QSci 292 • answers • Hmwk #5

Neu. 3rd ed, sec. 7.3, p. 350
Neu. 2nd ed, sec. 7.3.3, p424

#13) $F = \int \frac{1}{x^2 - 2x} dx = \int \frac{1}{x(x-2)} dx = A \int \frac{1}{x} dx + B \int \frac{1}{x-2} dx$

(#23 in 2nd ed) setup: $\frac{1}{x(x-2)} = A \frac{1}{x} + B \frac{1}{x-2}$

common denominator: $\frac{1}{x(x-2)} = \frac{A(x-2) + B(x)}{x(x-2)}$

equate numerators: $1 = A(x-2) + B(x) = (A+B)x - 2A$

solve for A + B

• either $A+B = 0$ & $-2A = 1$; so $A = -\frac{1}{2}$ & $B = -A = +\frac{1}{2}$

• or when $x=0$, $1 = -2A$, so that $A = -\frac{1}{2}$
when $x=2$, $1 = 2B$, so that $B = +\frac{1}{2}$

note: S.M. $z = x-2$

then:

$F = -\frac{1}{2}\int \frac{1}{x} dx + \frac{1}{2}\int \frac{1}{x-2} dx = \underbrace{-\frac{1}{2}\ln(x) + \frac{1}{2}\ln(x-2) + C}_{\text{acceptable answer}}$

or $\boxed{F = \ln\left(\sqrt{\frac{x-2}{x}}\right) + C}$ (P.F.)

#16) $F = \int \frac{1}{x^2 - 3x + 2} dx = \int \frac{1}{(x-1)(x-2)} dx = A \int \frac{1}{x-1} dx + B \int \frac{1}{x-2} dx$

(#24 in 2nd ed)

setup: $\frac{1}{(x-1)(x-2)} = A \frac{1}{x-1} + B \frac{1}{x-2} = \frac{A(x-2) + B(x-1)}{(x-1)(x-2)}$

numerator: $1 = A(x-2) + B(x-1)$

at $x = 1$, $1 = A(1-2) + B(1-1) = (-1)A \longrightarrow A = -1$
at $x = 2$, $1 = A(2-2) + B(2-1) = (+1)B \longrightarrow B = +1$

$\therefore F = -\int \frac{1}{x-1} dx + \int \frac{1}{x-2} dx = \underbrace{-\ln(x-1) + \ln(x-2) + C}_{\text{acceptable answer}}$

$\boxed{F = \ln\left(\frac{x-2}{x-1}\right) + C}$ (P.F.)

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#28) $F = \int \frac{2x-1}{x^2+5x+4}\,dx = \int \frac{2x-1}{(x+4)(x+1)}\,dx = A\int \frac{1}{x+4}\,dx + B\int \frac{1}{x+1}\,dx$

numerator: $2x-1 = A(x+1) + B(x+4)$

when $x=-4$; $-9=-3A$ & when $x=-1$, $-3=3B$ $\therefore A=3$ & $B=-1$

$\boxed{F = 3\ln(x+4) - \ln(x+1) + C}$ (P.F.)

#29) $F = \int \frac{1}{x^2-9}\,dx = \int \frac{1}{(x-3)(x+3)}\,dx = A\int \frac{1}{x-3}\,dx + B\int \frac{1}{x+3}\,dx$

numerator: $1 = A(x+3) + B(x-3)$

at $x=3$, $1=6A$ & at $x=-3$, $1=-6B$; $A=\frac{1}{6}$ & $B=-\frac{1}{6}$

$F = \frac{1}{6}\int \frac{1}{x-3}\,dx - \frac{1}{6}\int \frac{1}{x+3}\,dx = \underbrace{\frac{1}{6}\ln(x-3) - \frac{1}{6}\ln(x+3) + C}_{\text{acceptable answer}}$

$\boxed{F = \frac{1}{6}\ln\left(\frac{x-3}{x+3}\right) + C}$ (P.F.)

#31) $F = \int \frac{1}{x^2-x-2}\,dx = \int \frac{1}{(x-2)(x-1)}\,dx = A\int \frac{1}{x-2}\,dx + B\int \frac{1}{x-1}\,dx$

numerator: $1 = A(x-1) + B(x-2) \rightarrow A = \frac{1}{3}$ & $B = -\frac{1}{3}$

$\boxed{F = \frac{1}{3}\ln\left(\frac{x-2}{x-1}\right) + C}$ (P.F.)

#34) $F = \int \left(\frac{x^3+1}{x^2+3}\right) dx = \int x\,dx + \int \frac{1}{x^2+3}\,dx - 3\int \frac{x}{x^2+3}\,dx$

Since: $\frac{x^3+1}{x^2+3} = x + \frac{-3x+1}{x^2+3}$ by polynomial div.

