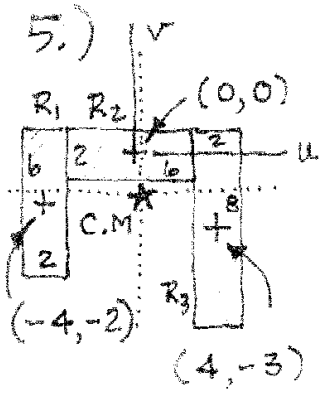
QSci 292 • answers • Hmwk #10

Center of Mass Problems

5.)



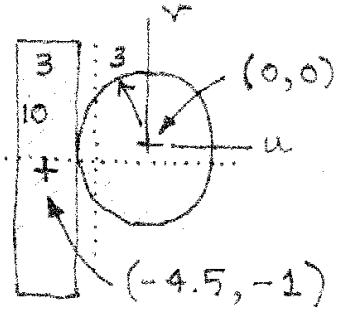
sub-R	$\bar{u}$	$\bar{v}$	A	$\bar{u}\cdot A$	$\bar{v}\cdot A$
1	0	0	12	0	0
2	-4	-2	12	-48	-24
3	4	-3	16	64	-48
sums:			40	16	-72
			I_1 (area)	I_2 (M_v)	I_3 (M_u)

$$\bar{u} = \frac{I_2}{I_1} = \frac{16}{40} = .0.4$$

$$\bar{v} = \frac{I_3}{I_2} = \frac{-72}{40} = -1.8$$

C of M = (0.4, -1.8)
(relative to u,v system)

6.)



sub-R		$\bar{u}$	$\bar{v}$	A	$\bar{u}\cdot A$	$\bar{v}\cdot A$
(circle)	1	0	0	9π	0	0
(rect.)	2	-4.5	-1	30	-135	-30
sums:				58.274	-135	-30
				I_1	I_2	I_3

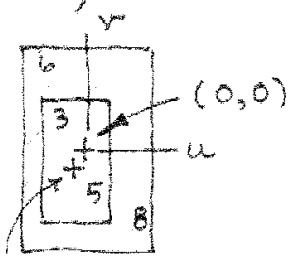
$$\bar{u} = \frac{-135}{58.274} = -2.31$$

$$\bar{v} = \frac{-30}{58.274} = -0.515$$

C of M = (-2.31, -0.515)
(u,v system)

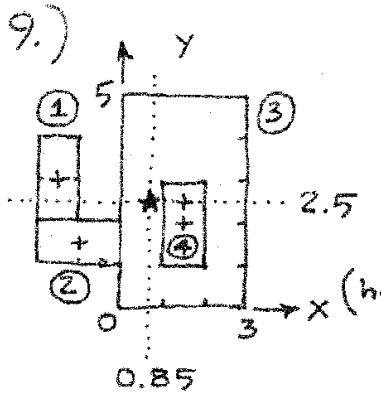
QSci 292 • answers • Hmwk #10

8.)



sub-R	$\bar{u}$	$\bar{v}$	A	$\bar{u}\cdot A$	$\bar{v}\cdot A$
1	0	0	48	0	0
(hole) 2	$-\frac{1}{3}$	$-\frac{1}{3}$	-15	+7.5*	+7.5*

9.)

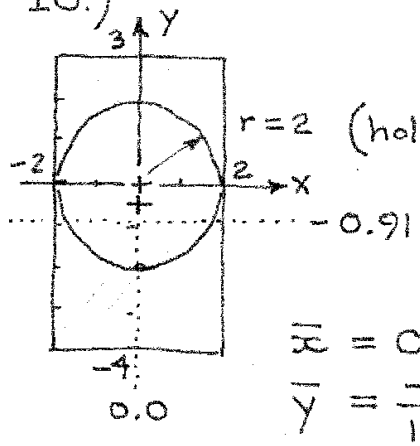


sub-R	$\bar{x}$	$\bar{y}$	A	$\bar{x}\cdot A$	$\bar{y}\cdot A$
1	-1.5	3.0	2	-3	6
2	-1.0	1.5	2	-2	3
3	1.5	2.5	15	22.5	37.5
(hole) 4	1.5	2.0	-2	-3	-4
sums :			17	14.5	42.5
			I_1	I_2	I_3

$$\bar{x} = \frac{14.5}{17} = 0.853$$

$$\bar{y} = \frac{42.5}{17} = 2.50 \qquad CM = (0.85, 2.50)$$

10.)



