QSci 291 • answers • Hmwk #9

parametric equation problems (on webpage)

#1) $x = \sqrt{t-4}$ & $y = t+1$

$\frac{dx}{dt} = \frac{1}{2\sqrt{t-4}}$; $\frac{dy}{dt} = 1$

$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$; $\boxed{y' = 2\sqrt{t-4}}$ ans

Note: $t = y-1$, so $x^2 = t-4 = y-5$ $\therefore y = x^2+5$ and $y' = 2x = 2\sqrt{t-4}$

#2) $x = \sqrt{t}+1$; $y = \sqrt[3]{t} = t^{1/3}$

$\frac{dx}{dt} = \frac{1}{2\sqrt{t}}$; $\frac{dy}{dt} = \frac{1}{3}t^{-2/3}$

$\frac{dy}{dx} = \frac{(1/3)\,t^{-2/3}}{(1/2)\,t^{-1/2}}$; $\boxed{y' = \frac{2}{3}t^{-1/6}}$ ans

Note: $t = (x-1)^2$ $\therefore y = (x-1)^{2/3}$

$y' = \frac{2}{3}(x-1)^{-1/3}$ or $y' = \frac{2}{3}\left[t^{1/2}\right]^{-1/3} = \frac{2}{3}t^{-1/6}$

#3) $x = 5(1-e^{-t})$; $y = 7(1-e^{-t}) - 3t$

$\dot{x} = 5e^{-t}$; $\dot{y} = 7e^{-t} - 3$

$\frac{dy}{dx} = \frac{7e^{-t}-3}{5e^{-t}}$ or $\boxed{y' = \frac{7}{5} - \frac{3}{5}e^{t}}$ ans

#4) $x = \frac{1-t^2}{1+t^2}$; $y = \frac{2t}{1+t^2}$

$\dot{x} = \frac{-4t}{(1+t^2)^2}$; $\dot{y} = \frac{2(1-t^2)}{(1+t^2)^2}$ Q.R.

$$\boxed{\frac{dy}{dx} = -\frac{1}{2}\left(\frac{1-t^2}{t}\right)}$$

note: since $1-t^2 = x(1+t^2)$ and $2t = y(1+t^2)$

$y' = -\frac{x}{y}$

QSci 291 • answers • Hmwk #9

Neu 3rd ed, sec. 10.3, p. 525
Neu 2nd ed, sec. 10.3.4, p631

#2) $z = f(x,y) = 2x\sqrt{y} - (3/xy^2)$

Let: $y \to b$, then

$z = (2\sqrt{b})x - (3/b^2)\cdot\frac{1}{x}$;

$\frac{dz}{dx} = 2\sqrt{b} - (3/b^2)(-1/x^2)$;

Replace $b \to y$, and

$$\boxed{\frac{\partial z}{\partial x} = 2\sqrt{y} - \left(\frac{3}{x^2 y}\right)}$$ ans

Let: $x \to a$, then

$z = (2a)\sqrt{y} - \left(\frac{3}{a}\right)\left(\frac{1}{y^2}\right)$;

$\frac{dz}{dy} = (2a)\left(\frac{1}{2\sqrt{y}}\right) - \left(\frac{3}{a}\right)\left(-\frac{2}{y^3}\right)$

Replace $a \to x$, and

$$\boxed{\frac{\partial z}{\partial y} = \frac{x}{\sqrt{y}} + \frac{6}{xy^3}}$$ ans

note: Neu 2nd ed: $z = 2x\sqrt{y} - (1/xy)$, then

$$\boxed{\frac{\partial z}{\partial x} = 2\sqrt{y} + \frac{1}{x^2 y}}$$ ans

$$\boxed{\frac{\partial z}{\partial y} = \frac{x}{\sqrt{y}} + \frac{1}{xy^2}}$$ ans

#6) $z = f(x,y) = \tan(x - 2y)$;

Let: $w = x - 2b$

$z = \tan(w)$ C.R.

$\frac{dz}{dx} = \frac{dz}{dw}\cdot\frac{dw}{dx} = \frac{1}{\cos^2(w)}\cdot(1)$

$$\boxed{\frac{\partial z}{\partial x} = \frac{1}{\cos^2(x-2y)} = \sec^2(x-2y)}$$ ans

Let: $w = a - 2y$

$z = \tan(w)$ C.R.

$\frac{dz}{dy} = \frac{dz}{dw}\cdot\frac{dw}{dy} = \frac{1}{\cos^2(w)}(-2)$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-2}{\cos^2(x-2y)} = -2\sec^2(x-2y)}$$ ans

note: Neu 2nd ed: $z = \tan(x - y)$

$$\boxed{\frac{\partial z}{\partial x} = \frac{1}{\cos^2(x-y)} = \sec^2(x-y)}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-1}{\cos^2(x-y)} = -\sec^2(x-y)}$$

QSci 291 • answers • Hmwk #9

#10) $z = f(x,y) = x^2 e^{-\frac{1}{2}xy}$

Let: $z = x^2 \cdot e^{-\frac{b}{2}y}$, then

$$\frac{dz}{dx} = (2x)e^{-\frac{b}{2}x} + (x^2)\left(-\frac{b}{2}e^{-\frac{b}{2}x}\right)$$

$$\boxed{\frac{\partial z}{\partial x} = 2x\left(1 - \frac{1}{4}xy\right)e^{-\frac{1}{2}xy}}$$ ans

