Homework #1

QSci 291
Fall 2000

Due date: 6 Oct 2000

jjohnson@u

$$L = \lim_{k \to \infty} (R_k)$$

- In each of the problems below compute some members of the sequence of ratios R_k to convince yourself that the limit value of the sequence is the same as that given.

Sequence of Ratios, R_k	Limit, L
1. $R_k = \left(1 + \frac{1}{k}\right)^k$	$e = 2.718\ldots$
2. $R_k = \dfrac{4^{\frac{1}{k}} - 1}{\left(\frac{1}{k}\right)}$	$\ln(4) = 1.38629\ldots$
3. $R_k = k \cdot \ln\left(\frac{k+1}{k}\right)$	1
4. $R_k = \dfrac{\tan\left(\frac{1}{k}\right)}{\left(\frac{1}{k}\right)}$	1

<u>note</u>: make sure to use radian mode for the tangent function

$$L = \lim_{n\to\infty} (S_n) = \lim_{n\to\infty} \left(\sum_{k=k_0}^{n} t_k\right)$$

- In each of the problems below compute enough terms in the sequence S_n to convince yourself that the infinite series, with terms t_k, converge to the limit value L.

Sequence of Sums, S_n	Limit, L
5. $S_n = \sum_{k=0}^{n} \left(\frac{1}{7}\right)^k$	$\frac{7}{6}$
6. $S_n = \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k$	3
7. $S_n = \sum_{k=1}^{n} \left(\frac{6}{k^2}\right)$	$\pi^2 = (3.1415927)^2$
8. $S_n = \sum_{k=1}^{n} \frac{4}{k(k+1)(k+2)}$	1
9. $S_n = \sum_{k=0}^{n} (-1)^k \frac{1}{(2k+1)\cdot 3^k}$	$\frac{\sqrt{3}}{6}\pi$
10. $S_n = \sum_{k=0}^{n} \left(\frac{1}{k!}\right)$	$e = 2.71828\ldots$

<u>note</u> : $k! = k \cdot k-1 \cdot k-2 \cdot \cdots 3 \cdot 2 \cdot 1$

$(0! \equiv 1)$

read as : k factorial