sub-R	$\bar{x}$	$\bar{y}$	A	$\bar{x}A$	$\bar{y}A$
(hole) 1	0	0	-4π	0	0
2	0	$-\frac{1}{2}$	28	0	-14
sums :			15.434	0	-14
			I_1	I_2	I_3

$$\bar{x} = 0$$

$$\bar{y} = \frac{-14}{15.434} = -0.908$$

QSci 292 • answers • Hmwk #10

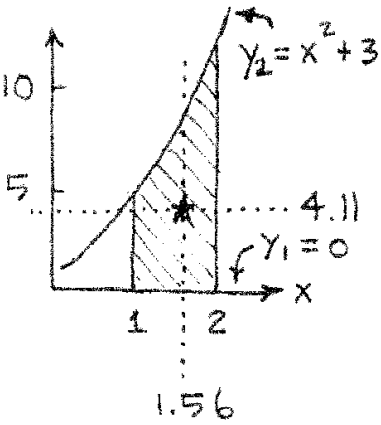
For problems 12, 13, 15 & 18, the 3 I's are:

$$I_1 = \int_a^b \int_0^{y_2} dy\,dx = \int_a^b y_2\,dx = \int_a^b f(x)\,dx$$

$$I_2 = \int_a^b \int_0^{y_2} x\,dy\,dx = \int_a^b x \cdot y_2\,dx = \int_a^b x \cdot f(x)\,dx$$

$$I_3 = \int_a^b \int_0^{y_2} y\,dy\,dx = \int_a^b \tfrac{1}{2} y_2^2\,dx = \tfrac{1}{2}\int_a^b [f(x)]^2\,dx$$

12.)



$$I_1 = \int_1^2 (x^2+3x+1)\,dx = \frac{47}{6}$$

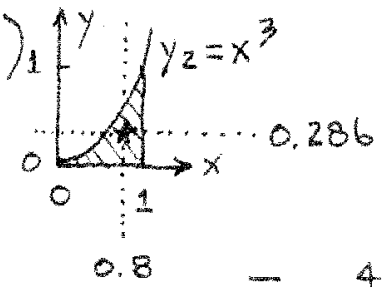
$$I_2 = \int_1^2 x(x^2+3x+1)\,dx = \frac{49}{4}$$

$$I_3 = \tfrac{1}{2}\int_1^2 (x^2+3x+1)^2\,dx = \frac{1931}{60}$$

note: $(x^2+3x+1)^2 = x^4+6x^3+11x^2+6x+1$

$$\bar{x} = \frac{49}{4}\cdot\frac{6}{47} = \underline{1.56}\;;\quad \bar{y} = \frac{1931}{60}\cdot\frac{6}{47} = \underline{4.11}$$

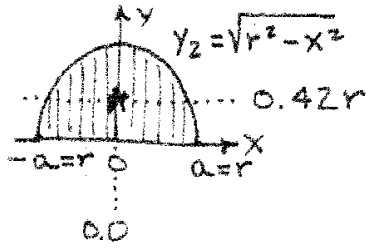
QSci 292 • answers • Hmwk #10

13.) 

$$I_1 = \int_0^1 x^3\,dx = \frac{1}{4}\;;\; I_2 = \int_0^1 x^4\,dx = \frac{1}{5}$$

$$I_3 = \frac{1}{2}\int_0^1 x^6\,dx = \frac{1}{14}$$

$$\bar{x} = \frac{4}{5} = \underline{0.8}\;;\; \bar{y} = \frac{4}{14} = \underline{0.286}$$

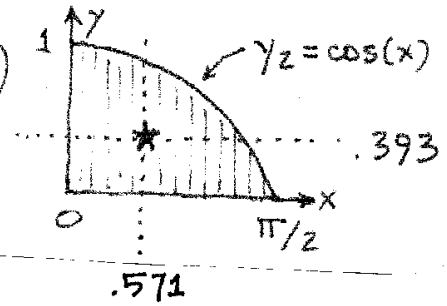
15.) 

$$I_1 = \frac{1}{2}\pi r^2$$

$$I_2 = 0 \quad \text{(by symmetry)}$$

$$I_3 = \frac{1}{2}\int_{-r}^{r}(r^2 - x^2)\,dx = \frac{2}{3}r^3$$

$$\underline{\bar{x} = 0}\;;\; \underline{\bar{y} = \frac{4}{3\pi}r}$$

18.) 