$F_1 = \int x\,dx = \frac{1}{2}x^2$; $F_2 = \int \frac{1}{x^2+3}\,dx = \frac{1}{\sqrt{3}}\int \frac{1}{z^2+1}\,dz$ S.M. $x=\sqrt{3}\,z$

$F_3 = \int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int \frac{1}{w}\,dw = \frac{1}{2}\ln(w) = \frac{1}{2}\ln(x^2+3)$ S.M. $w=x^2+3$

$\boxed{F = \frac{1}{2}x^2 + \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) - \frac{3}{2}\ln(x^2+3) + C}$

(poly. div., 2 S.M.)

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#36) $$F = \int \frac{x^4+3}{x^2-4x+3}\,dx = \int (x^2+4x+13)\,dx + 4\int \frac{10x-9}{(x-1)(x-3)}\,dx$$

$$F_1 = \int (x^2+4x+13)\,dx$$

$$F_1 = \tfrac{1}{3}x^3 + 2x^2 + 13x$$

poly. division

$$\begin{array}{r}
x^2+4x+13 \\
x^2-4x+3 \enclose{longdiv}{x^4+0\cdot x^3+0\cdot x^2+0x+3} \\
-(x^4-4x^3+3x^2) \\
\hline
4x^3-3x^2+0\cdot x \\
-(4x^3-16x^2+12x) \\
\hline
13x^2-12x+3 \\
-(13x^2-52x+39) \\
\hline
40x-36 \\
\text{or } 4(10x-9)
\end{array}$$

$$F_2 = \int \frac{10x-9}{(x-1)(x-3)}\,dx$$

$$= A\int \frac{1}{x-1}\,dx + B\int \frac{1}{x-3}\,dx \quad \text{(P.F.)}$$

numerator: $10x-9 = A(x-3) + B(x-1)$

when $x=1$, $1=-2A$; $A=-\frac{1}{2}$ & when $x=3$, $21=2B$; $B=21/2$

$$\boxed{F = \tfrac{1}{3}x^3 + 2x^2 + 13x - 2\ln(x-1) + 42\ln(x-3) + C}$$

#46) $$F = \int \frac{1}{x^2(x-1)^2}\,dx$$ <u>note</u>: repeated roots

setup: $$\frac{1}{x^2(x-1)^2} = A\frac{1}{x} + B\frac{1}{x^2} + C\frac{1}{x-1} + D\frac{1}{(x-1)^2}$$

num: $1 = x(x-1)^2A + (x-1)^2B + x^2(x-1)C + x^2D$

when $x=0$, $B=1$ & when $x=1$, $D=1$

<u>note</u>: $1 = (A+C)x^3 + (-2A+B-C+D)x^2 + (A-2B)x + B$

$\therefore$ $A-2B=0$ or $A=2$ and $A+C=0$ or $C=-2$

$$F = 2\int \frac{1}{x}\,dx + \int \frac{1}{x^2}\,dx - 2\int \frac{1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx$$

$$F = 2\ln(x) - \frac{1}{x} - 2\ln(x-1) - \frac{1}{x-1} + C$$

acceptable answer

or

$$\boxed{F = \ln\left[\left(\frac{x}{x-1}\right)^2\right] - \left(\frac{2x-1}{x(x-1)}\right) + C}$$ (P.F.)

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Neu 3rd ed, sec 7.3, p 350
Neu 2nd ed, sec 7.2, p 342

#56) (or #8, Neu. 2nd) $F = \int \frac{1}{x^2+5}\,dx$

note: no real roots in denomin.

Let: $x = \sqrt{5}\,u \leftrightarrow u = x/\sqrt{5}$, $dx = \sqrt{5}\,du$

$$F = \int \left(\frac{1}{5u^2+5}\right)\sqrt{5}\,du = \frac{\sqrt{5}}{5}\int \frac{1}{u^2+1}\,du = \frac{1}{\sqrt{5}}\arctan(u) + C$$

$$\boxed{F = \frac{1}{\sqrt{5}}\arctan\left(\frac{x}{\sqrt{5}}\right) + C}$$

(S.M.)

#64) (or #16, Neu. 2nd) $F = \int \left\{ \frac{\sin^2(x) - \cos^2(x)}{(\sin(x) - \cos(x))^2} \right\} dx$

with $s = \sin(x)$ & $c = \cos(x)$

$$\frac{s^2 - c^2}{(s-c)^2} = \frac{(s-c)(s+c)}{(s-c)(s-c)} = \frac{s+c}{s-c}$$

Let: $w = s - c$, $dw = (c - (-s))\,dx$, $dx = \frac{1}{s+c}\,dw$

$$F = \int \frac{s+c}{s-c}\,dx = \int \frac{s+c}{w}\left(\frac{1}{s+c}\right)dw = \int \frac{1}{w}\,dw = \ln w + C$$

$$\boxed{F = \ln\left(\sin(x) - \cos(x)\right) + C}$$

(E.C.E. & S.M.)