QSci 291 • answers • Hmwk #7

Neu 3^{rd} ed, sec. 5.7, p. 266
Neu 2^{nd} ed, sec. 5.7.1, p 324

#1) find root of $y = x^2 - 7$ to 6 digits

$$\hat{x} = X - \frac{f(X)}{f'(X)} \quad \text{or} \quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Neuhauser notation

- iteration formula:

$$x_{i+1} = x_i - \left(\frac{x_i^2 - 7}{2x_i}\right) = \frac{x_i^2 + 7}{2x_i}$$

- initial guess: $x_0 = 3$ (or $X = 3$)

EXCEL RESULTS

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$(x_{i+1} - x_i) \times 10^6$
0	3	2	6	2.666667	333333.3333
1	2.666667	0.111111	5.333333	2.645833	20833.33333
2	2.645833	0.000434	5.291667	2.645751	82.02099737
3	2.645751	6.73E-09	5.291503	2.645751	0.001271367
4	2.645751	0	5.291503	2.645751	0

note: roots are $\sqrt{7}$ & $-\sqrt{7}$

$\boxed{x_4 = 2.645751 = \sqrt{7}}$ ans

#2) find root of $y = e^{-x} - x$

$f(x) = e^{-x} - x$

$f'(x) = -(e^{-x} + 1)$

- iteration formula:

$$x_{i+1} = x_i + \left(\frac{e^{-x_i} - x_i}{e^{-x_i} + 1}\right)$$

- guess: $x_0 = \frac{1}{2}$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$(x_i - x_{i+1}) \times 10^6$
0	0.5	0.106531	-1.60653	0.566311	-66311.0032
1	0.566311	0.001305	-1.56762	0.567143	-832.1618376
2	0.567143	1.96E-07	-1.56714	0.567143	-0.125374919
3	0.567143	4.44E-15	-1.56714	0.567143	-2.88658E-09
4	0.567143	0	-1.56714	0.567143	0

EXCEL RESULTS

$\boxed{x_4 = 0.567143}$ ans

QSci 291 • answers • Hmwk #7

#3) find root of $y = \ln(x) + x^2$

$f(x) = \ln(x) + x^2$

$f'(x) = \frac{1}{x} + 2x$

• iteration formula:

$$x_{i+1} = x_i - \left(\frac{x_i \cdot \ln(x_i) + x_i^3}{1 + 2x_i^2}\right)$$

• guess: $x_0 = \frac{1}{2}$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$(x_i - x_{i+1}) \times 10^6$
0	0.5	-0.44315	3	0.647716	-147715.7269
1	0.647716	-0.01477	2.839319	0.652917	-5201.145472
2	0.652917	-5E-06	2.837422	0.652919	-1.768094033
3	0.652919	-5.4E-13	2.837422	0.652919	-1.90403E-07
4	0.652919	0	2.837422	0.652919	0

EXCEL RESULTS

$\boxed{x_4 = 0.652919}$ ans

#5) find root of $y = \sin(x) - \frac{1}{2}x$

$f(x) = \sin(x) - \frac{1}{2}x$

$f'(x) = \cos(x) - \frac{1}{2}$

• iteration formula:

$$x_{i+1} = x_i - \left(\frac{2 \cdot \sin(x_i) - x_i}{2\cos(x_i) - 1}\right)$$

• guess: $x_0 = 2$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$(x_i - x_{i+1}) \times 10^6$
0	2	-0.0907	-0.91615	1.900996	99004.4058
1	1.900996	-0.00452	-0.82423	1.895512	5483.948824
2	1.895512	-1.4E-05	-0.81904	1.895494	17.37817088
3	1.895494	-1.4E-10	-0.81902	1.895494	0.000174732
4	1.895494	0	-0.81902	1.895494	0

EXCEL RESULTS

$\boxed{x_4 = 1.895494}$ ans

note: -1.895494 is also a root

QSci 291 • answers • Hmwk #7

Neu 3rd ed., sec. 5.3, p. 235
Neu 2nd ed., sec. 5.3.4, p. 287

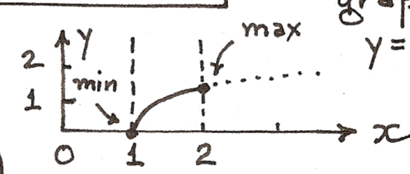
#2) $y = \sqrt{x-1}$; $1 \le x \le 2$

$$y' = \frac{1}{2\sqrt{x-1}} = 0$$ only when $x \to \infty$

no crit. pt.

$\boxed{\text{• NO local max. or min.}}$

graph: $y = \sqrt{x-1}$



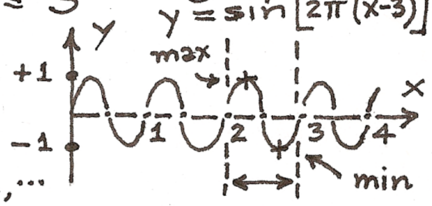
• absolute max. at $x = 2$ ($y = 1$)
• absolute min. at $x = 1$ ($y = 0$)

ans.

note: Neu 2nd ed $y = \sqrt{x}$; $0 \le x \le 4$
No local max or min
absol. max. at $x = 4$ ($y = 2$)
absol. min. at $x = 0$ ($y = 0$)

