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Neu. 3rd ed., ch. 7, p 387
Neu. 2nd ed., sec. 7.9, p 470

#5) $F = \int (1+\sqrt{x})^{1/3}\,dx = 2\int u^{1/3}(u-1)\,du$

S.M. : $u = 1+\sqrt{x}$, $\sqrt{x} = u-1$, $du = \frac{1}{2\sqrt{x}}\,dx$

$$F = 2\left\{\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\right\} + C$$

$$\boxed{F = \frac{6}{7}(1+\sqrt{x})^{7/3} - \frac{3}{2}(1+\sqrt{x})^{4/3} + C}$$

#10) $F = \int x^3 \ln x^2\,dx = \frac{1}{2}\int y \ln y\,dy$

S.M. : $y = x^2$, $dy = 2x\cdot dx$

$$F = \frac{1}{2}\left[\frac{1}{2}y^2 \ln y - \frac{1}{2}\int y^2 \frac{1}{y}\,dy\right] \quad \text{I.P.}$$

$$\therefore \boxed{F = \frac{1}{4}x^4\left(\ln x^2 - \frac{1}{2}\right) + C}$$

note : $F = 2\int x^3 \ln x\,dx$ (E.C.E.)
(use I.P. on this)

#20) $F = \int \ln(\sqrt{x})\,dx = \frac{1}{2}\int \ln(x)\,dx$ E.C.E.

$$\boxed{F = \frac{1}{2}x\left(\ln(x) - 1\right) + C}$$

note : a long way is to use $y=\sqrt{x}$ to obtain an integral in y then use I.P.

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#22) $F = \int \sin x \cos x \, e^{\sin x} dx$ S.M. $y = \sin x$

$F = \int y e^y dy = (y-1)e^y + C$

$$\boxed{F = (\sin(x) - 1)e^{\sin x} + C}$$

Trig. Sub.

#2) $F = \int \frac{1}{4+t^2} dt = \frac{1}{2} \int \frac{1}{1+u^2} du$

S.M. $u = t/2$, $t = 2u$, $du = \frac{1}{2} dt$

$$\boxed{F = \frac{1}{2} \arctan\left(\frac{t}{2}\right) + C}$$

note: $t = 2 \tan\theta$ could be used also

#3) $F = \int \frac{1}{\sqrt{9-y^2}} dy = \int d\theta = \theta + C$

T.S.: $y = 3\sin\theta$; $dy = 3\cos\theta \, d\theta$

$\sqrt{9-y^2} = 3\cos\theta$

$$\boxed{F = \arcsin\left(\frac{y}{3}\right) + C}$$

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#4) $F = \int \frac{1}{16-\theta^2}\, d\theta = \frac{1}{4}\int \sec\psi \, d\psi$

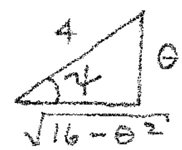
<u>T.S.</u>: $\theta = 4\sin\psi$; $d\theta = 4\cos\psi \cdot d\psi$

$16-\theta^2 = 16\cos^2\psi$

$F = \frac{1}{4}\ln(\sec\psi + \tan\psi) + C$

$\sin\psi = \frac{\theta}{4}$

$\cos\psi = \frac{\sqrt{16-\theta^2}}{4}$



$\sec\psi = 4/\sqrt{16-\theta^2}$

$\tan\psi = \theta/\sqrt{16-\theta^2}$

$$\boxed{F = \frac{1}{4}\ln\left(\frac{4+\theta}{\sqrt{16-\theta^2}}\right) + C}$$

or $F = \frac{1}{8}\ln\left(\frac{4+\theta}{4-\theta}\right) + C$

#5) $F = \int \frac{1}{x\sqrt{x^2-16}}\, dx = \frac{1}{4}\int d\theta$

<u>T.S.</u>: $x = 4\sec\theta$; $dx = 4\sec\theta\tan\theta\, d\theta$

$$\boxed{F = \frac{1}{4}\operatorname{arcsec}\left(\frac{x}{4}\right) + C}$$

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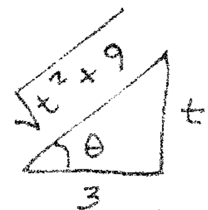
#7) $F = \int \frac{1}{(t^2+9)^{3/2}}\,dt = \frac{1}{9}\int \cos\theta\, d\theta$

T.S. : $t = 3\tan\theta$; $dt = 3\sec^2\theta\, d\theta$

$t^2+9 = 9\sec^2\theta$;

$(t^2+9)^{3/2} = 27\sec^3\theta$



$F = \frac{1}{9}\sin\theta + C$

$$\boxed{F = \frac{1}{9}\,\frac{t}{\sqrt{t^2+9}} + C}$$

$\tan\theta = \frac{t}{3}$

$\sin\theta = \frac{t}{\sqrt{t^2+9}}$

#10) $F = \int \frac{x}{\sqrt{9-x^2}}\,dx = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

S.M. : $u = 9-x^2$; $du = -2x \cdot dx$

$$\boxed{F = -\sqrt{9-x^2} + C}$$

note : $x = 3\sin\theta$ could also be used