$$I_1 = \int_0^{\pi/2}\cos(x)\,dx = 1$$

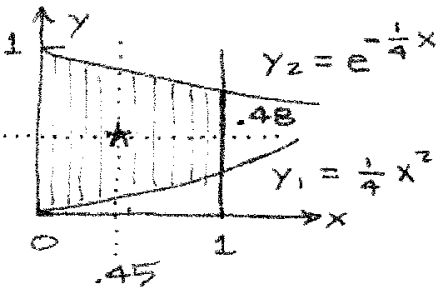
$$I_2 = \int_0^{\pi/2} x\cdot\cos(x)\,dx = \frac{\pi}{2} - 1$$

$$I_3 = \frac{1}{2}\int_0^{\pi/2}\cos^2(x)\,dx = \frac{\pi}{8}$$

$$\bar{x} = \frac{\pi}{2} - 1 = \underline{0.571}\;;\; y = \frac{\pi}{8} = \underline{0.393}$$

QSci 292 • answers • Hmwk #10

A)



$$I_1 = \int_0^1 \int_{y_1}^{y_2} dy\,dx$$

$$I_2 = \int_0^1 \int_{y_1}^{y_2} x\,dy\,dx$$

$$I_3 = \int_0^1 \int_{y_1}^{y_2} y\,dy\,dx$$

$$I_1 = \int_0^1 (y_2 - y_1)\,dx = \int_0^1 e^{-\frac{1}{4}x}\,dx - \int_0^1 \frac{1}{4}x^2\,dx$$

$$I_2 = \int_0^1 x(y_2 - y_1)\,dx = \int_0^1 x e^{-\frac{1}{4}x}\,dx - \int_0^1 \frac{1}{4}x^3\,dx$$

$$I_3 = \frac{1}{2}\int_0^1 (y_2^2 - y_1^2)\,dx = \frac{1}{2}\int_0^1 e^{-\frac{1}{2}x}\,dx - \frac{1}{2}\int \frac{1}{16}x^4\,dx$$

or

$$I_1 = 4\left(1 - \left(\tfrac{1}{e}\right)^{1/4}\right) - \frac{1}{12} = 0.8014$$

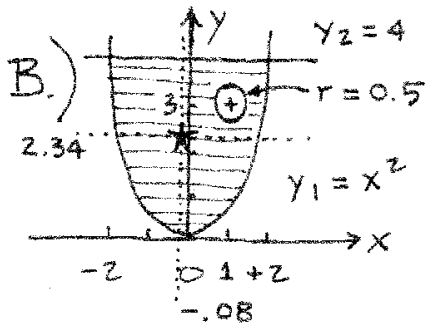
$$I_2 = 4\left(4 - 5\left(\tfrac{1}{e}\right)^{1/4}\right) - \frac{1}{16} = 0.3615$$

$$I_3 = \left(1 - \left(\tfrac{1}{e}\right)^{1/2}\right) - \frac{1}{160} = 0.3872$$

$$\bar{x} = \frac{.3615}{.8014} = \underline{0.45}\;;\quad \bar{y} = \frac{.3872}{.8014} = \underline{0.48}$$

QSci 292 • answers • Hmwk #10

B.)



parabolic boundary with a circular hole
[center of circle: (1, 3)]
area of circle: $\pi/4$

$$I_1 = \iint_{Q_1} dA - \iint_{Q_2} dA = \int_{-2}^{+2} (y_2 - y_1)\,dx - \frac{\pi}{4}$$

$$I_2 = \iint_{Q_1} x\,dA - \iint_{Q_2} x\,dA = \int_{-2}^{+2} x(y_2 - y_1)\,dx - \underbrace{(1)\frac{\pi}{4}}_{\bar{x}\cdot A}$$

$$I_3 = \iint_{Q_1} y\,dA - \iint_{Q_2} y\,dA = \frac{1}{2}\int_{-2}^{+2} (y_2^2 - y_1^2)\,dx - \underbrace{(3)\frac{\pi}{4}}_{\bar{y}\cdot A}$$

$$I_1 = 2\int_0^2 (4 - x^2)\,dx - \frac{1}{4}\pi = \frac{32}{3} - \frac{\pi}{4} = 9.88$$

$$I_2 = \underbrace{\int_{-2}^{0} x(4 - x^2)\,dx}_{-F} + \underbrace{\int_{0}^{+2} x(4 - x^2)\,dx}_{+F} - \frac{1}{4}\pi = -F + F - \frac{\pi}{4} = -0.7853$$

$$I_3 = 2\cdot\frac{1}{2}\int_0^2 (4^2 - x^4)\,dx - \frac{3}{4}\pi = \frac{128}{5} - \frac{3}{4}\pi = 23.14$$

$$\bar{x} = -0.079 \;;\; \bar{y} = 2.34$$