

## EXERCISES 4.5

Find  $dy$  for the following functions:

1.  $y = x^2 + 7x + 1$ .
2.  $y = (t^2 + 1)^4$ .
3.  $y = t \ln t$ .
4.  $y = ue^{-u}$ .
5.  $y = \ln(z^2 + 1)$ .
6.  $y = \frac{x+1}{x^2+1}$ .
7.  $xy + x^2 = 3$ .
8.  $yz^2 + z^2 - 4yz = 1$ .
9.  $\ln(yz) = 1 + z$ .
10.  $xe^y + e^x = 1$ .
11. Find  $dy$  for  $y = x^3$  when  $x = 2$ ,  $dx = 0.01$ .
12. Find  $du$  for  $u = t^2 + 3t + 1$  when  $t = -1$ ,  $dt = 0.02$ .
13. Find  $dx$  for  $x = y \ln y$  when  $y = 1$ ,  $dy = 0.003$ .
14. Find  $df$  for  $f(x) = xe^x$  when  $x = 0$ ,  $\Delta x = -0.01$ .

Find  $dy$  and  $\Delta y$  for the following functions:

15.  $y = 3x^2 + 5$  when  $x = 2$ ,  $\Delta x = 0.01$ .
16.  $y = \sqrt{t}$  when  $t = 4$ ,  $dt = 0.41$ .
17.  $y = \ln u$  when  $u = 3$ ,  $du = 0.06$ .
18.  $y = \sqrt{x+2}$  when  $x = 2$ ,  $dx = 0.84$ .
19. Use differentials to approximate the cube root of 9.
20. Use differentials to approximate the fourth root of 17.
21. Use differentials to approximate the fifth root of 31.
22. Use differentials to approximate the value of  $(4.01)^3 + \sqrt{4.01}$ .
- \* 23. The radius of a sphere is equal to 8 cm with a possible error of  $\pm 0.002$  cm. The volume is calculated assuming that the radius is exactly 8 cm. Use differentials to estimate the maximum error in the calculated volume.
- \* 24. Use differentials to determine the approximate increase in the surface area of a soap bubble when its radius increases from 2 cm to 2.001 cm.
25. If the volume of a sphere is to be determined to within a percentage error that does not exceed 2%, what is the maximum percentage error that can be allowed in the measured value of the radius?
26. The acceleration due to gravity,  $g$ , is determined by measuring the period of swing of a pendulum. If the length of the pendulum is  $l$  and the measured period is  $T$ , then  $g$  is given by the formula

$$g = \frac{4\pi^2 l}{T^2}$$

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Find the percentage error in  $g$  if:

- (a)  $l$  is measured accurately but  $T$  has an error of 1%, and
- (b)  $T$  is measured accurately but  $l$  has an error of 2%.

\* volume of a sphere is :  $V = \frac{4}{3} \pi r^3$   
 \* surface area of a sphere is :  $A = 4\pi r^2$