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Neu 3rd ed., sec. 8.1, p. 403
Neu. 2nd ed., sec. 8.1.4, p. 491

#2) $y' = e^{-3x}$; I.C.: $y(0) = 10$ (or at $x=0, y=10$)

step 1: $y' \cdot dx = e^{-3x}$; step 2: $\int y' dx = \int e^{-3x} dx$, thus:

$\boxed{y = -\frac{1}{3}e^{-3x} + C}$ general solution

apply I.C.
$1 = -\frac{1}{3}e^{0} + C$; $C = \frac{4}{3}$

$\boxed{y = \frac{1}{3}(4 - e^{-3x})}$ particular solution

#2) $y' = e^{-x}$; I.C.: $y(0) = 10$
(Neu. 2nd ed)

$\int y' dx = \int e^{-x} dx$ or $\boxed{y = -e^{-x} + C}$ general soln

at $x = 0, y = 10$ hence $10 = -e^{-0} + C$

thus $\boxed{y = 11 - e^{-x}}$ particular soln   or $\underline{C = 11}$

#4) $y' = \frac{1}{1+x^2}$; I.C.: $y(0) = 1$

$y = \int y' dx = \int \frac{1}{1+x^2} dx = \arctan(x) + C$

at $x = 0, y = 1$; $1 = \arctan(0) + C$

$\therefore \underline{C = 1}$

thus $\boxed{y = 1 + \arctan(x)}$ ans.

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#10) rate of phosphorous production

$$\frac{dP}{dt} = 3t + 1 \quad ; \quad \text{I.C.}: P(0) = 0$$

$$P = \frac{3}{2}t^2 + t + C \quad ; \quad \underline{C = 0}$$

$$\boxed{P = \frac{3}{2}t^2 + t} \text{ ans.}$$

#11) $\frac{dy}{dx} = 3y$; I.C. : $y(0) = 2$

$$\ln y \text{ or } = \int \frac{1}{y}\,dy = \int \frac{1}{y}\frac{dy}{dx}\,dx = \int 3\,dx = 3x + C$$

$$y = e^{\ln y} = e^{3x+C} = e^{3x} \cdot e^{C} = ke^{3x}$$

at $x = 0$, $y = 2$; $2 = ke^{3(0)}$ or $\underline{k = 2}$

$$\boxed{y = 2e^{3x}} \text{ ans.}$$

#12) $y' = 2(1-y)$; I.C. : $y(0) = 2$

$$-\ln(1-y) = \int \frac{1}{1-y}\,dy = 2\int 1\,dx = 2x + C$$

$$1 - y = e^{\ln(1-y)} = e^{-(2x+C)} = e^{-2x} \cdot e^{-C} = ke^{-2x}$$

e^{-C}

$$\boxed{y = 1 - ke^{-2x}}$$ general solution

apply I.C.

$$2 = 1 - ke^{-0}$$

or $k = -1$

$$\therefore \boxed{y = 1 + e^{-2x}}$$ particular solution

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#20) radioactivity

$$\frac{dW}{dt} = -\lambda W \; ; \; I.C.: W(0) = W_0$$

$$\ln W = \int \frac{1}{W} dW = -\lambda \int dt = -\lambda t + C$$

a)

or $\boxed{W = k e^{-\lambda t}}$; $W_0 = k e^{-\lambda(0)}$

b)

$W = W_0 e^{-\lambda t}$ or $\boxed{W = 123 e^{-\lambda t}}$ $\therefore \underline{k = W_0}$

→ at $t = t_{1/2}$ (half-life), $W = \frac{1}{2} W_0$ (definition)

$$\therefore \frac{W_0}{2} = W_0 e^{-\lambda t_{1/2}} \text{ or } \ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

hence $t_{1/2} = \dfrac{\ln 2}{\lambda}$

when $t = 5$, $W = 20$; $20 = 123 e^{-\lambda \cdot 5}$

or $\ln\left(\frac{20}{123}\right) = -\lambda \cdot 5$ $\therefore \lambda = \frac{1}{5} \ln\left(\frac{123}{20}\right)$

$\boxed{\lambda = 0.36329}$ & $\boxed{t_{1/2} = 1.9 \text{ min}}$

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#37) population size model

$$\frac{dN}{dt} = 0.34N\left(1 - \frac{N}{200}\right); \text{ I.C.}: N(0) = 50$$

$$\int \frac{1}{N\left(1-\frac{N}{200}\right)} \frac{dN}{dt} \cdot dt = 0.34 \int dt$$

or $$\int \frac{200}{N(200-N)}\, dN = 0.34t + C$$

P.F.

