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Neu 3rd ed., sec. 4.7, p. 192
Neu 2nd ed., sec. 4.7, p 233

#20) Find $d(f^{-1}(x))/dx$ at $x=1$, if $f(*)=(*)+e^{-(*)^2}$

<u>Note</u>: $*$ can bereplaced by any variable, i.e. x, y, t etc.

Use y for $*$, then let $x = f(y) = y + e^{-y^2}$

The inverse function of $f(*)$ is then $\underline{y = g(x) = f^{-1}(x)}$

Since $g(x)$ is not easy to find by normal algebraic manuvers, use the inv. func. rule $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$ I.R.

$\frac{dx}{dy} = 1 - 2ye^{-y^2}$, hence $\frac{dy}{dx} = \frac{1}{1-2ye^{-y^2}}$

But what is the value of y when $x = 1$?

Or $1 = y + e^{-y^2}$, what is y? Try $y = 0$, $1 = 0 + e^0$, it works! So when $x = 1$, $y = 0$. Use $y = 0$ to find dy/dx at $y = 0$. Since $1 - 2(0)e^0 = 1$, then $dy/dx = 1/1$ at $y = 0$

$\therefore \boxed{d(f^{-1}(x))/dx = 1}$ at $x = 1$
ans.

(This problem is not easy! It requires a solid understanding of inverse functions and the ambiguous notation, $f^{-1}(*)$ for inverse func., i.e. functions occur in pairs, $f(*)$ and $g(*)$; and if $y = f(g(y))$ or $x = g(f(x))$ then f and g are said to be inverses of each other; more over $f^{-1}(*)$ is just another "name" for $g(*)$)

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#31) $y = (\ln(x))^2 \equiv \ln^2(x)$ — $\underline{\text{C.R.}}$ $u = \ln(x)$, $y = u^2$

$u'(x) = \frac{1}{x}$; $y'(u) = 2u$

$\therefore \boxed{y' = \frac{2}{x} \cdot \ln(x)}$ ans

#52) $y = h(s) = \ln(\ln s)$ — $\underline{\underline{\text{C.R.}}}$ $u = \ln(s)$, $y = \ln(u)$

$u'(s) = \frac{1}{s}$; $y'(u) = \frac{1}{u}$

$\therefore \boxed{y' = \frac{1}{s \cdot \ln(s)}}$ ans

#65) $y = (\ln(x))^x \equiv \ln^x(x)$ use ECE "trick"

$y = u^v = e^{\ln(u^v)} = e^{v \cdot \ln(u)} = e^p \qquad y = e^{\ln(y)}$

where $u = \ln(x)$, $v = x$ and $p = x \cdot \ln(\ln(x))$

Now $y' = p' \cdot e^p$ and $p' = \ln(\ln(x)) + x\left(\frac{1}{x \cdot \ln x}\right)$, $\underline{\underline{\text{P.R.}}}$

then $\boxed{y' = \left(\ln(\ln(x)) + \frac{1}{\ln(x)}\right) \cdot (\ln(x))^x}$ ans

Neu. 3$^{\underline{rd}}$ ed, sec. 4.4, p. 172
Neu. 2$^{\underline{nd}}$ ed, sec. 4.4.5, p. 208

#50) $x \cdot y - y^3 = 1$, find y' — $\underline{\text{note}}$: it is hard to solve for y

Let $F(x,y) = x \cdot y - y^3 - 1 = p(x,y) - q(y) - 1$, then

$$\frac{dF}{dx} = \frac{dp}{dx} - \frac{dq}{dx} = \underbrace{(x \cdot y' + 1 \cdot y)}_{\text{P.R.}} - \underbrace{3y^2(y')}_{\text{C.R.}} = 0$$

Solve for y': $(x - 3y^2)\,y' = -y$

$\therefore \boxed{y' = -\frac{y}{x - 3y^2}}$ ans.

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#74) $y=(2x^2+4)^3$, find y'' <u>C.R.</u> for y'

$$y'=3(2x^2+4)^2\cdot 2(2x)=12x(2x^2+4)^2$$

$$y''=12\{(2x^2+4)^2+x(2x^2+4)\cdot 2(2x)\}$$ <u>P.R.</u> & <u>C.R.</u> for y''

or $\boxed{y''=48(x^2+2)(3x^2+2)}$ ans

*Neu 2nd ed $y=(x^2-3)^2$; $y'=4x(x^2-3)$

$$y''=12(x^2-1)$$

#78) $y=\frac{1}{x^2}+x-x^3$ find y''

$$y'=1-\frac{2}{x^3}-3x^2;\quad y''=\frac{6}{x^4}-6x=\boxed{6\left(\frac{1}{x^4}-x\right)}$$

*Neu 2nd ed $y=\frac{1}{x^2}+x-1$; $y'=1-\frac{2}{x^3}$; $y''=\frac{6}{x^4}$ ans

Neu 3rd ed, Review problems, p200
Neu 2nd ed, Review problems, p243 ⇦

#6) $y=\frac{\sin(3s+1)}{\cos(3s)}$ <u>Q.R.</u> $u=\sin(3s+1)$; $v=\cos(3s)$

$$\boxed{y'=\frac{3\{\cos(3s)\cos(3s+1)-\sin(3s)\sin(3s+1)\}}{\cos^2(3s)}}$$ ans.

#8) $y=e^{-x}\cdot\ln(x+1)$ <u>P.R.</u> $u=e^{-x}$

$v=\ln(x+1)$

$$\boxed{y'=\left(\frac{1}{x+1}-\ln(x+1)\right)e^{-x}}$$ ans.

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#30) (a) $y = \sinh(x) = \frac{1}{2}(e^{x} - e^{-x})$

$y' = \frac{1}{2}e^{x} - \frac{1}{2}(-e^{-x})$

or $y' = \frac{1}{2}(e^{x} + e^{-x}) = \cosh(x)$

$$\boxed{\therefore \frac{d}{dx}(\sinh x) = \cosh x}$$ ans

$y = \cosh(x) = \frac{1}{2}(e^{x} + e^{-x})$

$y' = \frac{1}{2}e^{x} - \frac{1}{2}e^{-x} = \frac{1}{2}(e^{x} - e^{-x})$

$$\boxed{\frac{d}{dx}(\cosh x) = \sinh x}$$ ans

(b) $\frac{d}{dx}(\tanh x) = \frac{d}{dx}\left(\frac{\sinh x}{\cosh x}\right)$

Q.R. $u = \sinh x \;;\; u' = \cosh x$

$v = \cosh x \;;\; v' = \sinh x$

$vu - u \cdot v = \cosh^2 x - \sinh^2 x = 1$

and $\frac{vu' - u \cdot v}{v^2} = \frac{1}{\cosh^2 x}$

$$\boxed{\therefore \frac{d}{dx}(\tanh x) = \frac{1}{\cosh^2 x}}$$ ans

note: the functions $\frac{1}{2}(e^{x} - e^{-x})$ and $\frac{1}{2}(e^{x} + e^{-x})$ have derivative patterns similar to trig. function and that is why they are called hyperbolic sine and hyperbolic cosine