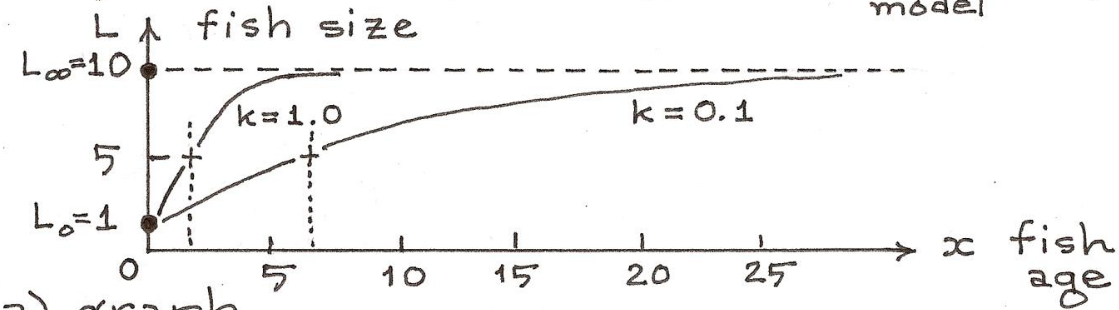
note: Neu 2nd $y = e^{-4x}$, hence $y' = -4e^{-4x}$

#47) $y = h(t) = 5^{\sqrt{t}}$; C.R.; $v = \sqrt{t}$, $y = 5^{v}$

$$\frac{dy}{dt} = \frac{dy}{dv} \cdot \frac{dv}{dt} = \left(\ln(5) \cdot 5^{v}\right) \cdot \left(\frac{1}{2\sqrt{t}}\right) \quad \therefore \boxed{h'(t) = \left(\frac{\ln(5)}{2}\right) \frac{5^{\sqrt{t}}}{\sqrt{t}}}$$ ans

QSci 291 • answers • Hmwk #4

#65) $L(x) = L_\infty - (L_\infty - L_0)e^{-kx}$ von Bertalanffy model



a) graph

b) $L_0 = 1$, initial fish size & $L_\infty = 10$, final fish size

c) $k=1.0$ curve reaches $L=5$ first

<u>note</u>: von Bertalanffy eq. can be expressed as:

$$x = \frac{1}{k} \ln\left(\frac{L_\infty - L_0}{L_\infty - L(x)}\right)$$

so if $L=5$, $L_0 = 1$ and $L_\infty = 100$,
$x = 0.588$ when $k = 1.0$ & $x = 5.88$ when $k = 0.1$

d.) $$\frac{dL}{dx} = -(L_\infty - L_0)\frac{d}{dx}\left(e^{-kx}\right) = k(L_\infty - L_0)e^{-kx}$$

but $L_\infty - L(x) = (L_\infty - L_0)e^{-kx}$, hence

$$\boxed{\frac{dL}{dx} = -k(L_\infty - L)}$$

von Bertalanffy differential equation model for fish growth

e) when the value of the parameter k becomes large, then growth is rapid (most of the growth occurs in a short period of time)