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Neu 3rd ed, sec 7.2, p. 342
Neu 2nd ed, sec 7.2.1, p. 414

#6) $F = \int x \cdot \sin(1-2x)\,dx$ (1st, S.M.; 2nd, I.P.)

Let: $z = 1-2x \leftrightarrow x = -\frac{1}{2}(1-z)$, $dz = -2\,dx$, $dx = -\frac{1}{2}dz$

$$\therefore F = -\frac{1}{4}\int (1-z)\sin(z)\,dz = -\frac{1}{4}\left(F_1 - F_2\right)$$

$$F_1 = \int \sin(z)\,dz = -\cos(z)$$

$$F_2 = \int z\sin(z)\,dz = -z\cdot\cos(z) - \underbrace{\int\left(-\cos(z)\right)dz}_{+\sin(z)}$$

where: $u = z$, $du = dz$; $dv = \sin(z)dz$, $v = -\cos(z)$

$$\therefore F = -\frac{1}{4}\left\{-\cos(z) - \left[-z\cos(z) + \sin(z)\right]\right\} + C$$

$$F = \frac{1}{4}\left\{(1-z)\cos(z) - \sin(z)\right\} + C$$

or $\boxed{F = \frac{1}{2}x\cos(1-2x) + \frac{1}{4}\sin(1-2x) + C}$ (S.M. & I.P.)

#6) Neu 2nd ed $F = \int x\cdot\sin(2x)\,dx$

Let: $w = 2x$, $dw = 2\,dx$, $dx = \frac{1}{2}dw$, then $F = \frac{1}{4}\int w\sin(w)\,dw$

With $u = w$; $du = dw$ & $dv = \sin(w)dw$; $v = -\cos(w)$

$$F = \frac{1}{4}\left\{-w\cdot\cos(w) - \int\left(-\cos(w)\right)dw\right\} = \frac{1}{4}\left\{-w\cos(w) + \sin(w)\right\}$$

$$F = \frac{1}{4}\sin(2x) - \frac{1}{2}x\cdot\cos(2x) + C$$ (S.M. & I.P.)

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#10) $F = \int 2x^2 e^{-x} dx = -2x^2 e^{-x} + 4\int x e^{-x} dx$

with: $u = x^2$; $du = 2x\,dx$ & $dv = e^{-x} dx$; $v = -e^{-x}$

$F_1 = \int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x}$

with: $u = x$; $du = x\,dx$ & $dv = e^{-x} dx$; $v = -e^{-x}$

$F = -2x^2 e^{-x} + 4\left\{-x e^{-x} - e^{-x}\right\} + C$

$\therefore \boxed{F = -2\left(x^2 + 2x + 2\right) e^{-x} + C}$ (I.P. twice)

#12) $F = \int x^2 \ln(x) dx = \frac{1}{3}x^3 \cdot \ln(x) - \int \frac{1}{3}x^3 \left(\frac{1}{x}\right) dx$

with: $u = \ln(x)$; $du = \frac{1}{x} dx$ & $dv = x^2 dx$; $v = \frac{1}{3}x^3$

$\boxed{F = \frac{1}{3}x^3 \left(\ln(x) - \frac{1}{3}\right) + C}$ (I.P.)

#15) $F = \int x \cdot \sec^2(x)\, dx = x \cdot \tan(x) - \int \tan(x)\, dx$

with: $u = x$; $du = dx$ & $dv = \sec^2(x) dx$; $v = \tan(x)$

$\boxed{F = x \cdot \tan(x) + \ln(\cos(x)) + C}$ (I.P.)