$$\int \frac{200}{N(200-N)}\, dN = \int \frac{1}{N}\, dN + \int \frac{1}{200-N}\, dN$$

$$\therefore \ln N - \ln(200-N) = 0.34t + C$$

or $$\frac{N}{200-N} = e^{\ln\left(\frac{N}{200-N}\right)} = e^{0.34t} \cdot k$$

• rearrange with algebra manuvers

$$N = 200\left(\frac{ke^{.34t}}{1+ke^{.34t}}\right)$$ general soln

at $t = 0$, $N = 50$; $50 = 200\left(\frac{k}{1+k}\right)$

$\therefore k = \frac{200}{3}$

$$\boxed{N = 200 \, \frac{1}{1 + \frac{3}{200}e^{-.34t}}}$$ ans

$N \to 200$ * when $t \to \infty$

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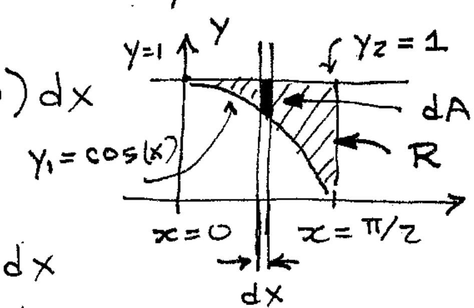
Neu. 3rd ed., sec. 6.3, p 321
Neu. 2nd ed., sec. 6.3.6, p 392

#4) $R = \{(x,y): 0 \le x \le \frac{\pi}{2},\ y_1 < y < y_2\}$

with $y_1 = \cos(x)$ and $y_2 = 1$

$dA = (y_2 - y_1)\,dx$

$dA = (1 - \cos(x))\,dx$



$A = \int dA$

$A = \int_0^{\pi/2} (1 - \cos(x))\,dx$

$A = \left[x - \sin(x) \Big|_0^{\pi/2} \right] = \left[\left(\frac{\pi}{2} - 1\right) - (0 - 0) \right]$

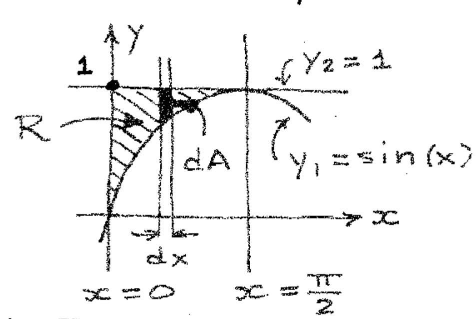
$\boxed{A = \frac{\pi}{2} - 1}$ ans or $A = 0.57$ sq. units

#2) $R = \{(x,y): 0 < x < \pi/2,\ y_1 < y < y_2\}$ $y_1 = \sin(x)$
(Neu. 2nd ed) $y_2 = 1$

$A = \int_0^{\pi/2} (1 - \sin(x))\,dx$

$A = \left[x + \cos(x) \Big|_0^{\pi/2} \right]$

$A = \left[\left(\frac{\pi}{2} - 0\right) - (0 + 1) \right]$



$\boxed{A = \frac{\pi}{2} - 1}$ ans $A = 0.57$ sq. units

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#2.) $R = \{(x,y) : -1 \le x \le +1,\ y_1 < y < y_2\}$

with $y_1 = 2x^2 - 1$ and $y_2 = 2 - x^4$

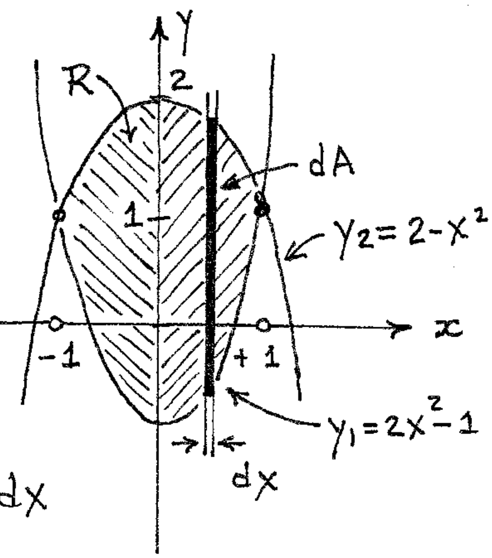
<u>note</u>: $y_1 = y_1$ implies

$2x^2 - 1 = 2 - x^4$ or

$x^4 + 2x^2 - 3 = (x^2+3)(x^2-1) = 0$

$\therefore x = \pm 1$ (limits of int.)

