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Neu 3rd ed, p. 192, sec. 4.7
Neu 2nd ed, p. 233, sec. 4.7.4

#37) $y = \ln[x/(x+1)] = \ln(x) - \ln(x+1)$ E.C.E.

$$y' = \frac{1}{x} - \frac{1}{x+1} \quad \text{or} \quad \boxed{y' = \frac{1}{x(x+1)}} \text{ ans}$$

• another way: C.R. $z = x/(x+1)$; $y = \ln(z)$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{z}\left[\frac{(x+1)(1) - x(1)}{(x+1)^2}\right] = \frac{1}{\left(\frac{x}{x+1}\right)} \cdot \frac{1}{(x+1)^2}$$

Q.R.

hence $\boxed{y' = \frac{1}{x(x+1)}}$ ans

#51) $y = \sin(\ln(3t))$ Let: $z = 3t$; $\frac{dz}{dt} = 3$

C.R. $w = \ln(z)$; $\frac{dw}{dz} = \frac{1}{z}$

$$y' \equiv \frac{dy}{dt} = \frac{dy}{dw} \cdot \frac{dw}{dz} \cdot \frac{dz}{dt}$$

$y = \sin(w)$; $\frac{dy}{dw} = \cos(w)$

$$y' = \cos(w) \cdot \frac{1}{z} \cdot 3 = 3 \cdot \frac{1}{3t} \cdot \cos(\ln(3t))$$

$$\therefore \boxed{y' = \frac{1}{t} \cdot \cos(\ln(3t))} \text{ ans}$$

#58) $y = \log\left[\sqrt[3]{\tan(x^2)}\right] = \frac{1}{3}\log(\tan(x^2))$

C.R. Let $p = x^2$; $dp/dx = 2x$

$q = \tan(p)$; $dq/dp = 1/\cos^2(p) = \sec^2(p)$

$y = \frac{1}{3}\log(q)$; $dy/dq = \frac{1}{3} \cdot \frac{1}{\ln(10)} \cdot \frac{1}{q}$

$$\frac{dy}{dx} = \frac{dy}{dq} \cdot \frac{dq}{dp} \cdot \frac{dp}{dx} = \left(\frac{1}{3} \cdot \frac{1}{\ln(10)} \cdot \frac{1}{\tan(p)}\right) \cdot \left(\frac{1}{\cos^2(p)}\right) \cdot (2x)$$

$$\therefore \boxed{y' = \left(\frac{2}{3 \cdot \ln(10)}\right) \cdot \left(\frac{x}{\cos(x)}\right)} \text{ ans}$$

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Neu 3rd ed, p 177, sec 4.5
Neu 2nd ed, p 215, sec 4.5.1

#15) $y=f(x)=3\cdot\sin(x^2)$; C.R. $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

Let: $u=x^2$; $y=3\cdot\sin(u)$, then $\frac{du}{dx}=2x$; $\frac{dy}{du}=3\cdot\cos(u)$

$\boxed{y'=f'(x)=6\cdot x\cdot\cos(x^2)}$ ans

#17) $y=f(x)=\sec(t^2-3)$; C.R. $u=t^2-3$; $\frac{du}{dt}=2t$

$y=\sec(u)$; $\frac{dy}{du}=\frac{\sin(u)}{\cos^2(u)}$

$\boxed{y'=f'(x)=2t\cdot\frac{\sin(t^2-3)}{\cos^2(t^2-3)}}$ ans

note: another form $y'=2t\cdot\sec(t^2-3)\cdot\tan(t^2-3)$ E.C.E.

#36) $y=f(x)=\sin(x)\cdot\cos(x)$; P.R. $u=\sin(x)$, $v=\cos(x)$

$\boxed{y'=f'(x)=\cos^2(x)-\sin^2(x)}$ ans

#44) $y=g(x)=\frac{1}{\sin(3x)}=\csc(3x)$; C.R. $z=3x$, $y=\csc(z)$

$y'=g'(x)=3\cdot\csc(3x)\cdot\cot(3x)$

or

$\boxed{y'=3\cdot\frac{\cos(3x)}{\sin^2(3x)}}$ ans

#48) $y=h(x)=\csc(3x)\cdot\cot(3x)=\frac{\cos(3x)}{\sin^2(3x)}$

Let: $z=3x$, $u=\frac{\cos(z)}{\sin(z)}$, $v=\frac{1}{\sin(z)}$ use C.R. & P.R.

$$y'=\frac{dy}{dz}\cdot\frac{dz}{dx}=\left[\left(\frac{\cos(z)}{\sin(z)}\right)\cdot\left(\frac{-\cos(z)}{\sin^2(z)}\right)+\left(\frac{1}{\sin(z)}\right)\cdot\left(\frac{-1}{\sin^2(z)}\right)\right]\cdot(3)$$

$\therefore \boxed{y'=-3\cdot\left(\frac{1+\cos^2(3x)}{\sin^3(3x)}\right)}$ ans

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Neu 3^{rd} ed, p. 181, sec 4.6
Neu 2^{nd} ed, p. 221, sec 4.6.1

#21) $y = f(x) = \sin(e^x)$; C.R. $w = e^x$, $y = \sin(w)$

$$y' = \frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{dx} = \cos(w) \cdot e^x$$

$$\therefore \boxed{y' = f'(x) = e^x \cdot \cos(e^x)} \text{ ans}$$

#28) $y = g(s) = \exp[\tan(s^3)]$ note: $e^x \equiv \exp(x)$

C.R.
$z = s^3$; $dz/ds = 3s^2$
$w = \tan(z)$; $dw/dz = 1/\cos^2(z) = \sec^2(z)$
$y = e^w$; $dy/dw = e^w$

$$y' = g'(s) = \frac{dy}{ds} = \frac{dy}{dw} \cdot \frac{dw}{dz} \cdot \frac{dz}{ds} = e^w \cdot \sec^2(z) \cdot 3s^2$$

$$\therefore \boxed{y' = g'(s) = 3s^2 \left(\frac{\exp[\tan(s^3)]}{\cos^2(s^3)} \right)} \text{ ans}$$