#30) $F = \int \cos(\ln(x))\, dx = \int \cos(y)\, e^y\, dy$

where: $y = \ln(x) \leftrightarrow x = e^y$; $dy = \frac{1}{x} dx$; $dx = e^y dy$

$F = \int \cos(y) e^y dy = \sin(y) e^y - \int \sin(y) e^y dy$

with: $u = e^y$; $du = e^y dy$ & $dv = \cos(y) dy$; $v = \sin(y)$

$F_1 = \int \sin(y) dy = -\cos(y) e^{-y} - \int (-\cos(y) e^y dy = -\cos(y) e^y + F$

with: $u = e^y$; $du = e^y dy$ & $dv = \sin(y) dy$; $v = -\cos(y)$

$F = \sin(y) e^y - \left\{\cos(y) e^y + F\right\}$ (S.M. I.P. twice)

$\boxed{F = \frac{1}{2}x\left\{\sin(\ln(x)) + \cos(\ln(x))\right\} + C}$

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#32) $F = \int \sin^2(x)\,dx = -\sin(x)\cdot\cos(x) + \int \cos^2(x)\,dx$

where: $u = s$; $du = c\,dx$ & $dv = s\,dx$; $v = -c$

with: $s = \sin(x)$ & $c = \cos(x)$ <u>note</u>: $s^2 + c^2 = 1$

but $F = -s\cdot c + \int (1 - s^2)\,dx = -sc + x - F$

$\therefore$ $\boxed{F = \frac{1}{2}\left(x - \sin(x)\cdot\cos(x)\right) + C}$ (I.P. & E.C.E.)

#33) $F = \int \arcsin(x)\,dx = x\cdot\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$

where: $u = \arcsin(x)$; $du = \frac{1}{\sqrt{1-x^2}}\,dx$ & $dv = 1\cdot dx$; $v = x$

but $F_1 = \int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{1}{\sqrt{w}}\cdot dw = -\frac{1}{2}\left(2\sqrt{w}\right) = -\sqrt{1-x^2}$

with: $w = 1 - x^2$, $dw = -2x\,dx$; $dx = -\frac{1}{2x}\,dw$

$\therefore$ $\boxed{F = x\cdot\arcsin(x) - \sqrt{1-x^2} + C}$ (I.P. & S.M.)

#39) $F = \int \cos(\sqrt{x})\,dx = 2\int y\cos(y)\,dy$

with: $y = \sqrt{x} \leftrightarrow x = y^2$; $dy = \frac{1}{2\sqrt{y}}\,dy$; $dx = 2y\cdot dy$

$F = 2\left\{y\cdot\sin(y) - \int \sin(y)\,dy\right.$

where: $u = y$; $du = dy$ & $dv = \cos(y)\,dy$; $v = \sin(y)$

$F = 2\left\{y\cdot\sin(y) + \cos(y)\right\} + C$

$\therefore$ $\boxed{F = 2\left\{\sqrt{x}\sin(\sqrt{x}) + \cos(\sqrt{x}) + C\right.}$ (S.M. & I.P.)

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#41) $F = \int x^3 e^{-\frac{1}{2}x^2} dx$

Let: $w = -\frac{1}{2}x^2 \leftrightarrow x^2 = -2w$, $dw = -x\,dx$, $dx = -\frac{1}{x} \cdot dw$

$$F = \int x^3 e^w \left(\frac{-1}{x}\right) dw = -\int x^2 e^w dw = 2\int w e^w dw$$

with $u = w$; $du = dw$ & $dv = e^w dw$; $v = e^w$

$$F = 2\left\{w e^w - \int e^w dw\right\} = 2(w-1)e^w + C$$

$$\therefore \boxed{F = -(2+x^2)e^{-\frac{1}{2}x^2} + C}$$

(S.M. & I.P.)

#45) $F = \int e^{\sqrt{x}} dx$

Let: $y = \sqrt{x} \leftrightarrow x = y^2$, $dy = \frac{1}{2\sqrt{x}} dx$, $dx = 2\sqrt{x}\,dw$

$F = 2\int y e^y dy = 2(y-1)e^y + C$ <u>note</u>: see #41

$$\boxed{F = 2\left(\sqrt{x} - 1\right)e^{\sqrt{x}} + C}$$

(S.M. & I.P.)