Let: $z = a^2 \cdot e^{-\frac{a}{2}y}$

$$\frac{dz}{dy} = a^2\left(-\frac{a}{2}e^{-\frac{a}{2}y}\right)$$

$$\boxed{\frac{\partial z}{\partial y} = -\frac{1}{2}x^3 \cdot e^{-\frac{1}{2}xy}}$$ ans

note: Neu. 2nd ed.: $z = x^2 e^{-xy}$

$$\boxed{\frac{\partial z}{\partial x} = 2x\left(1 - \frac{1}{2}xy\right)e^{-xy}}$$ ans

$$\boxed{\frac{\partial z}{\partial y} = -x^3 e^{-xy}}$$ ans

#12) $z = f(x,y) = \cos(x^2 - y^2)\, e^{-y^2}$

Let: $w = x^2 - b^2$

$$z = \cos(w)e^{-b^2}$$

$$\frac{dz}{dx} = \left(e^{-b^2}\right)\left(\frac{d\cos(w)}{dw} \cdot \frac{dw}{dx}\right)$$

$$\boxed{\frac{\partial z}{\partial x} = -2x \cdot \sin(x^2 - y^2) \cdot e^{-y^2}}$$ ans

Let: $w = a^2 - y^2$

$$z = \cos(w)e^{-y^2}$$

$$\frac{dz}{dy} = \left[\left(\frac{d\cos(w)}{dw} \cdot \frac{dw}{dy}\right) \cdot e^{-y^2} + \cos(w) \cdot \left(-2y e^{-y^2}\right)\right]$$

$$\boxed{\frac{\partial z}{\partial y} = 2y \cdot \left[\sin(x^2 - y^2) - \cos(x^2 - y^2)\right] \cdot e^{-y^2}}$$ ans

note: Neu 2nd ed.: $z = \cos(x^2 + y^2)\, e^{-y^2}$

$$\boxed{\frac{\partial z}{\partial x} = -2x \cdot \sin(x^2 + y^2) \cdot e^{-y^2}}$$ ans

$$\boxed{\frac{\partial z}{\partial y} = -2y\left[\sin(x^2 + y^2) + \cos(x^2 + y^2)\right]e^{-y^2}}$$ ans

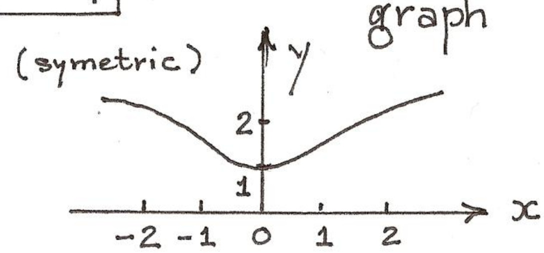
QSci 291 • answers • Hmwk #9

Neu. 3rd ed., sec. 5.3, p. 235
Neu. 2nd ed., sec. 5.3.4, p. 287

#16) $y = \sqrt{1+x^2}$

$$y' = x/\sqrt{1+x^2}$$

$$y'' = 1/\left(\sqrt{1+x^2}\right)^3$$



Set $y'=0$, then crit. pt. is at $x=0$
Since $y''(0)=+1$, y has a local MIN at $x=0$
coordinates of MIN pt.: $(0,1)$
Note: No MAX pts; local MIN is the same as the absolute MIN for this problem.
(for $-\infty < x \le 0$, y decreases; for $0 \le x < +\infty$, y increases)

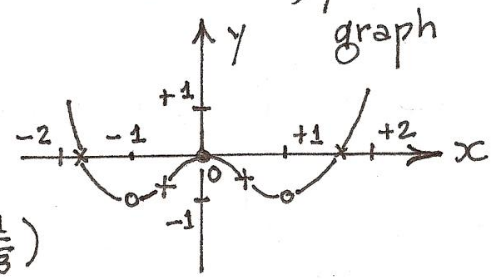
#28) $y = x^4 - 2x^2$ or

$$y = x^2(x-\sqrt{2})(x+\sqrt{2})$$

$$y' = 4x^3 - 4x = 4x(x-1)(x+1)$$

$$y'' = 12x^2 - 4 = 12\left(x-\tfrac{1}{\sqrt{3}}\right)\left(x+\tfrac{1}{\sqrt{3}}\right)$$

$$y''' = 24x$$



Set $y'=0$, three crit. pts: 1) $x=0$, 2.) $x=-1$ & 3) $x=+1$
at $x=0$, $y=0$ & $y''(0)=-4$; y has a local MAX
at $x=-1$, $y=-1$ & $y''(-1)=+8$; y has a local MIN
at $x=+1$, $y=-1$ & $y''(+1)=+8$; y has a local MIN
The absolute MIN is at either $x=-1$ or $x=+1$
There is no absolute MAX
(y dec. in $-\infty < x < -1$ and in $0 < x < +1$;
y inc. in $-1 < x < 0$ and in $1 < x < +\infty$)
note: infl. pts at $x = \pm 1/\sqrt{3}$