(ignor $x = \pm 3i$; $i = \sqrt{-1}$)



$dA = (y_2 - y_1)\, dx$

$dA = \{(2 - x^4) - (2x^2 - 1)\}\, dx$

$dA = (3 - 2x^2 - x^4)\, dx$

$$A = \int dA = \int_{-1}^{+1} (3 - 2x^2 - x^4)\, dx$$

or by symmetry

$$A = 2 \int_0^1 (3 - 2x^2 - x^4)\, dx$$

$$A = 2\left\{ 3x - \frac{2}{3}x^3 - \frac{1}{5}x^5 \Big|_0^1 \right\} = 2\left\{ 3 - \frac{2}{3} - \frac{1}{5} \right\}$$

$\boxed{A = \frac{64}{15}}$ ans $\quad A = 4.27$ sq. units

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#4) $R = \{(x,y): -1 \le x \le 1,\ y_1 \le y \le y_2\}$

(Neu. 2nd ed)

with $y_1 = x^2$ & $y_2 = 2 - x^4$

<u>note</u>:

at $x = \pm 1$,

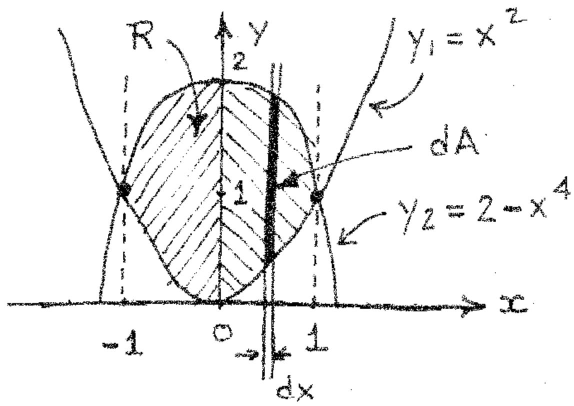
$y_1 = y_2$

i.e.

$x^2 = 2 - x^4$

or

$x^4 + x^2 - 2 = 0$



$(x^2+2)(x^2-1) = 0$ or $x = \pm 1$

$$dA = (y_2 - y_1)\,dx = (2 - x^4 - x^2)\,dx$$

$$A = \int dA = \int_{-1}^{+1} (2 - x^4 - x^2)\,dx$$

or by symmetry $A = 2\int_0^1 (2 - x^4 - x^2)\,dx$

$$A = 2\left[2x - \frac{1}{5}x^5 - \frac{1}{3}x^3 \Big|_0^1\right] = 2\left[2 - \frac{1}{5} - \frac{1}{3}\right]$$

$\boxed{A = \frac{44}{15}}$ ans or $A = 2.93$ sq. units

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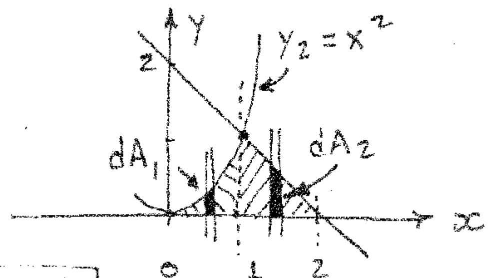
#6) $R = \{(x,y): 0 \le x \le 2,\ y_1 \le y \le y_2\}$

with $y_1 = 0$ and $y_2 = x^2$ for $0 \le x \le 1$
and
$y_2 = 2 - x$ for $1 \le x \le 2$

<u>note</u>: at $x = 1$
$x^2 = 2 - x$
since
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$\therefore x = -2$ or $\boxed{x = 1}$
not in 1^{st} quadrant



$dA_1 = (y_2 - y_1)\,dx = x^2\,dx$

$dA_2 = (y_2 - y_1)\,dx = (2 - x)\,dx$

$$A_1 = \int_0^1 x^2\,dx \ ; \ A_2 = \int_1^2 (2 - x)\,dx$$

$$A_1 = \left[\frac{1}{3}x^3 \Big|_0^1\right] = \frac{1}{3}$$

$$A_2 = \left[2x - \frac{1}{2}x^2 \Big|_1^2\right] = \left[(4-2) - \frac{1}{2}(4-1)\right] = \frac{1}{2}$$

$A = \frac{1}{3} + \frac{1}{2}$ or $\boxed{A = \frac{5}{6}}$ ans $A = 0.833$ sq. units