#10) $y = \sin[2\pi(x-3)]$; $2 \le x \le 3$

graph: $y = \sin[2\pi(x-3)]$



$$y' = 2\pi \cos[2\pi(x-3)] = 0$$

note: $\cos(z) = 0$ if $z = (2n+1)\frac{\pi}{2}$ for $n = \cdots, -2, -1, 0, +1, \cdots$

for $n = -1$, $z = -\frac{\pi}{2} = 2\pi(x-3)$ or $x = 3 - \frac{1}{4} = 2.75$
for $n = -2$, $z = -\frac{3\pi}{2} = 2\pi(x-3)$ or $x = 3 - \frac{3}{4} = 2.25$

crit. pts.

$$y'' = -(2\pi)^2 \sin[2\pi(x-3)]$$

at $x = 2.25$, $y'' = -1$ $\therefore$ y is a maximum
at $x = 2.75$, $y'' = +1$ $\therefore$ y is a minimum

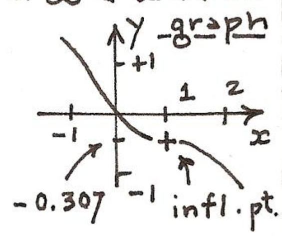
• local max. at $x = 2.25$ (also absol. max)
• local min. at $x = 2.75$ (also absol. min)

ans

QSci 291 • answers • Hmwk #7

#10) cont'd   note Neu 2nd ed $y = \ln(1+x^2) - x$ for $-\infty < x < +\infty$

$$y' = \frac{2x}{x^2+1} - 1 = \frac{-(x-1)^2}{x^2+1}$$



$$y'' = -\frac{(x^2+1)2(x-1)-(x-1)^2(2x)}{(x^2+1)^2} = 2\frac{(x-1)^2}{(x^2+1)^2}$$

crit. pt.

• $y' = 0$ at $x = 1$ but $y''(1) = 0$

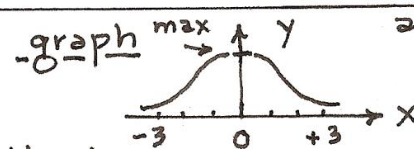
since $y'(.99)$ is neg. and $y'(1.01)$ is neg.,

then $y = \ln(1+x^2) - x$ has an Infl. Pt. at $x = 1$

$\boxed{\text{There are no local or absolute max's or min's}}$

ans

#12) $y = e^{-\frac{1}{4}x^2}$



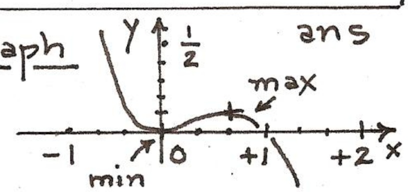
$y' = -\frac{1}{2}x \cdot e^{-\frac{1}{4}x^2}$ crit. pt.

$y'' = \frac{1}{2}\left(\frac{1}{2}x^2 - 1\right)e^{-\frac{1}{4}x^2}$; $y' = 0$ at $x = 0$; $y''(0) = -1$ neg.

$\boxed{\therefore\ y = e^{-\frac{1}{4}x^2} \text{ has a local and absolute max at } x = 0}$

ans

#14) $y = x^2(1-x) = x^2 - x^3$ - graph



$y' = x(2-3x)$

$y'' = 2(1-3x)$

crit. pts.

$y' = 0$ at $x = 0$ and $x = \frac{2}{3}$; $y''(0) = +2$ pos. ; $y''\left(\frac{2}{3}\right) = -2$ neg.

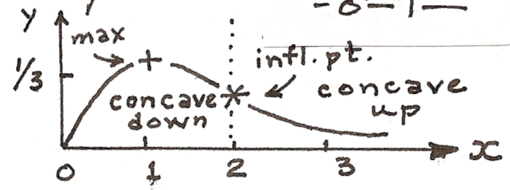
$\boxed{\begin{array}{r} \therefore\ y = x^2(1-x) \text{ has a local minimum at } x = 0 \\ \text{and has a local maximum at } x = \frac{2}{3} \end{array}}$

ans

QSci 291 • answers • Hmwk #7

#22) find inflection pt. of $y = xe^{-x}$

-graph



$y' = (1-x)e^{-x}$

$y'' = -(2-x)e^{-x}$

$y''' = (3-x)e^{-x}$

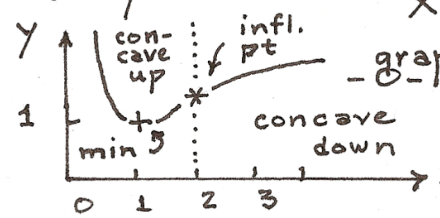
$y'' = 0$ at $x = 2$ (crit. pt. of y'); $y'''(2) = \frac{1}{e^2} \neq 0$

$$\boxed{\therefore\ y = x \cdot e^{-x} \text{ has an inflection point at } x = 2}$$

ans

#24) find inflection pt. of $y = \ln x + \frac{1}{x}$

-graph



$y' = \frac{1}{x} - \frac{1}{x^2} = \frac{1}{x^2}(x-1)$

$y'' = -\frac{1}{x^2} + \frac{2}{x^3} = -\frac{1}{x^3}(x-2)$

$y''' = \frac{2}{x^3} - \frac{6}{x^4} = \frac{2}{x^4}(x-3)$

$y'' = 0$ at $x = 2$ (crit. pt. of y'); $y'''(2) = -\frac{1}{8} \neq 0$

$$\boxed{\therefore\ y = \ln(x) + \frac{1}{x} \text{ has an inflection pt at } x = 2}$$

ans